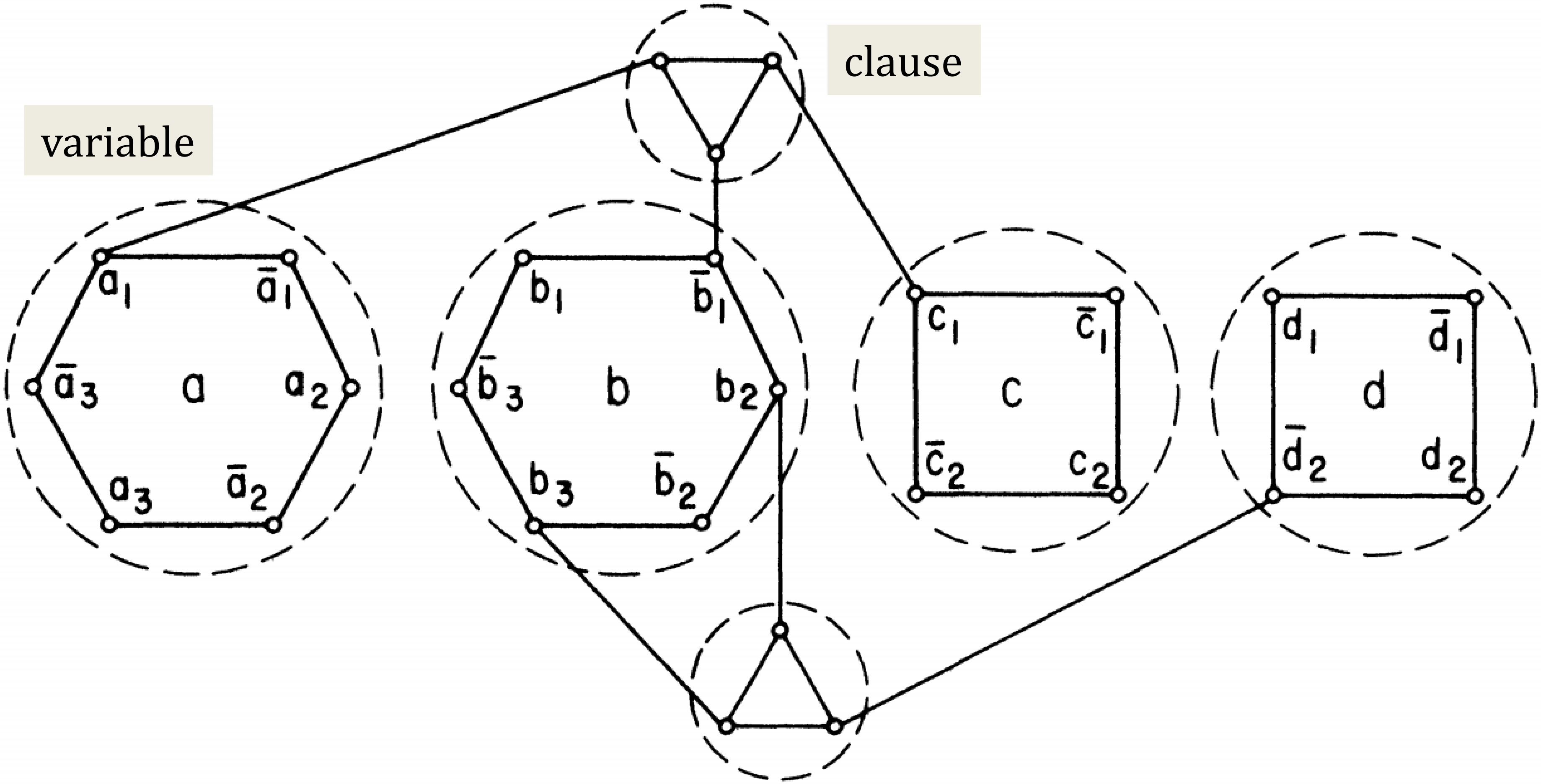
Planar Vertex Cover

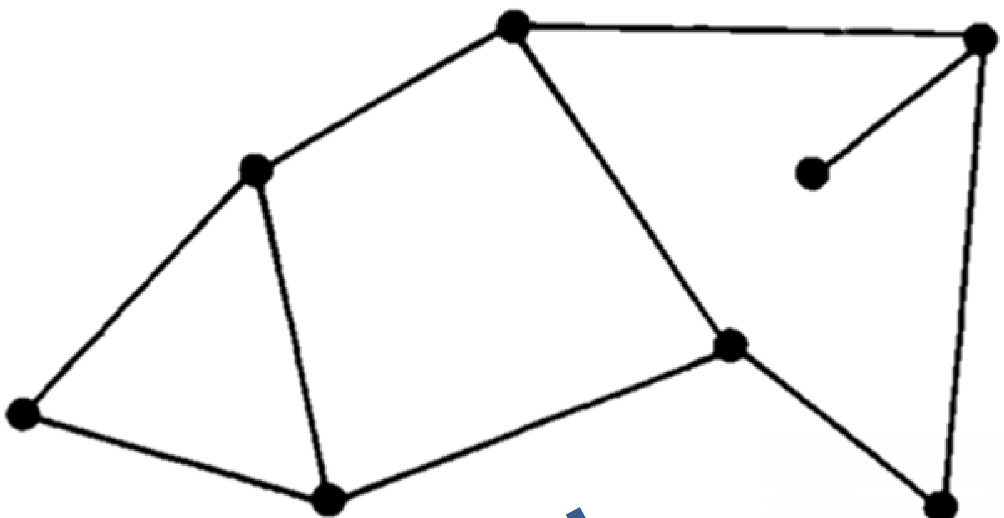
[Lichtenstein 1982]

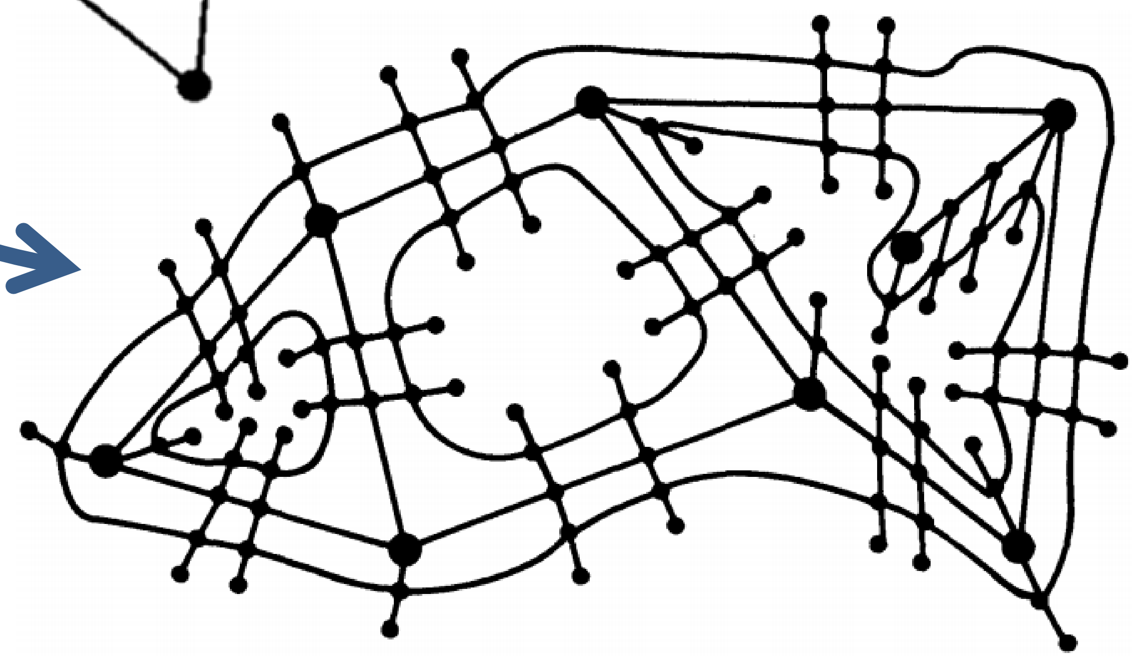


Example : $B = (a + \bar{b} + c)(b + b + \bar{d})$

Planar Connected Vertex Cover

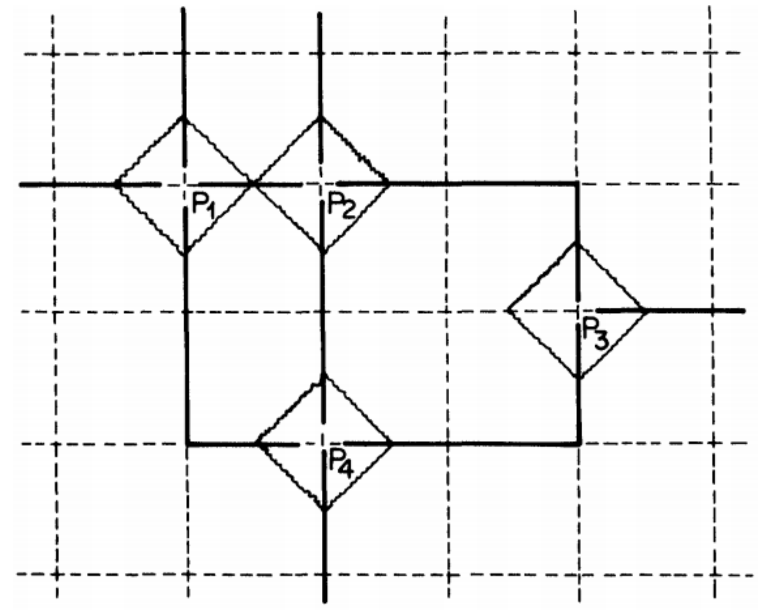
[Garey & Johnson 1977]

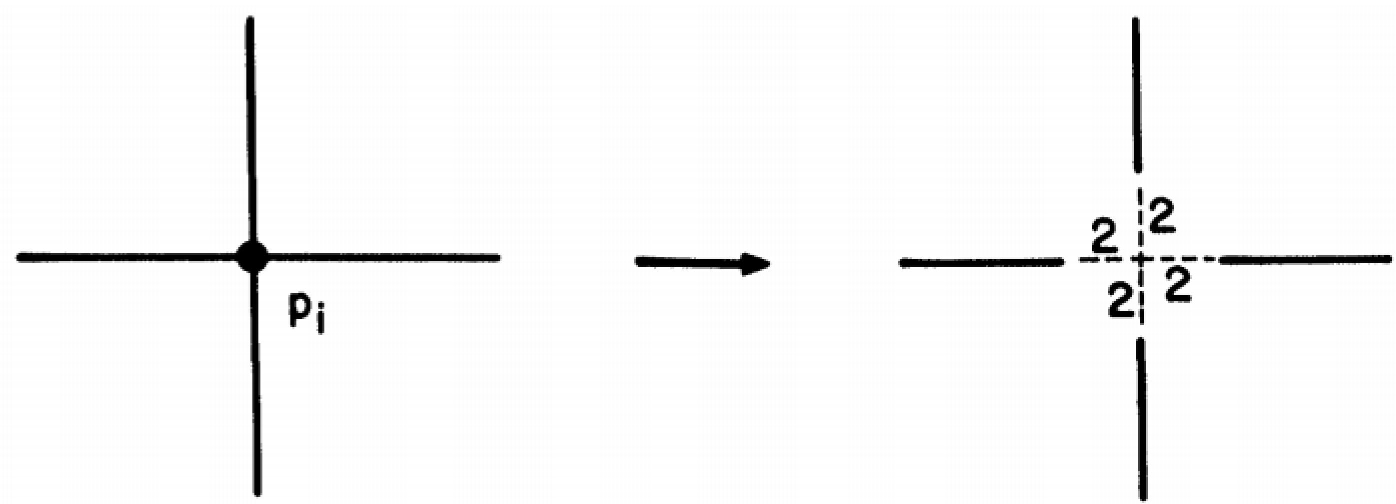




Rectilinear Steiner Tree

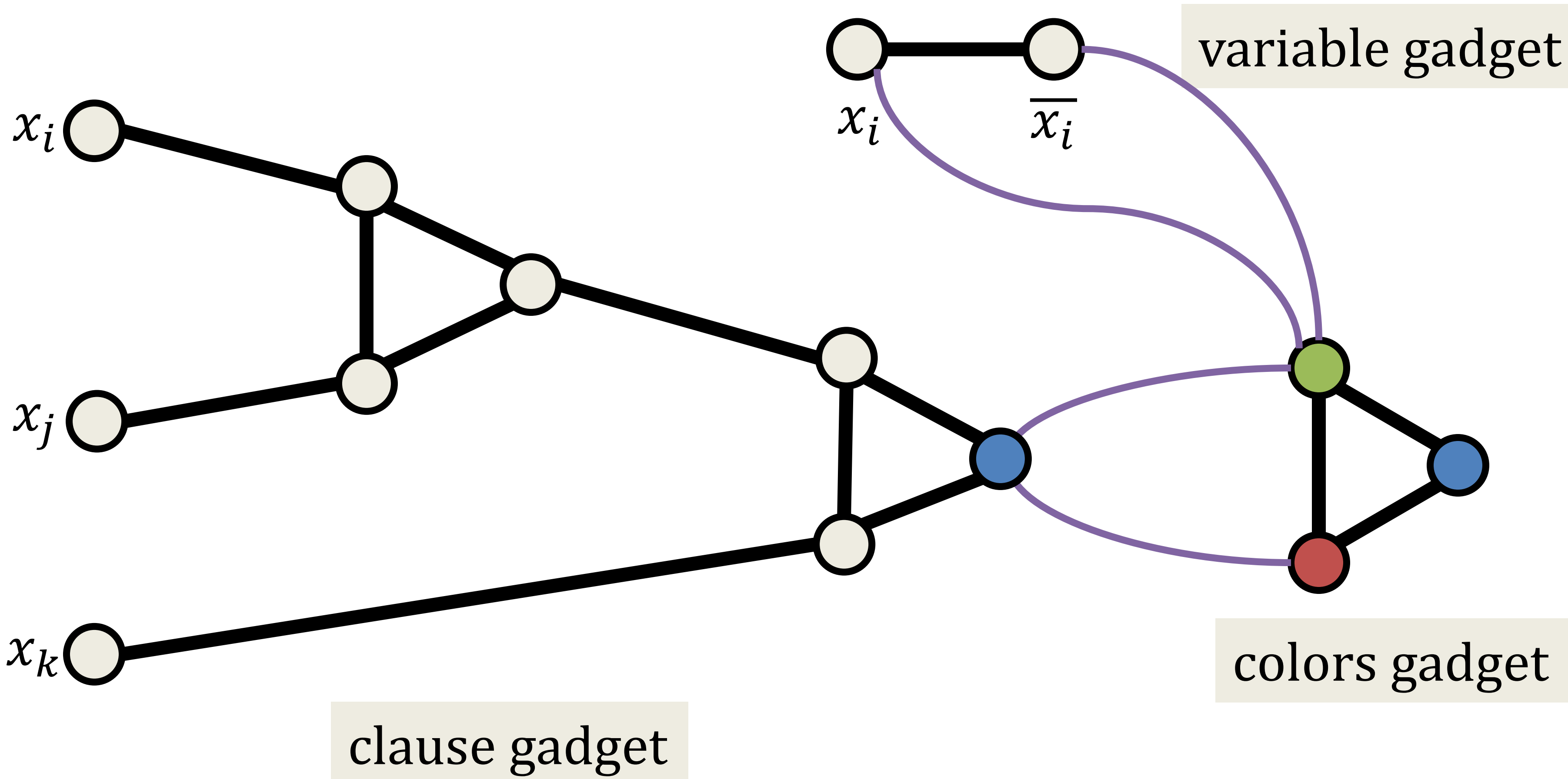
[Garey & Johnson 1977]





