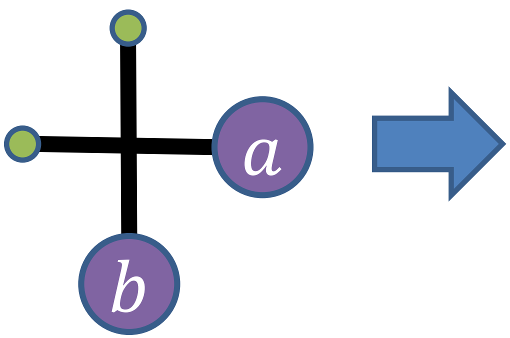
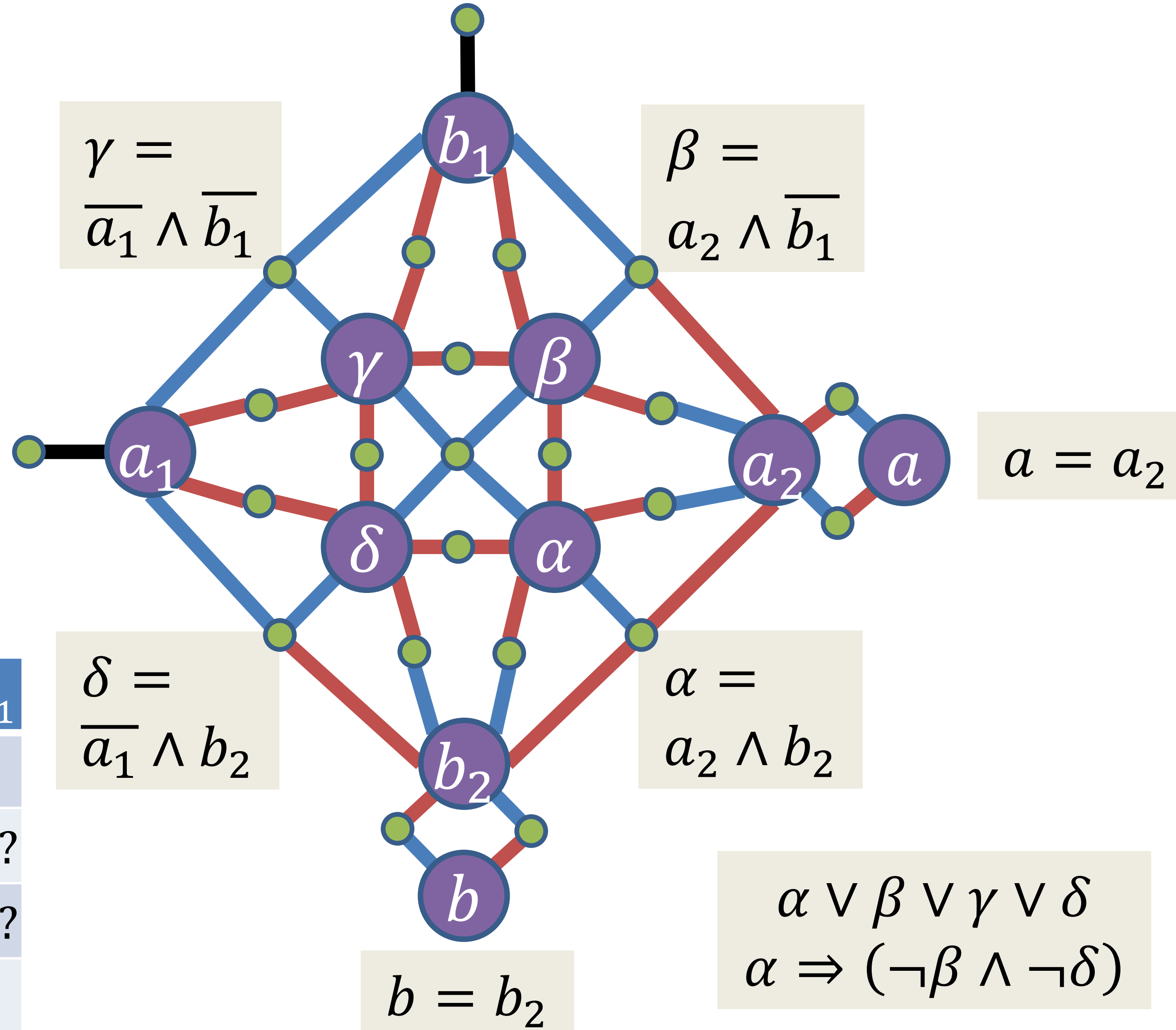
Planar 3SAT is NP-hard

[Lichtenstein 1982]