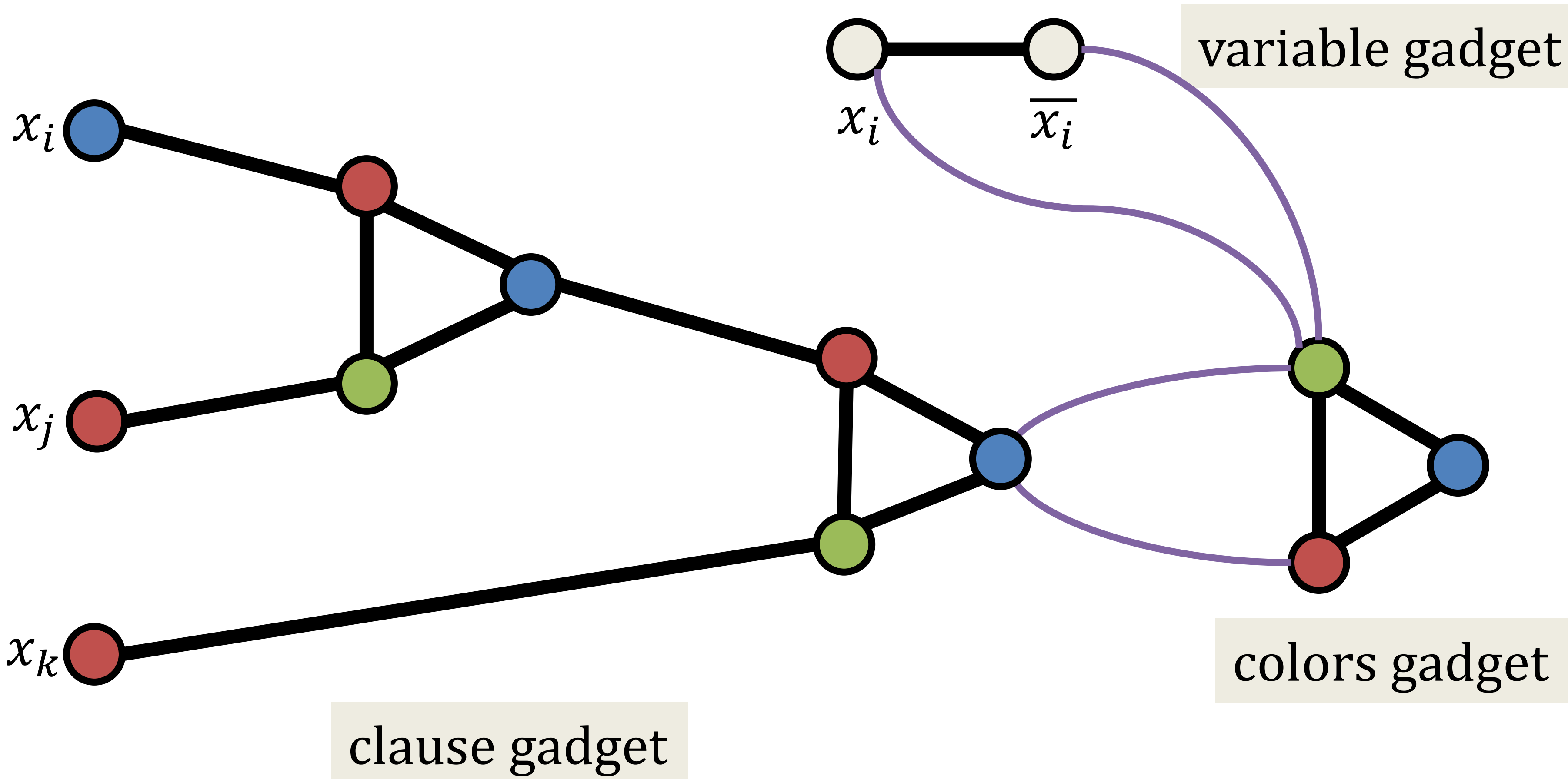
Vertex 3-Coloring

[Garey, Johnson, Stockmeyer 1976]



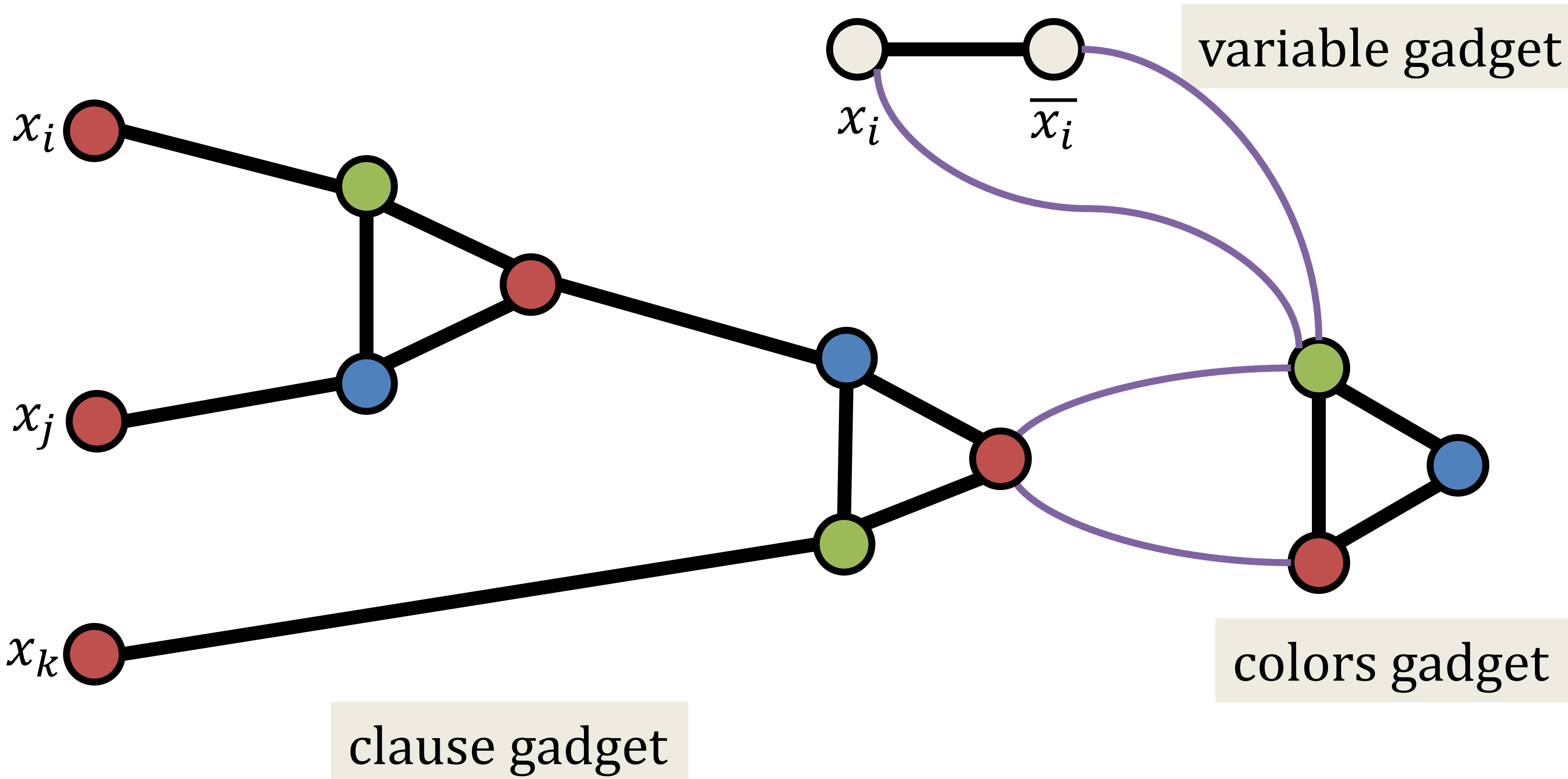
Vertex 3-Coloring

[Garey, Johnson, Stockmeyer 1976]



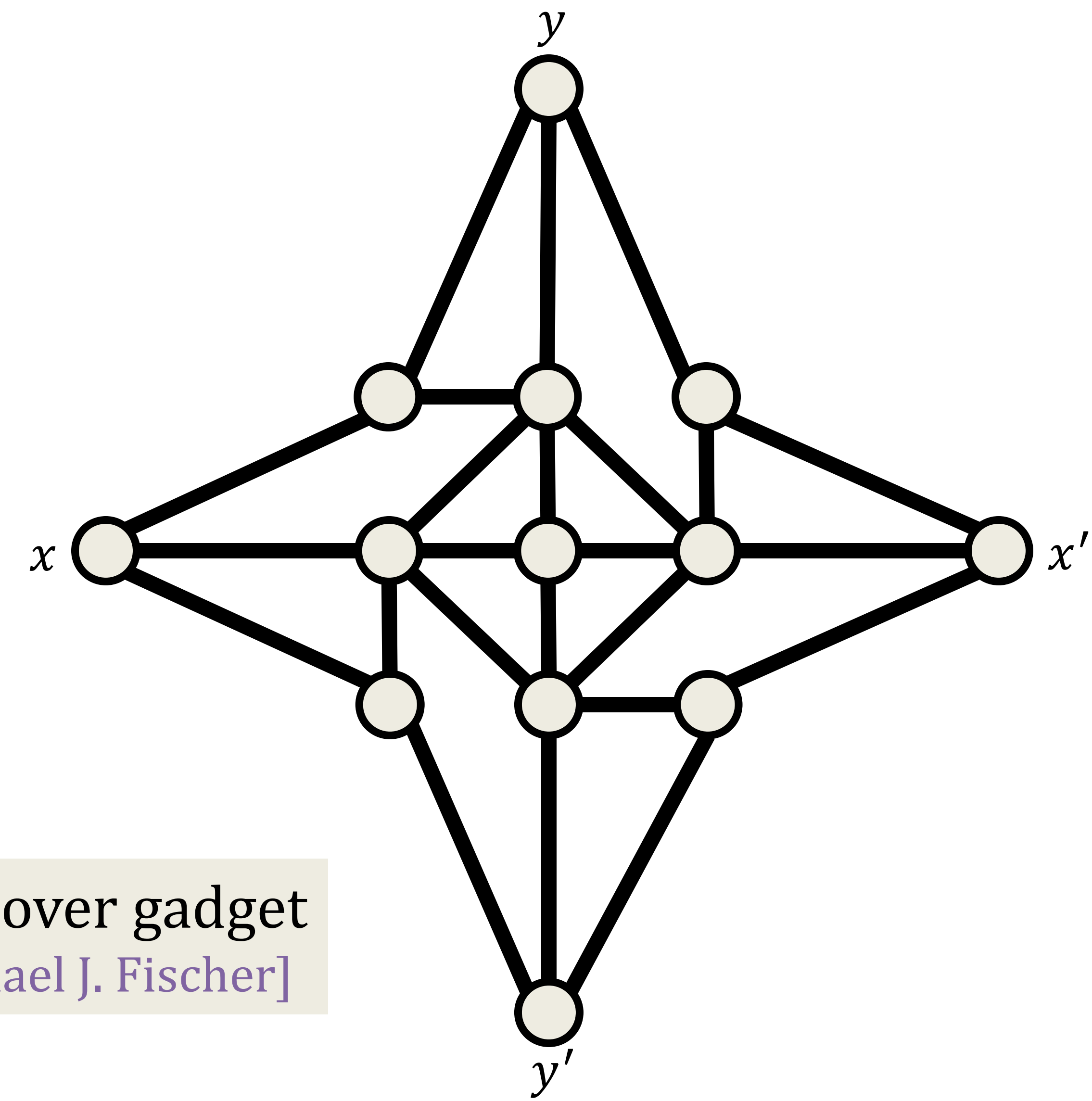
Vertex 3-Coloring

[Garey, Johnson, Stockmeyer 1976]



Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]

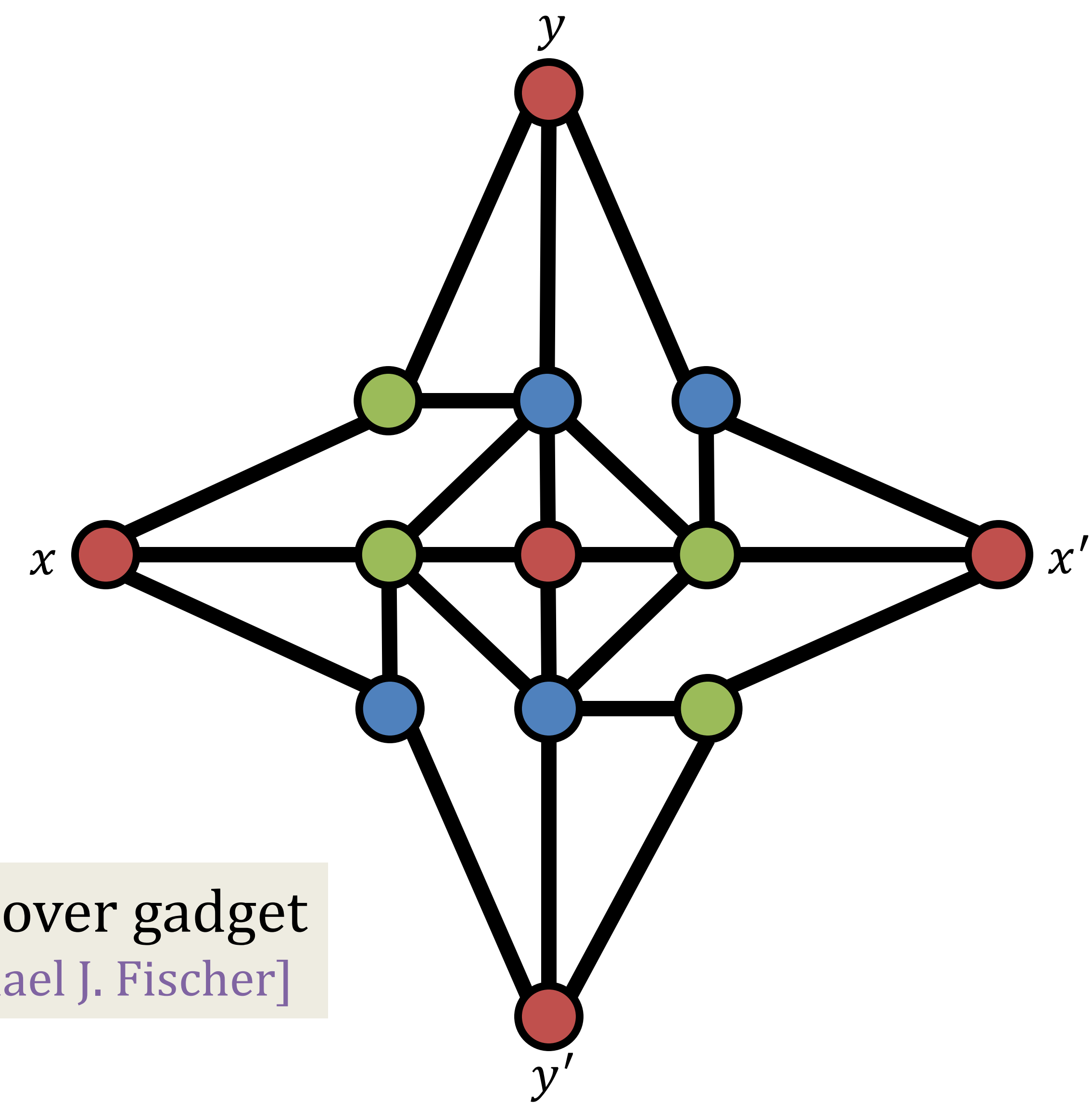


crossover gadget

[Michael J. Fischer]

Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]

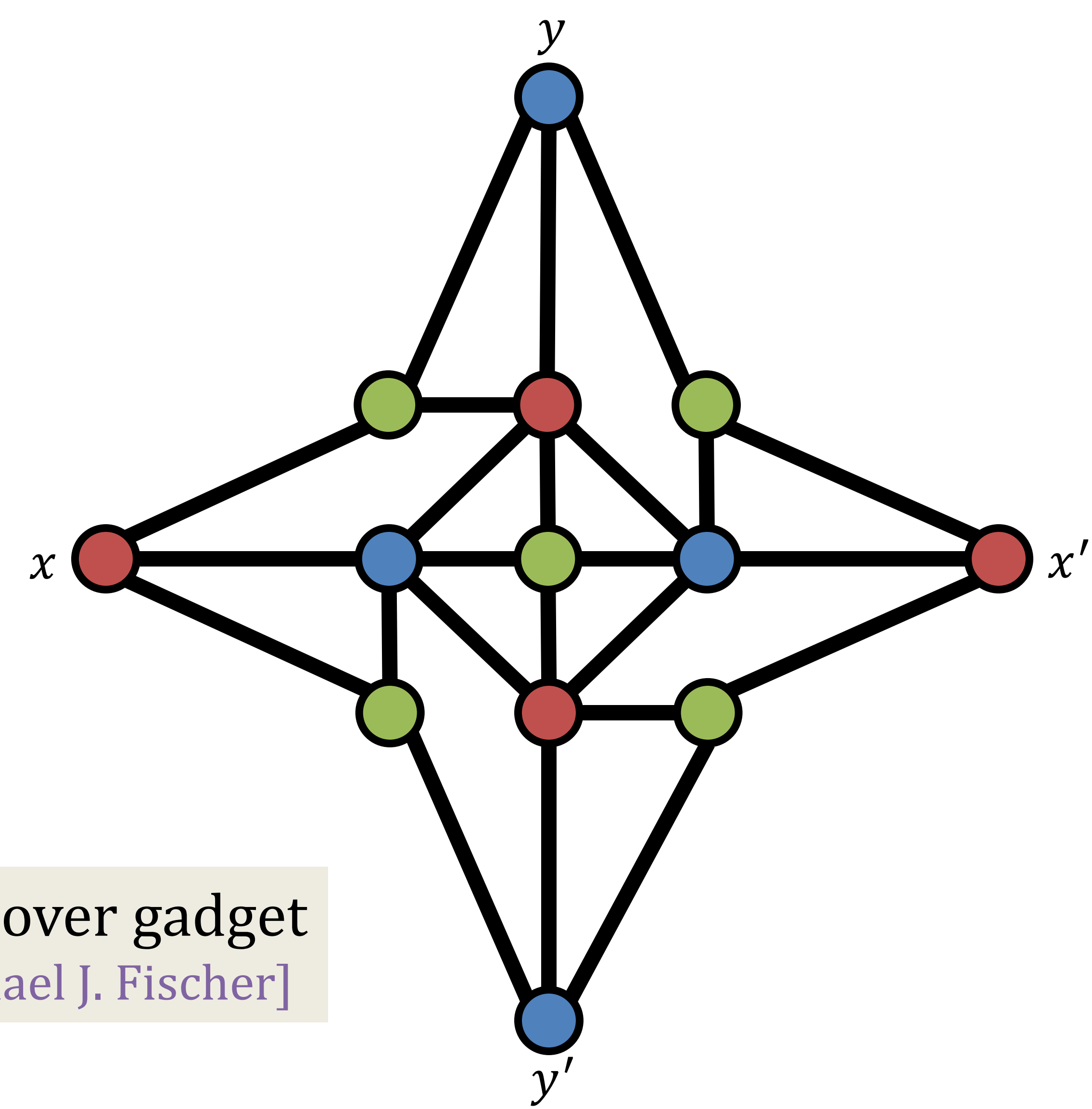


crossover gadget

[Michael J. Fischer]

Planar 3-Coloring

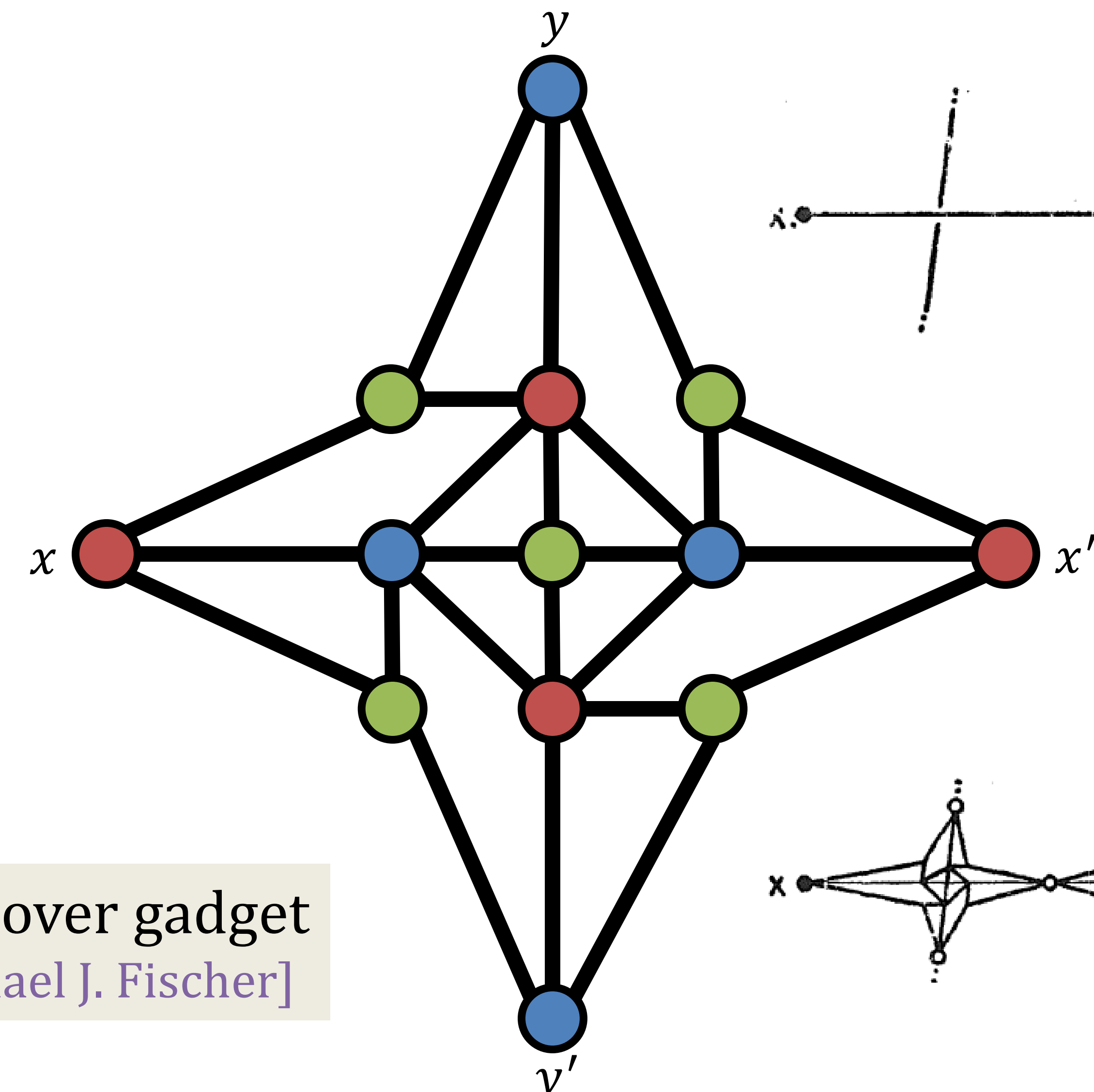
[Garey, Johnson, Stockmeyer 1976]

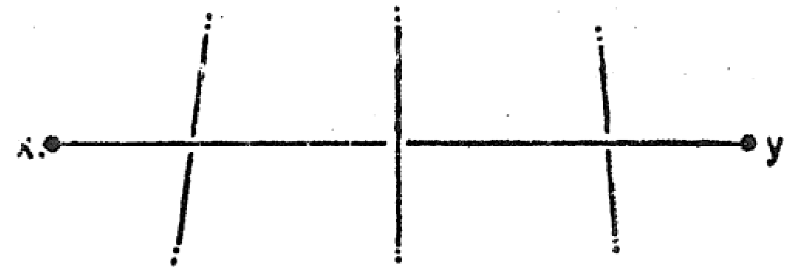


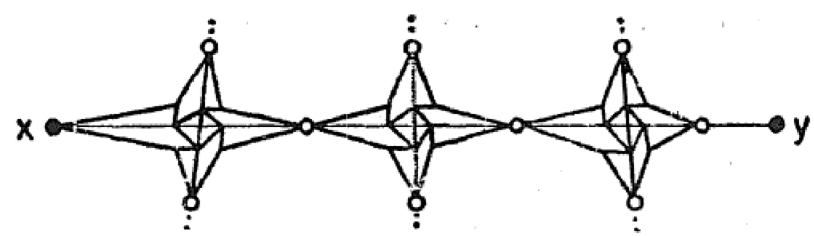
crossover gadget
[Michael J. Fischer]

Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]



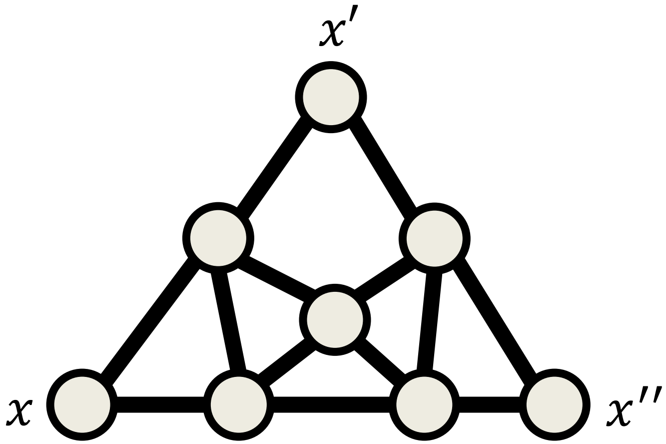




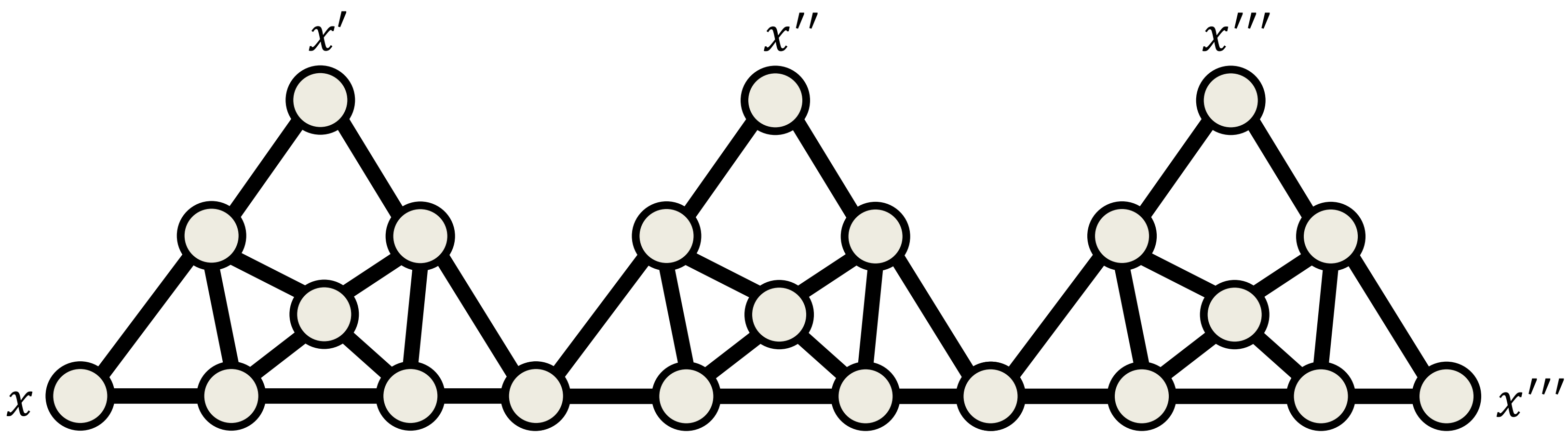
crossover gadget
[Michael J. Fischer]

Planar 3-Coloring, Max Degree 4

[Garey, Johnson, Stockmeyer 1976]

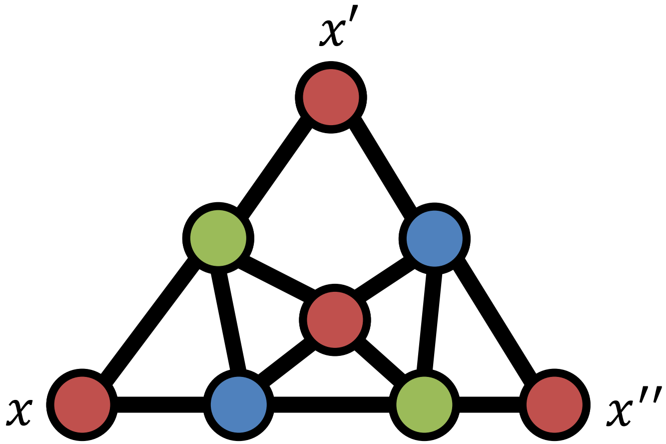


high-degree gadget

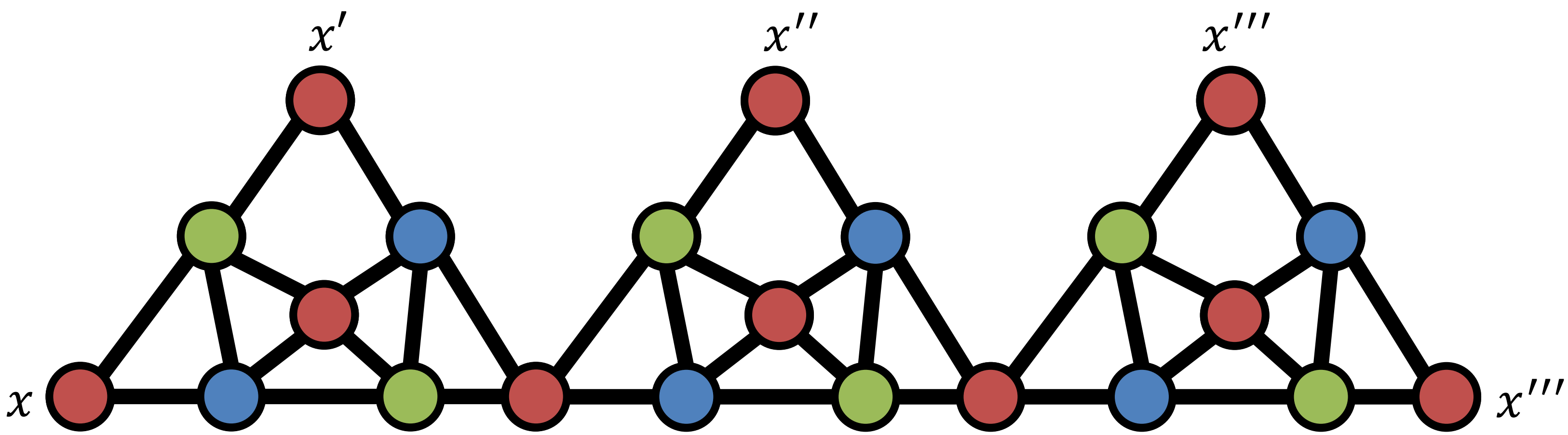


Planar 3-Coloring, Max Degree 4

[Garey, Johnson, Stockmeyer 1976]



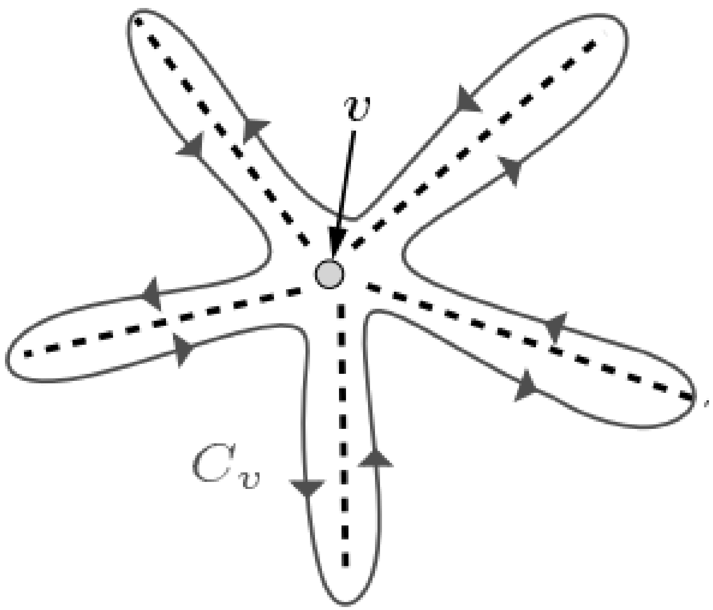
high-degree gadget

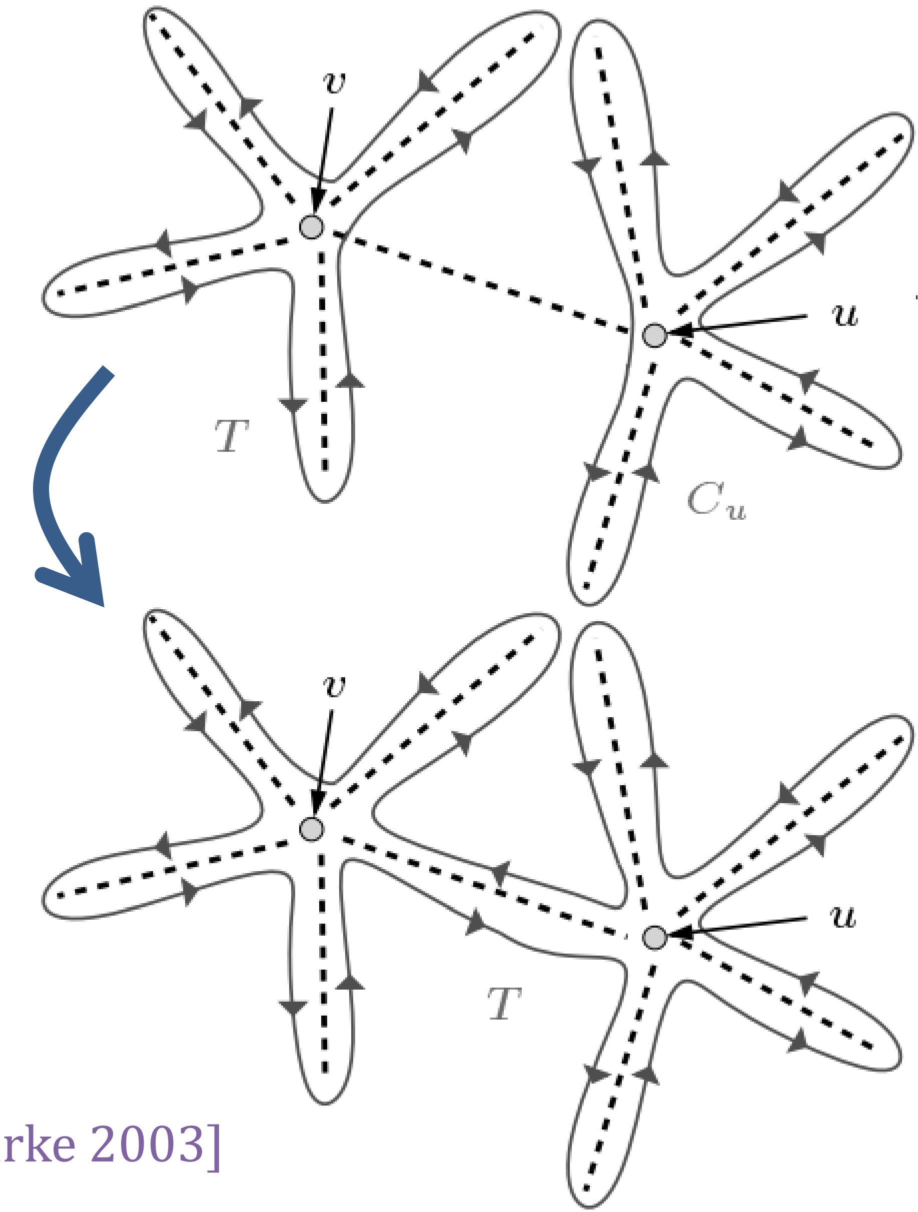


Pushing 1×1 Blocks Complexity

Name	Push	Fixed	Slide	Goal	Complexity	Reference
Push-k	$k \geq 1$	no	min	path	NP-hard	D, D, O'Rourke 2000
Push-$*$	∞	no	min	path	NP-hard	Hoffmann 2000
PushPush-k	$k \geq 1$	no	max	path	PSPACE-complete	D, Hoffmann, Holzer 2004
PushPush-$*$	∞	no	max	path	NP-hard	Hoffmann 2000
Push-1F	1	yes	min	path	NP-hard	DDO 2000
Push-kF	$k \geq 2$	yes	min	path	PSPACE-complete	D, Hearn, Hoffmann 2002
Push-$*$F	∞	yes	min	path	PSPACE-complete	Bremner, O'Rourke, Shermer 1994
Push-kX	$k \geq 1$	no	min	simple path	NP-complete	D, Hoffmann 2001
Push-$*$X	∞	no	min	simple path	NP-complete	Hoffmann 2000
Sokoban	1	yes	min	storage	PSPACE-complete	Culberson 1998

Planar Euler Tours

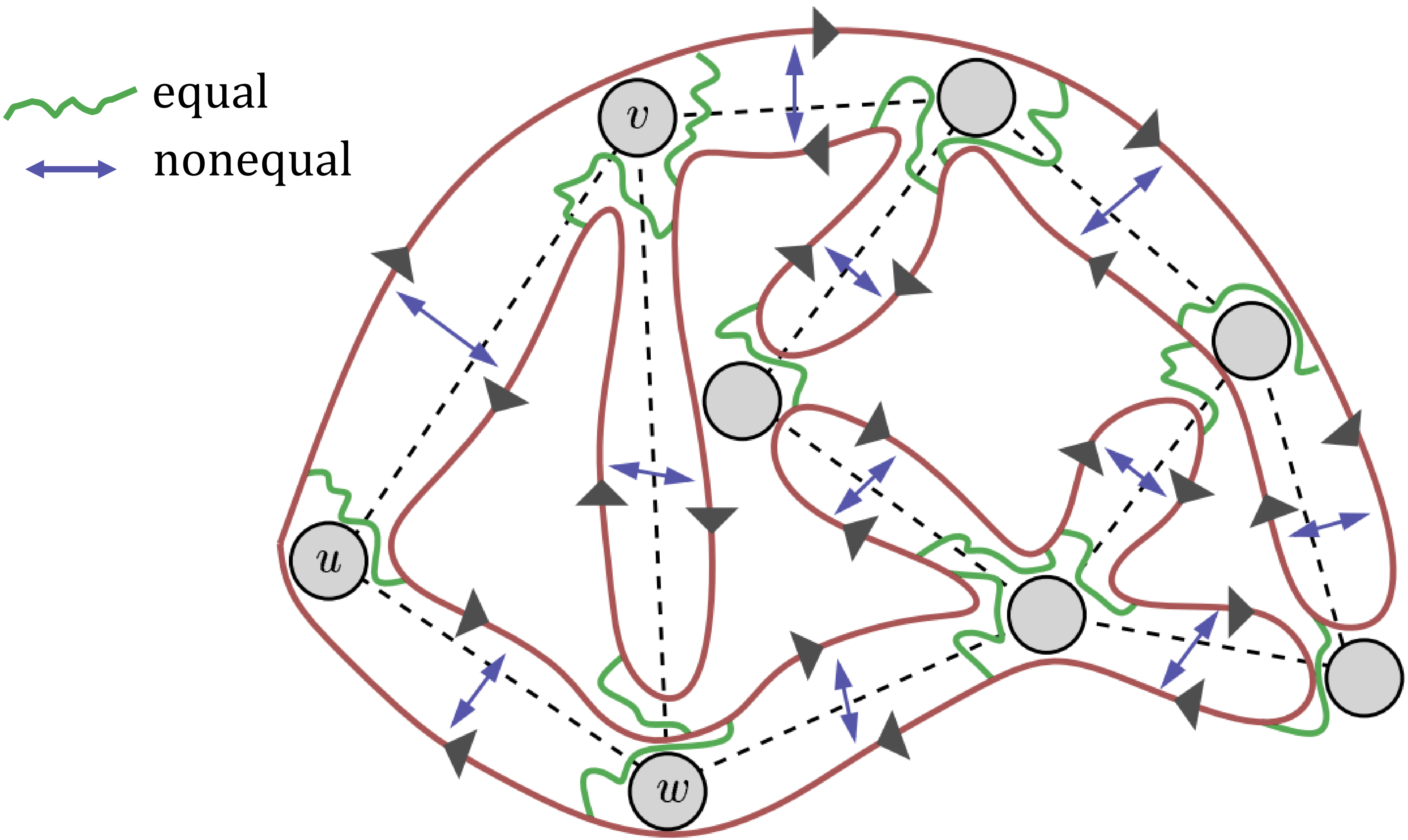




[Demaine, Demaine, Hoffmann, O'Rourke 2003]

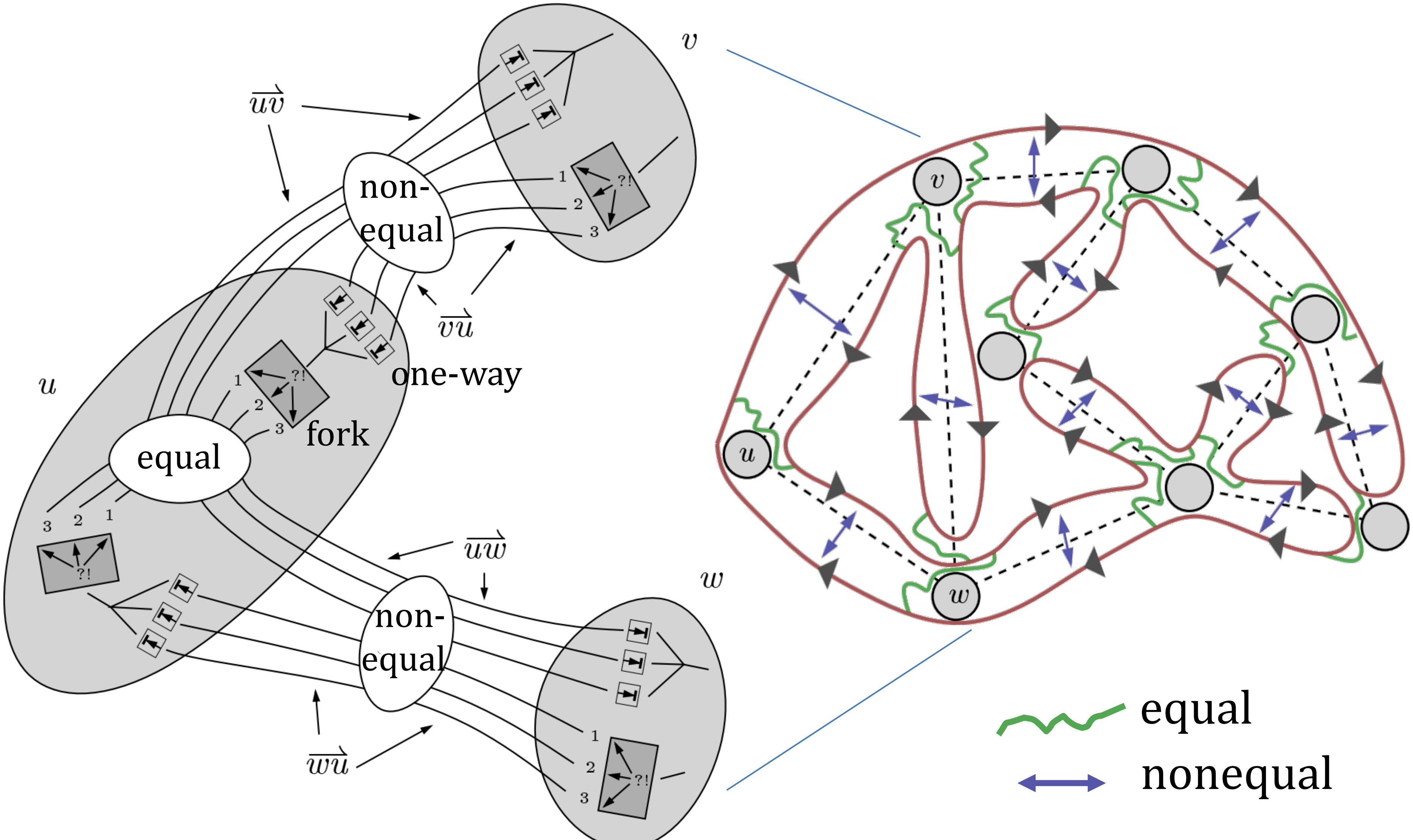
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]



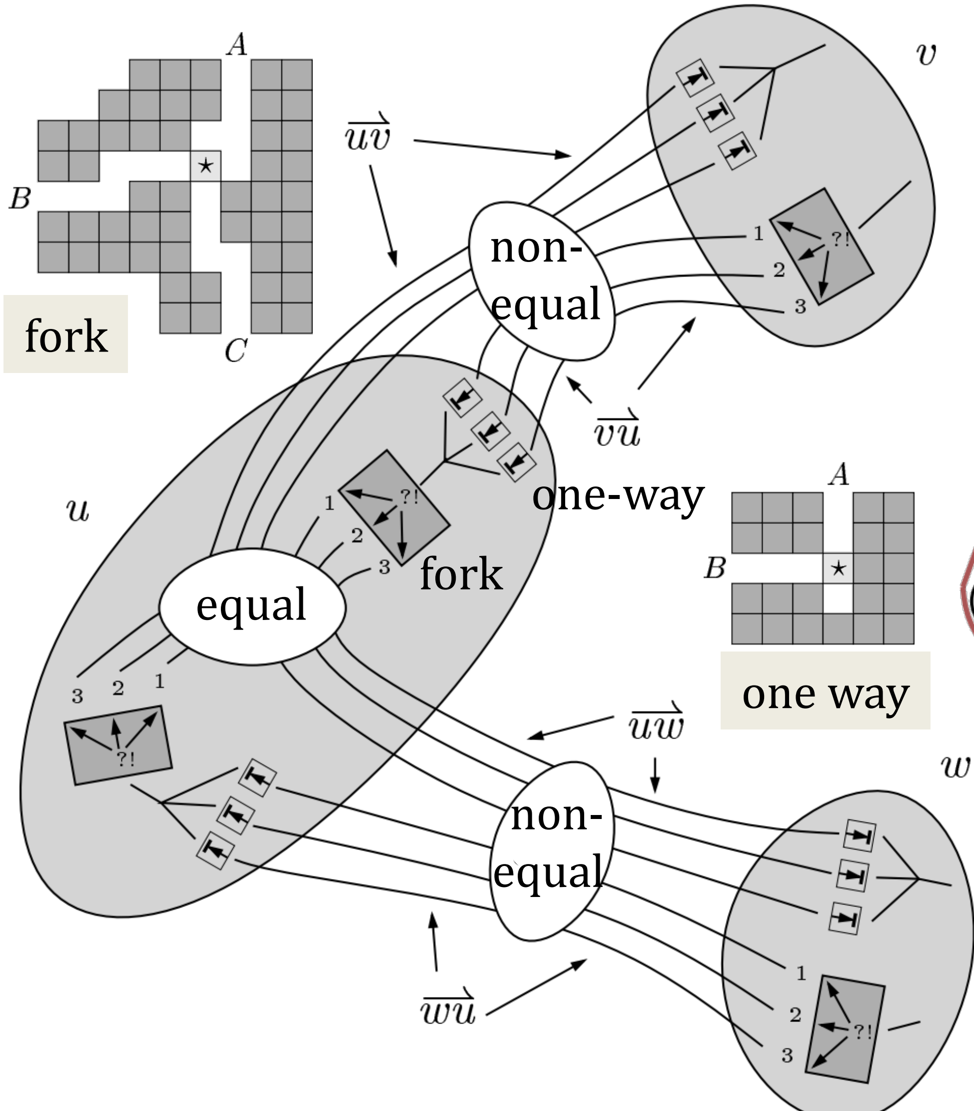
Push-1X is NP-complete

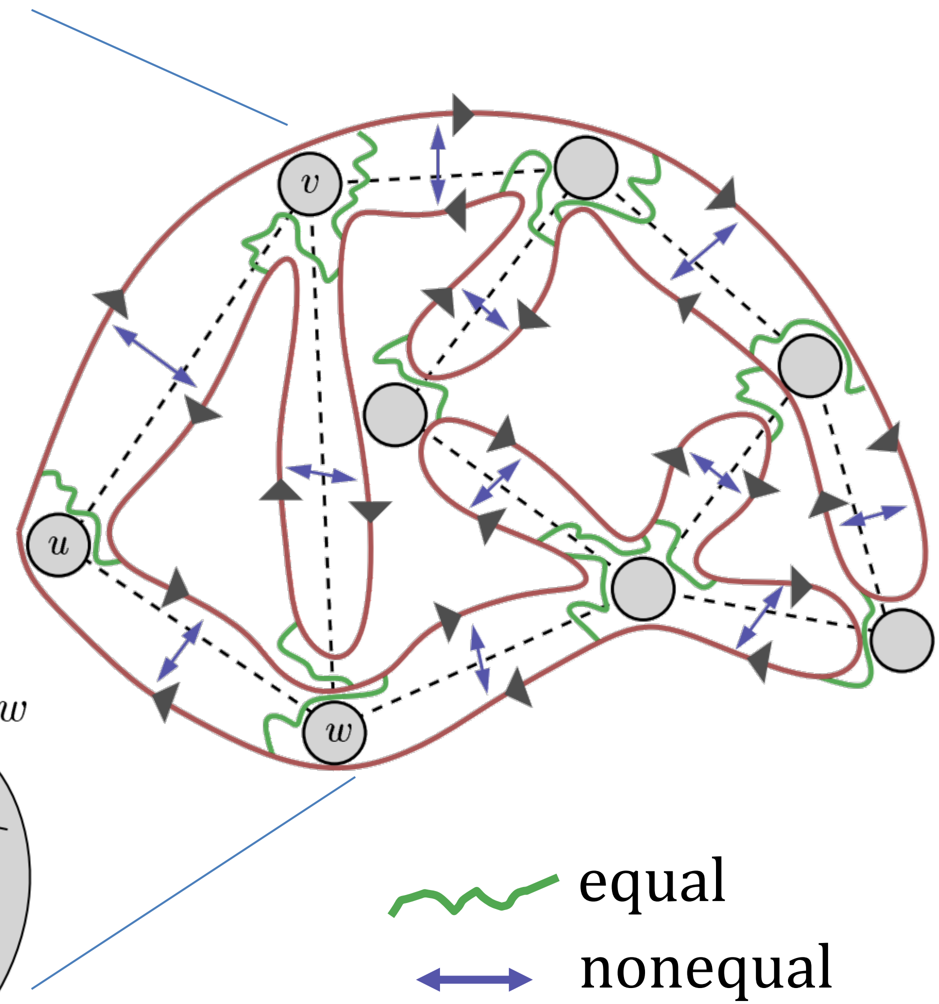
[Demaine, Demaine, Hoffmann, O'Rourke 2003]



Push-1X is NP-complete

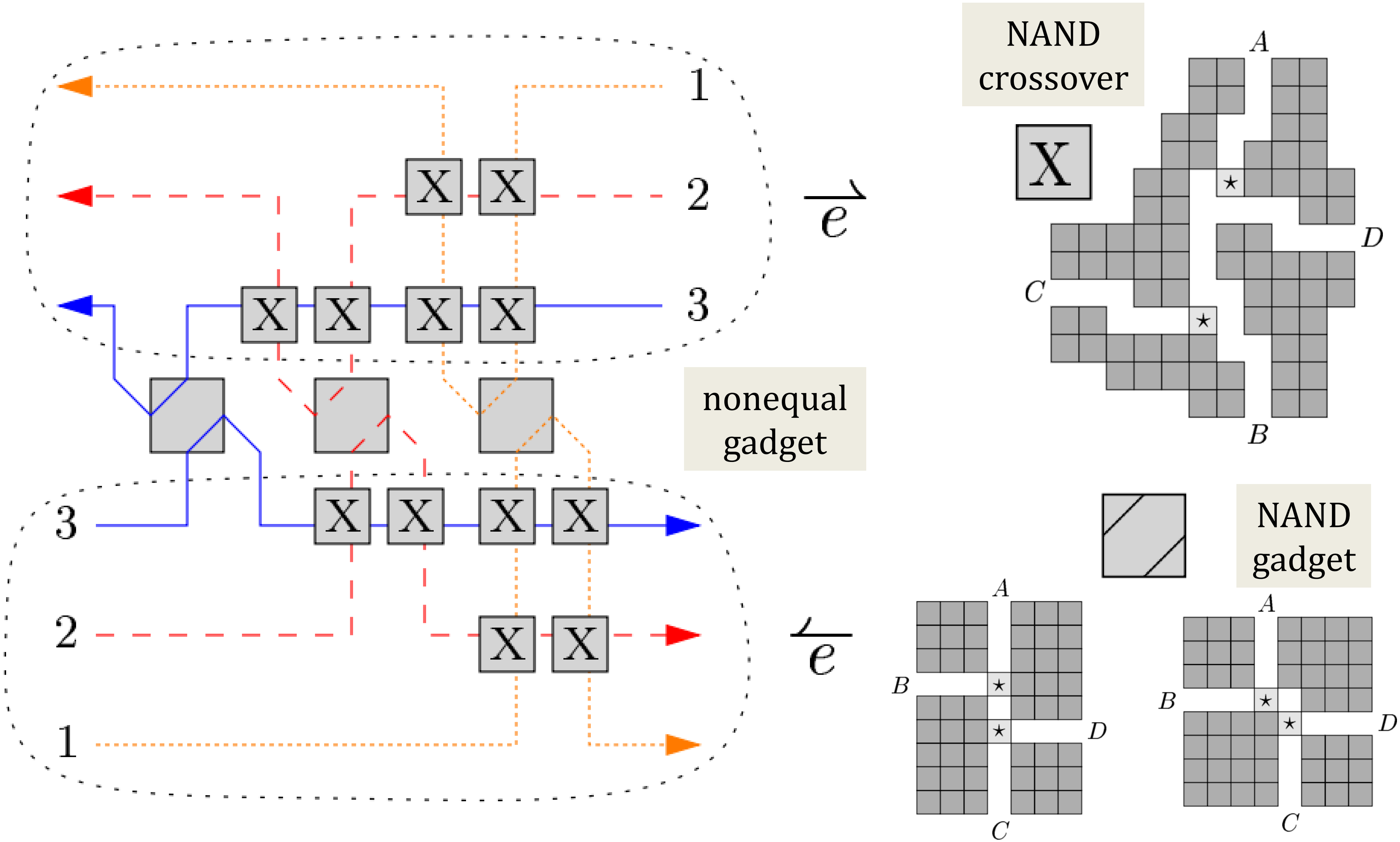
[Demaine, Demaine, Hoffmann, O'Rourke 2003]





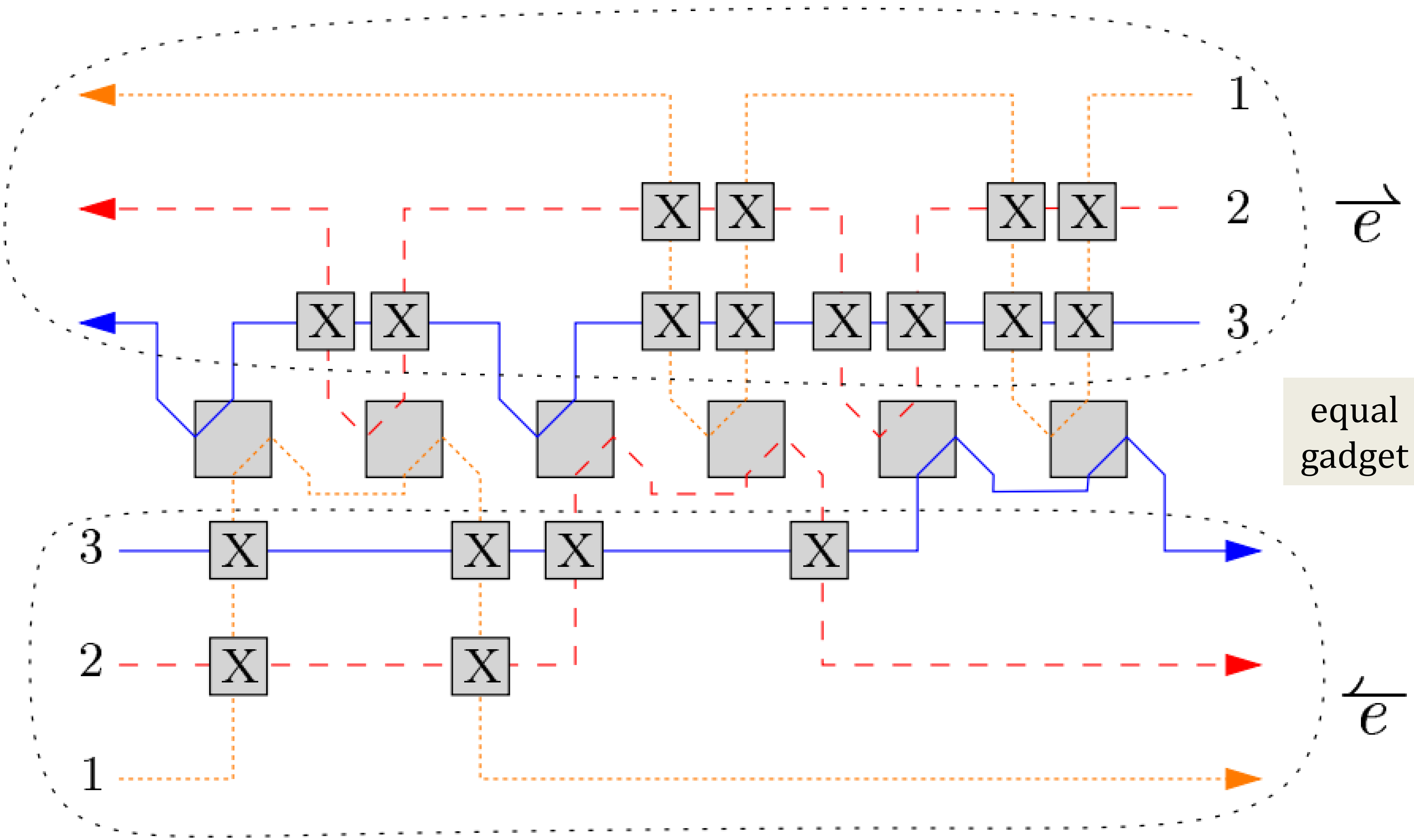
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]



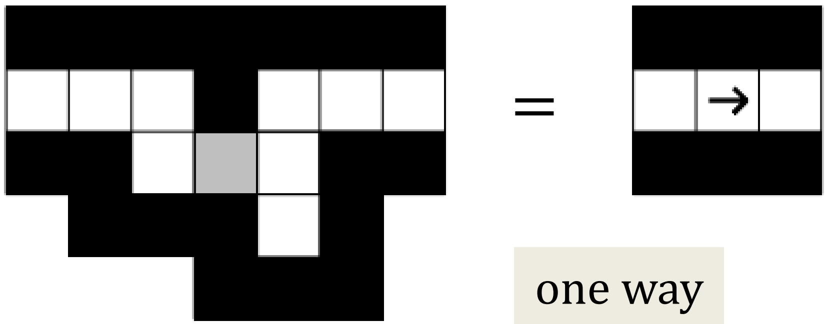
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]



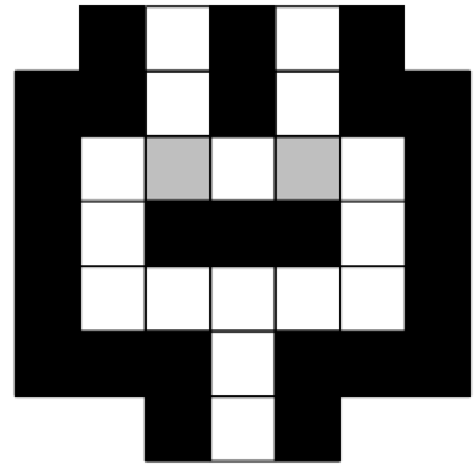
Push-1G is NP-hard

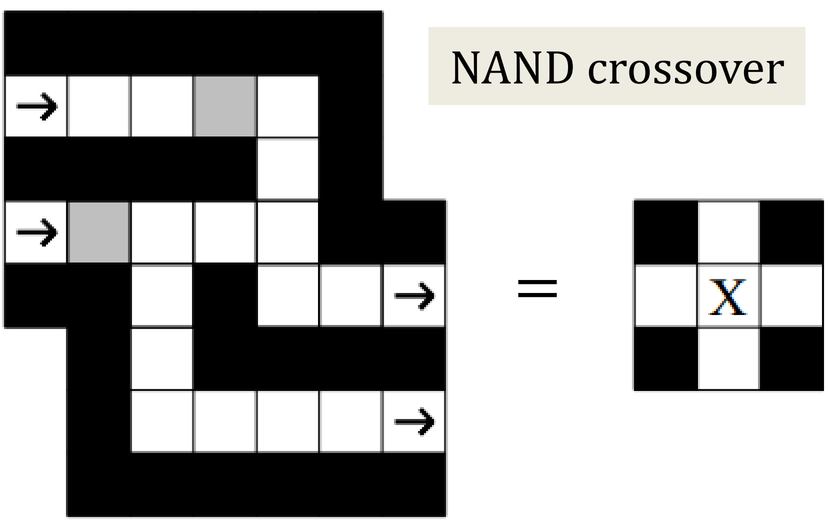
[Friedman 2002]



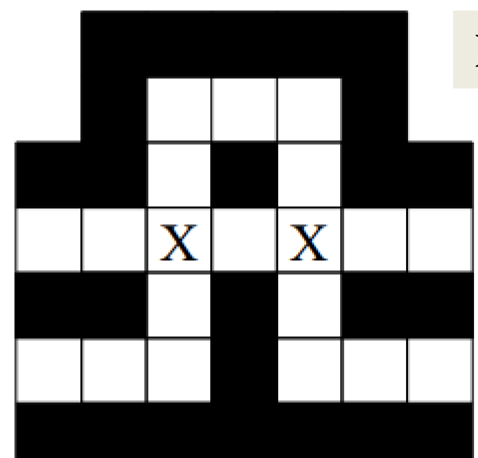
one way

fork





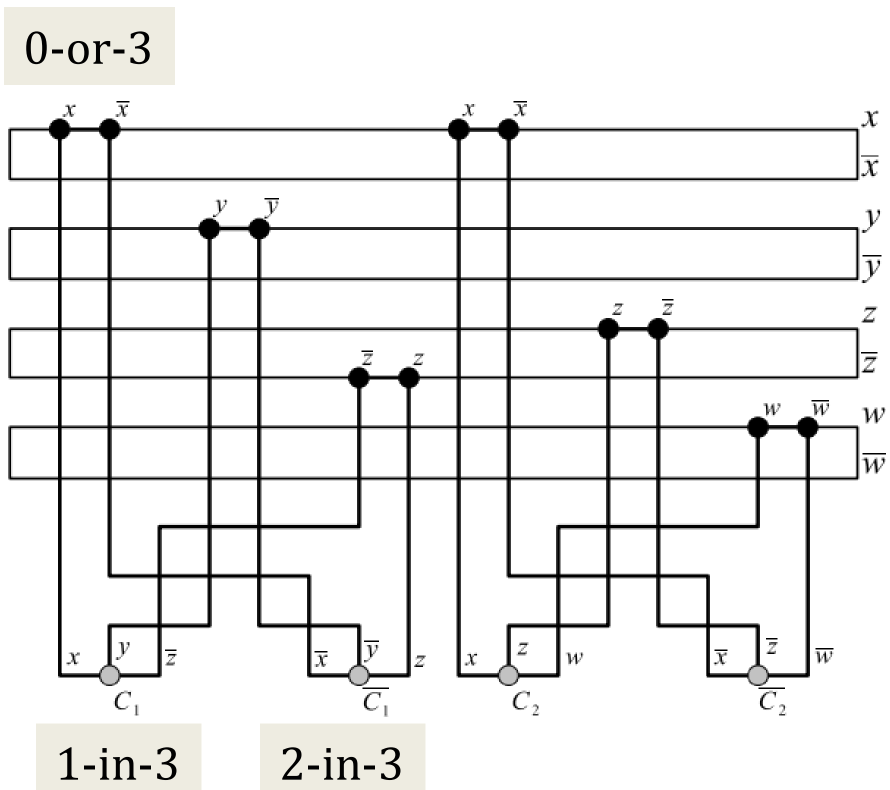
NAND crossover

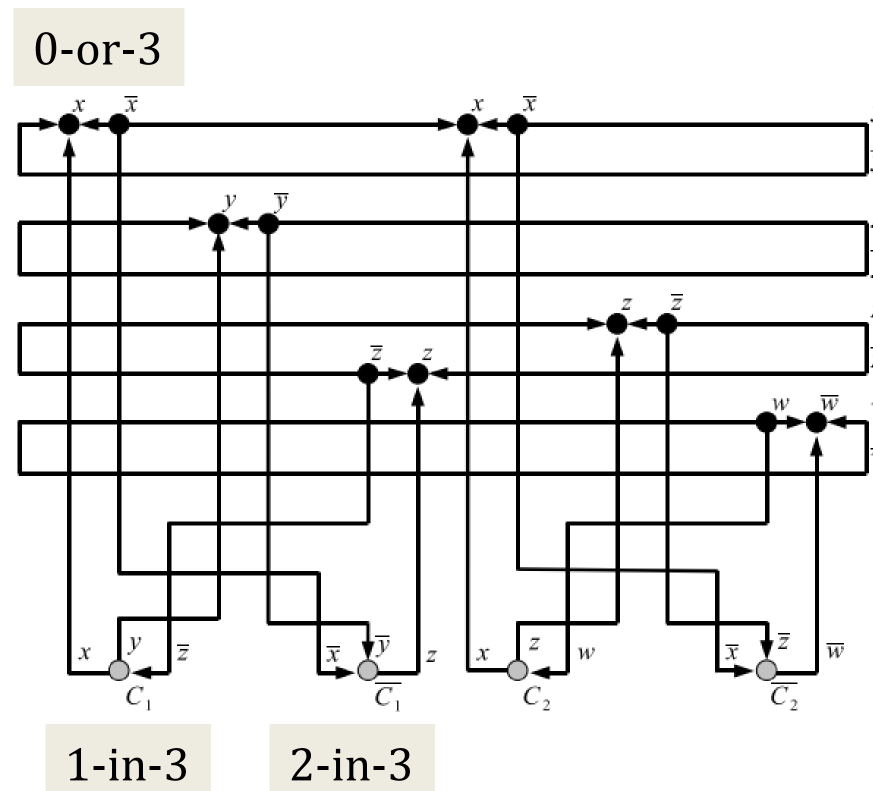


NAND

Graph Orientation

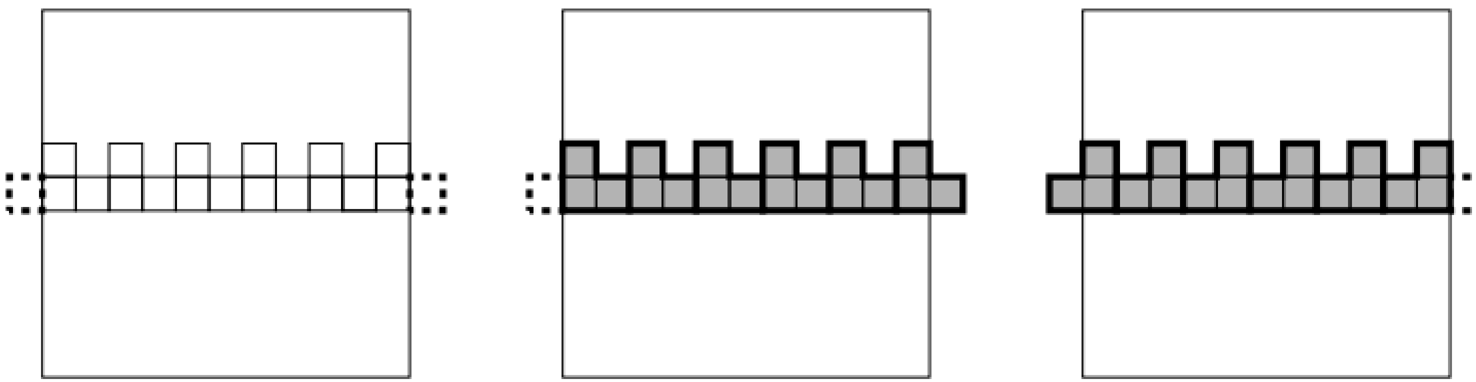
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]



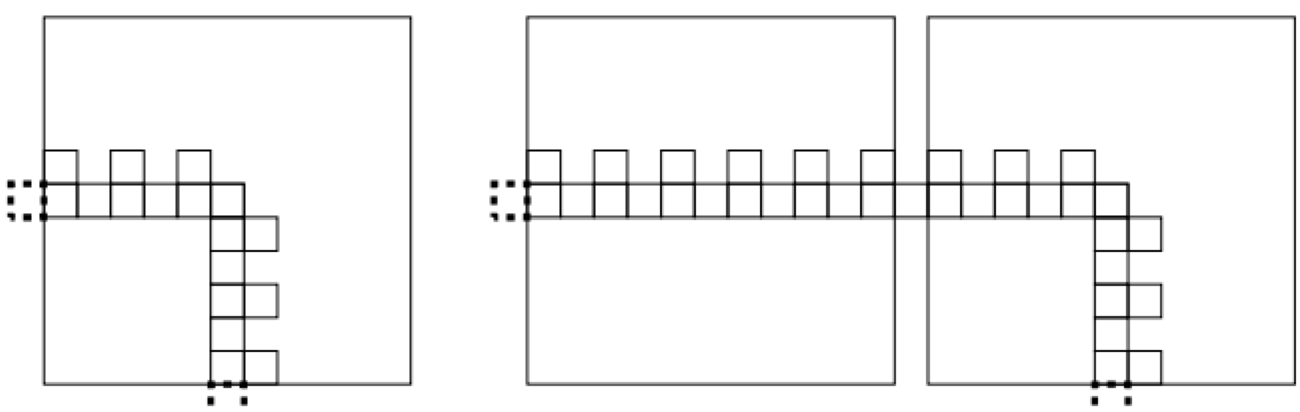


Packing L Trominoes into Polygon

[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

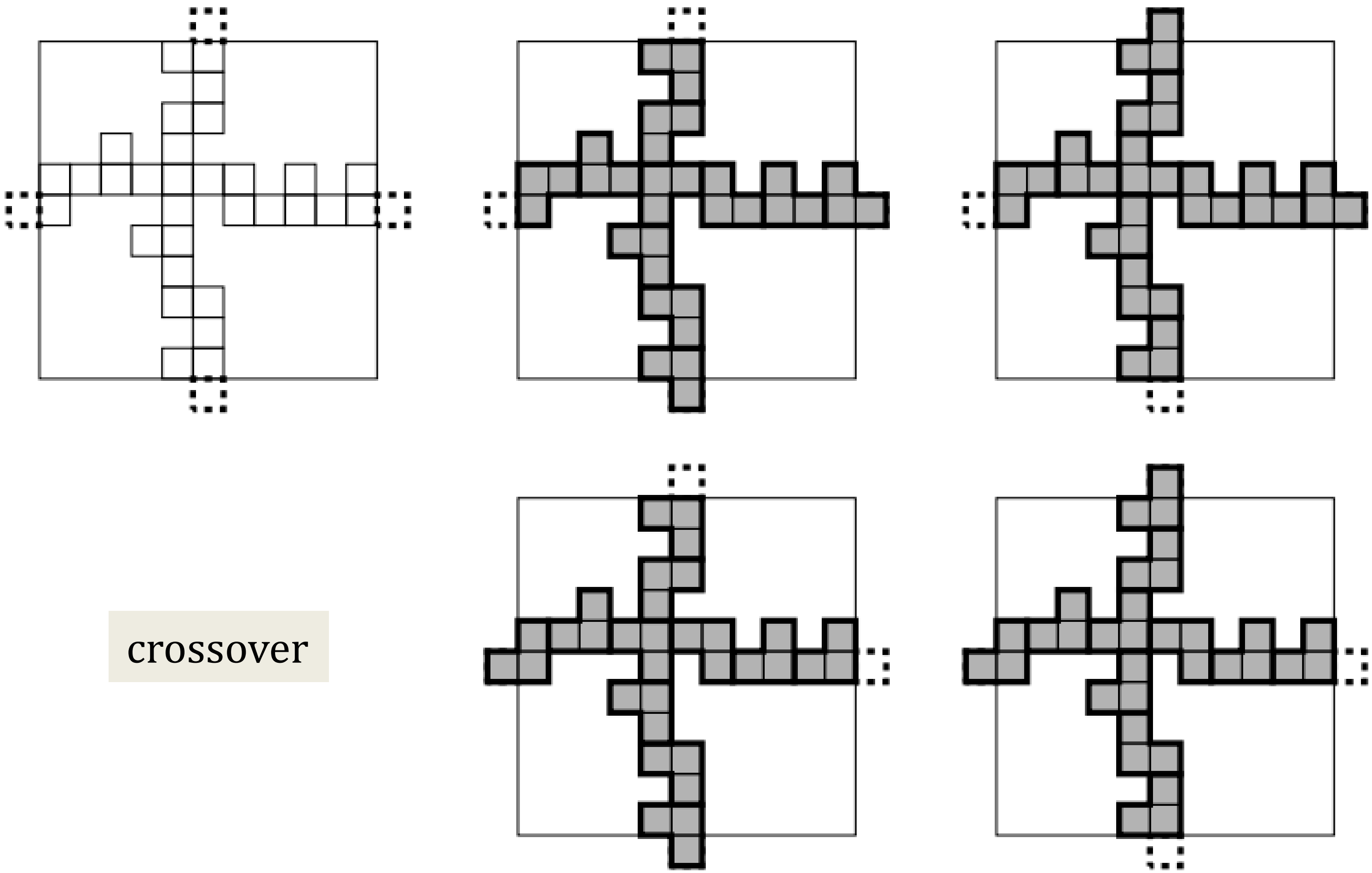


edge gadget



Packing L Trominoes into Polygon

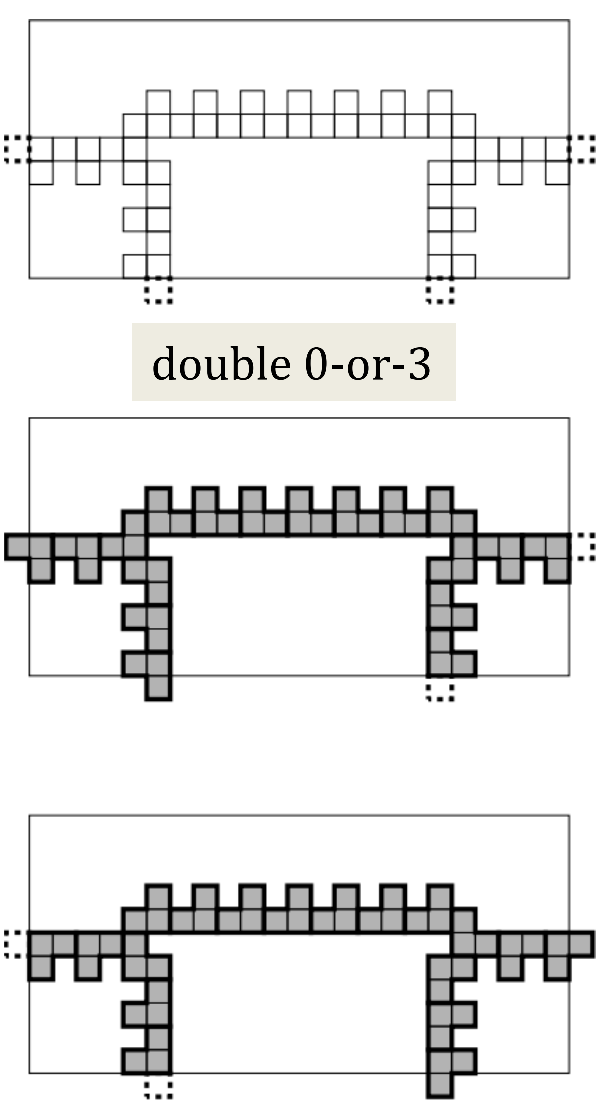
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

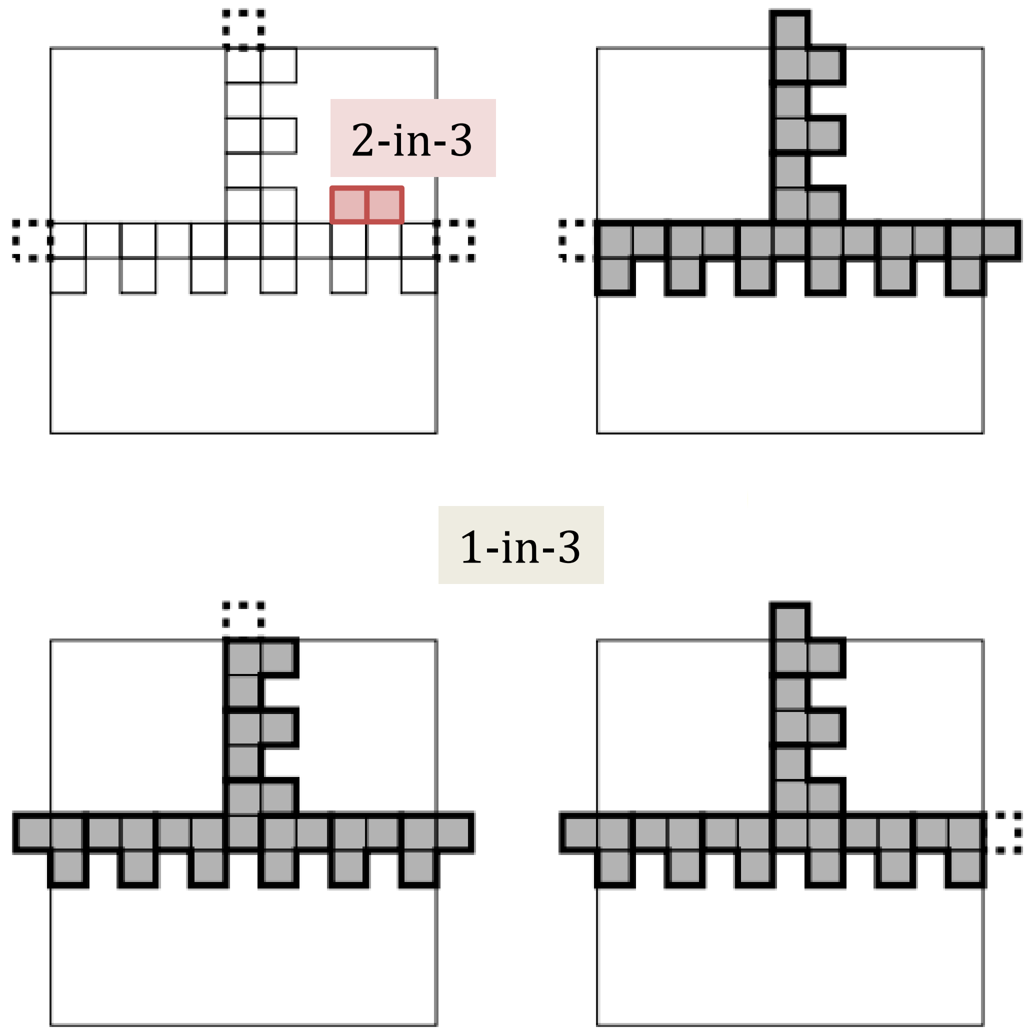


crossover

Packing L Trominoes into Polygon

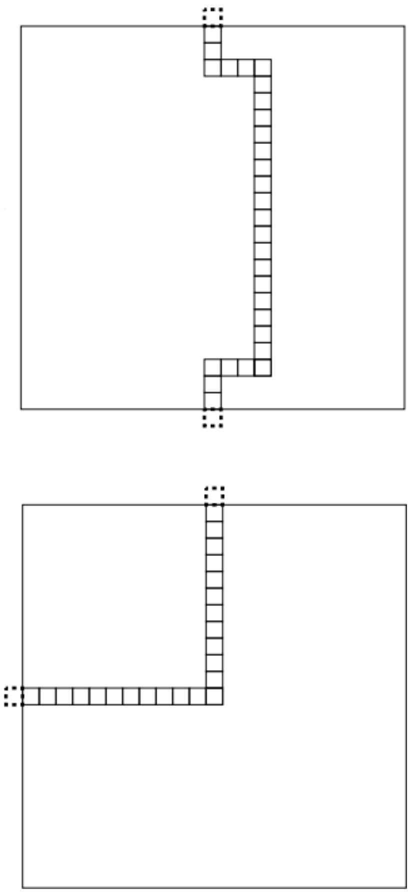
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]



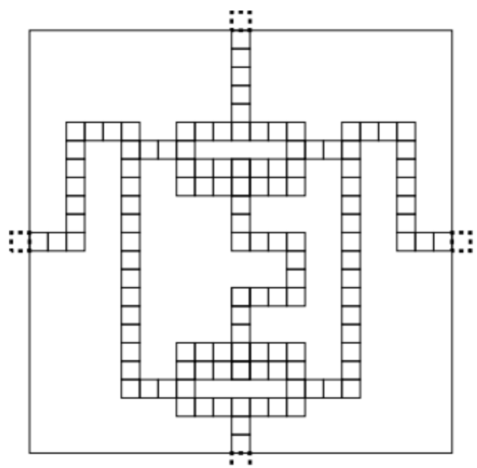


Packing I Trominoes into Polygon

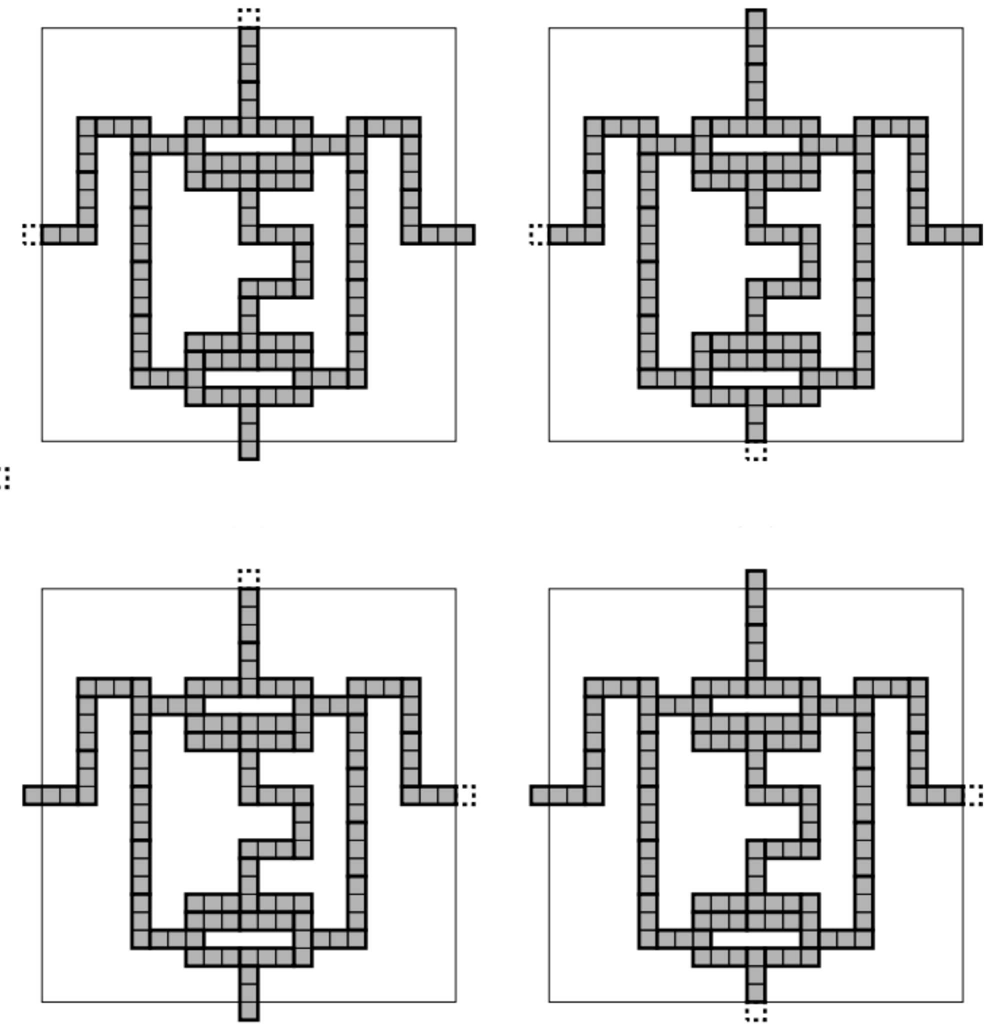
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]



edge gadget

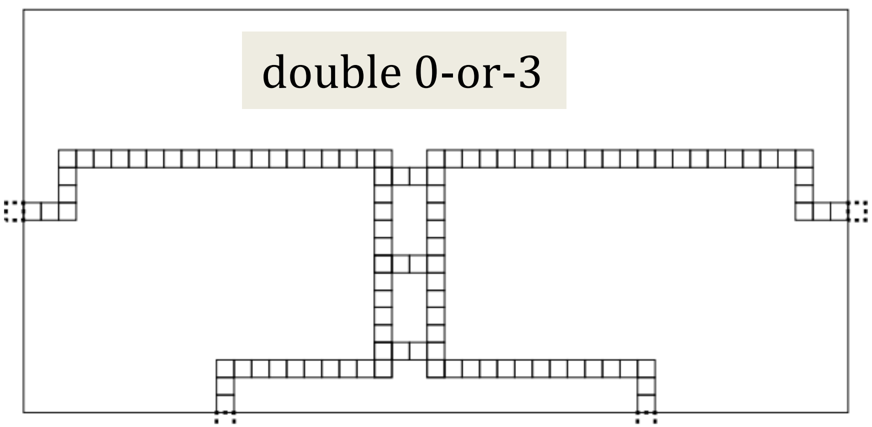


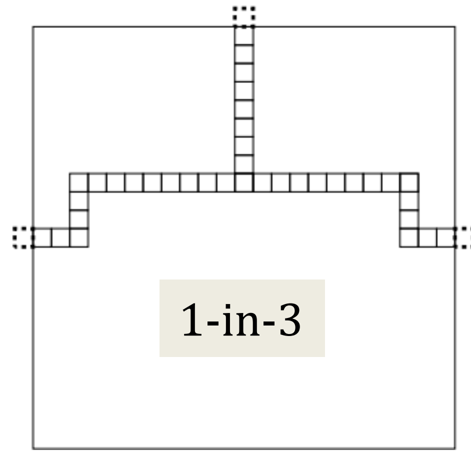
crossover

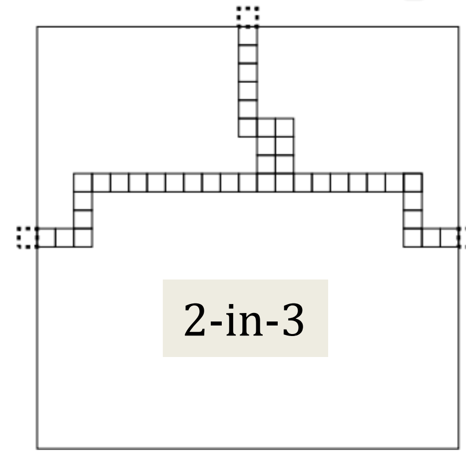


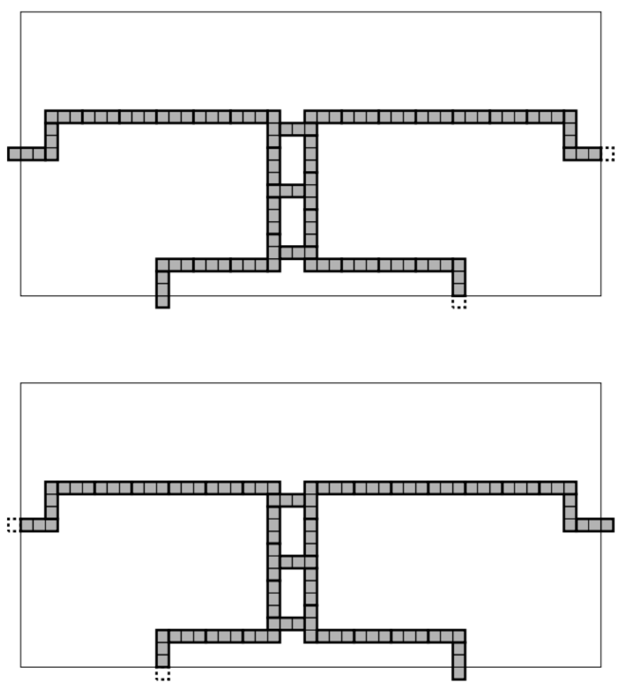
Packing I Trominoes into Polygon

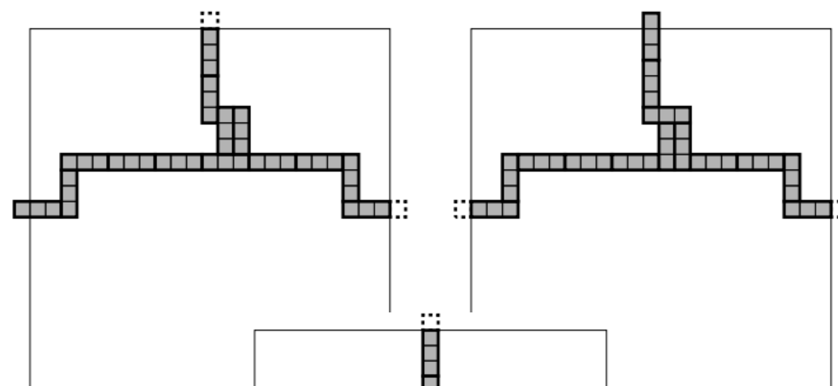
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

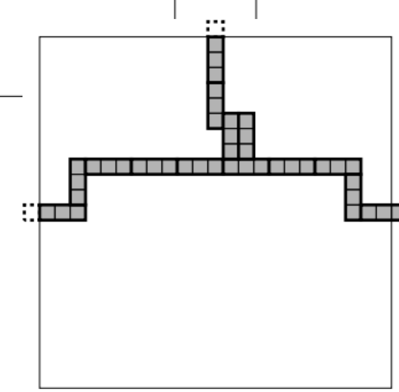








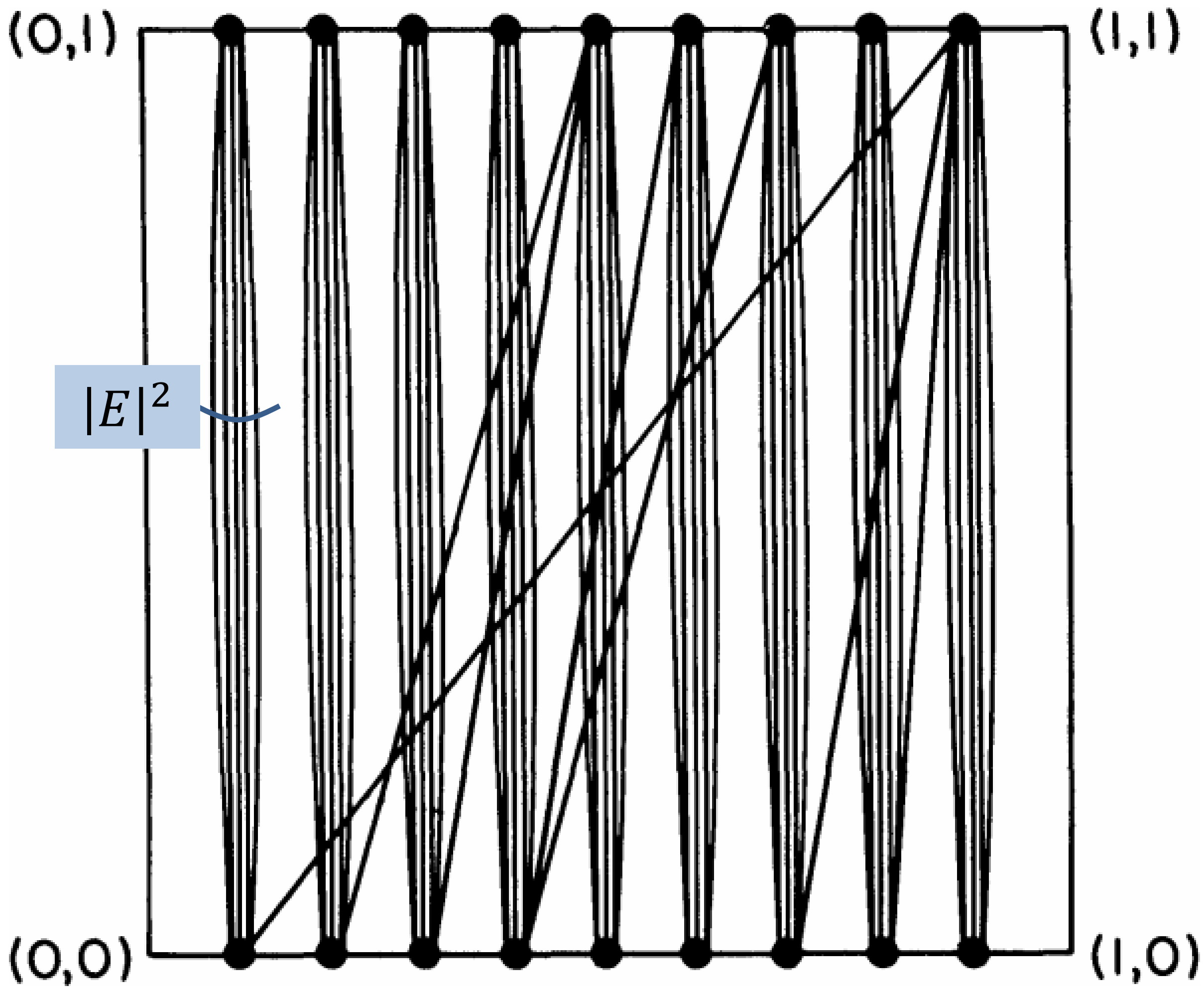




Problem	NP-complete	[Díaz, Petit, Serna 2002]
BANDWIDTH	in general	[Papadimitriou 1976]
	for trees with maximum degree 3	[Garey et al. 1978]
	for caterpillars with hair-length ≤ 3	[Monien 1986]
	for caterpillars with ≤ 1 hair per backbone vertex	[Monien 1986]
	for cyclic caterpillars with hair-length 1	[Muradyan 1999]
	for grid graphs and unit disk graphs	[Díaz et al. 2001a]
MINLA	in general	[Garey et al. 1976]
	for bipartite graphs	[Even and Shiloach 1975]
CUTWIDTH	in general	[Gavril 1977]
	for graphs with maximum degree 3	[Makedon et al. 1985]
	for planar graphs with maximum degree 3	[Monien and Sudborough 1988]
	for grid graphs and unit disk graphs	[Díaz et al. 2001a]
MODCUT	for planar graphs with maximum degree 3	[Monien and Sudborough 1988]
VERTSEP	in general	[Lengauer 1981]
	for planar graphs with maximum degree 3	[Monien and Sudborough 1988]
	for chordal graphs	[Gustedt 1993]
	for bipartite graphs	[Goldberg et al. 1995]
	for grid graphs and unit disk graphs	[Díaz et al. 2001a]
SUMCUT	in general	[Díaz et al. 1991]
		[Lin and Yuan 1994b]
		[Golovach 1997]
	for cobipartite graphs	[Yuan et al. 1998]
EDGEBIS	in general	[Garey et al. 1976]
	for graphs with maximum degree 3	[MacGregor 1978]
	for graphs with maximum degree bounded	[MacGregor 1978]
	for d-regular graphs	[Bui et al. 1987]

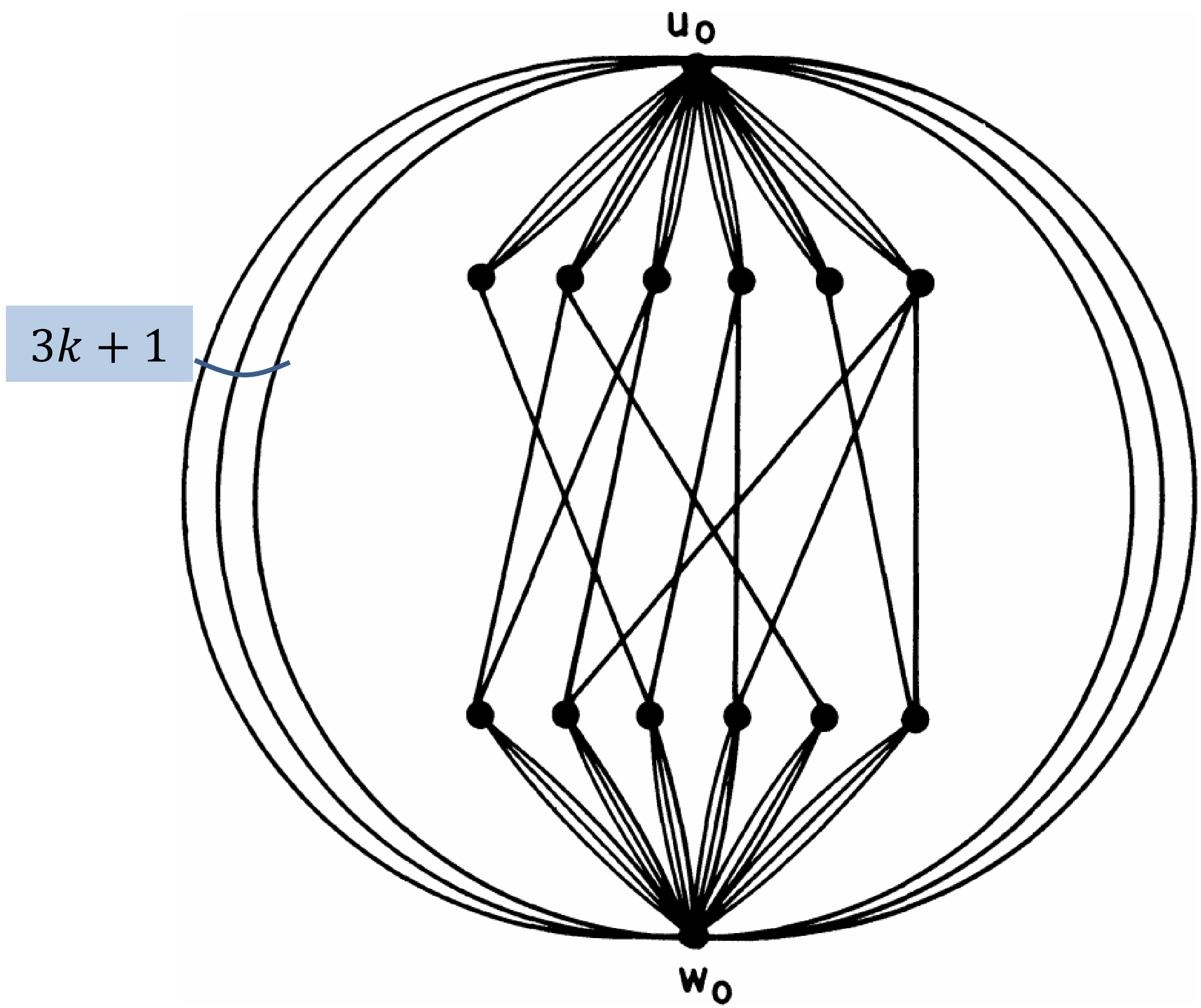
Bipartite Crossing Number

[Garey & Johnson 1983]



Crossing Number is NP-Complete

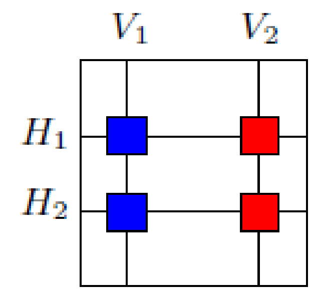
[Garey & Johnson 1983]



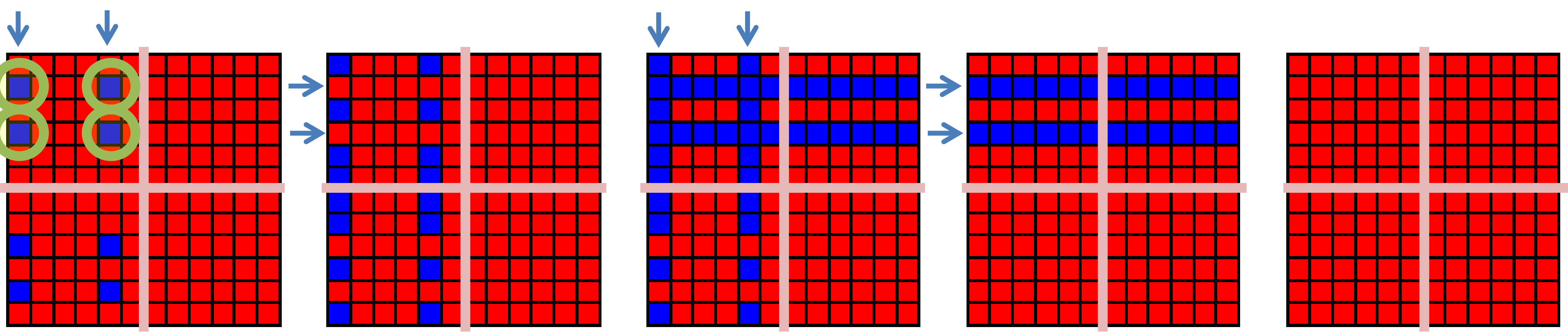
How To Solve Rubik's Cube Faster

[Demaine, Demaine, Eisenstat, Lubiw, Winslow 2011]

- Kill $\Theta(\log n)$ birds with $\Theta(1)$ stones
- Look for cubies arranged in a grid that have the same solution sequence
  - $X \times Y$ grid can be solved in $\Theta(X + Y)$ moves instead of the usual $\Theta(X \cdot Y)$ moves
  - Can always find $\Theta(\log n)$-factor savings like this



(b) V_1, H_1, V_1, H_1.



Optimal Rubik's Cube Solutions

[Demaine, Demaine, Eisenstat, Lubiw, Winslow 2011]

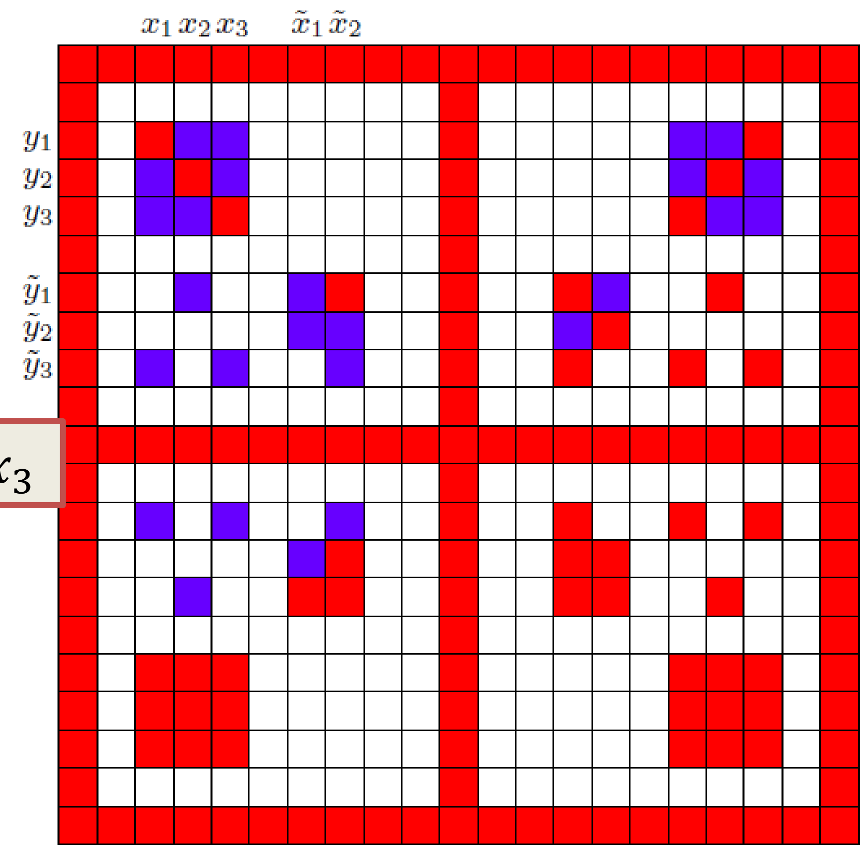
- NP-hard to solve a specified subset of $n \times n \times 1$ "Rubik's Square" using fewest possible moves

important & solved

important & unsolved

unimportant / don't care / chameleon

first x_2 between first x_1 & first x_3



- <u>Open:</u> NP-hard if all cubies are important? [Erickson 2010]