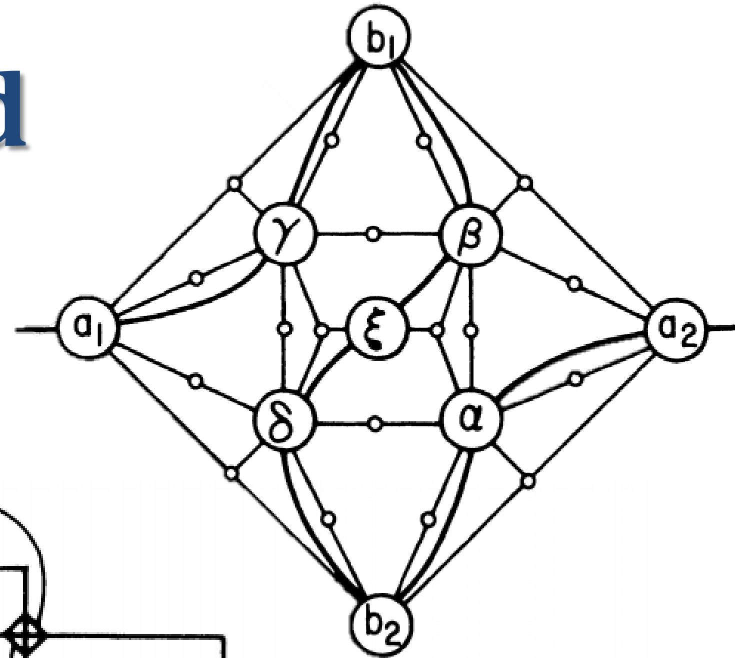


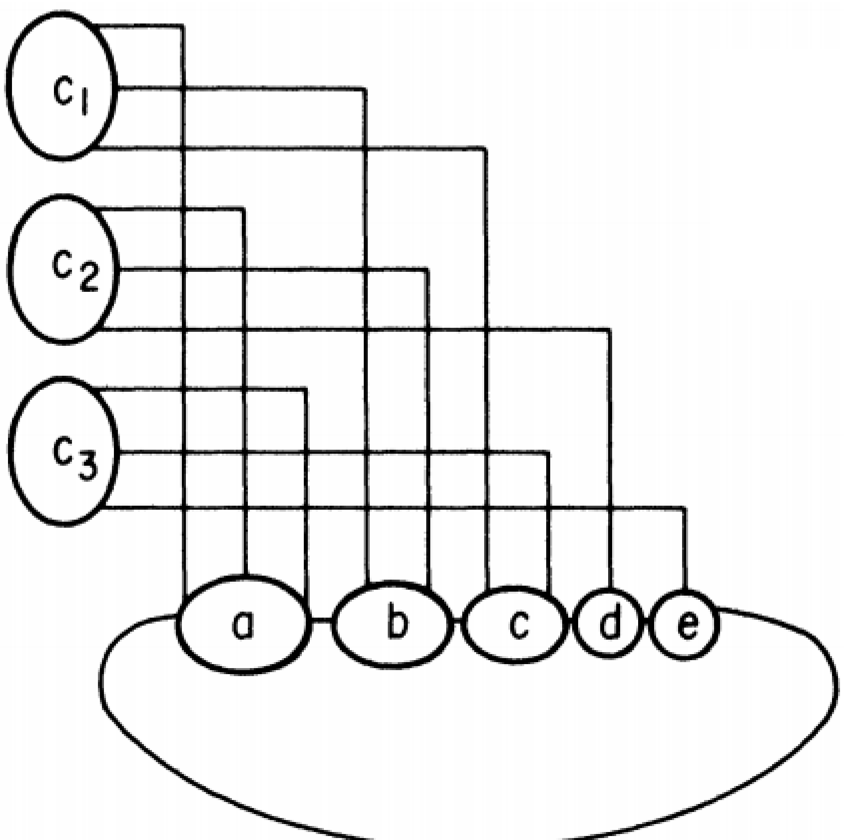


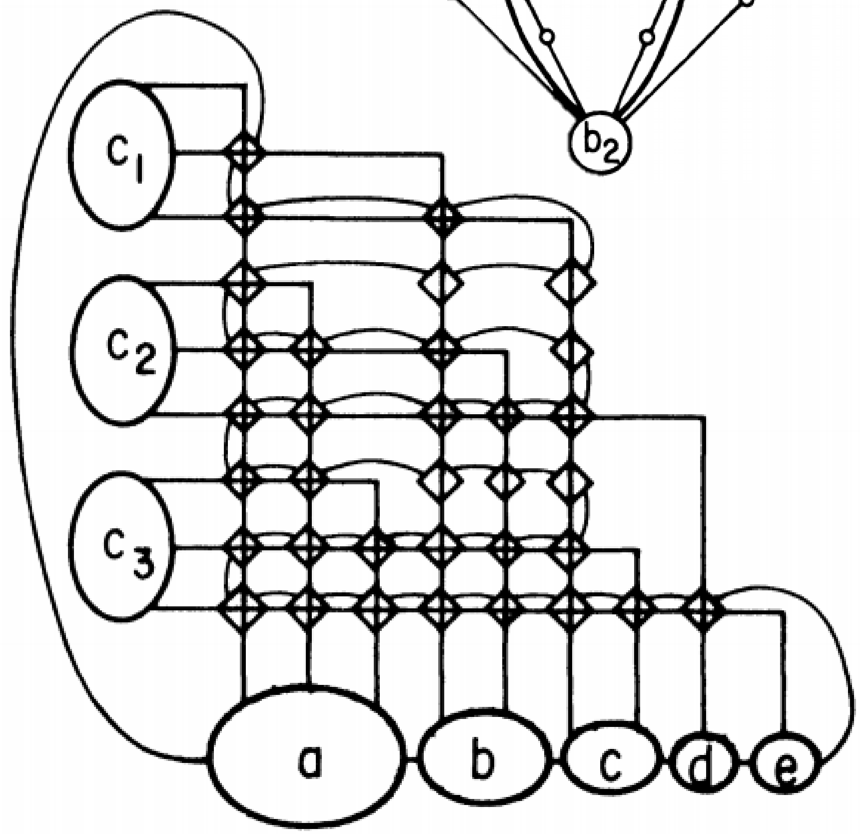
a_2	b_2	α	β	δ	γ	a_1	b_1
0	0	0	0	0	1	0	0
0	1	0	0		1?	0?	0?
1	0	0		0	1?	0?	0?
1	1	1	0	0		1	1

Planar 3SAT is NP-hard

[Lichtenstein 1982]

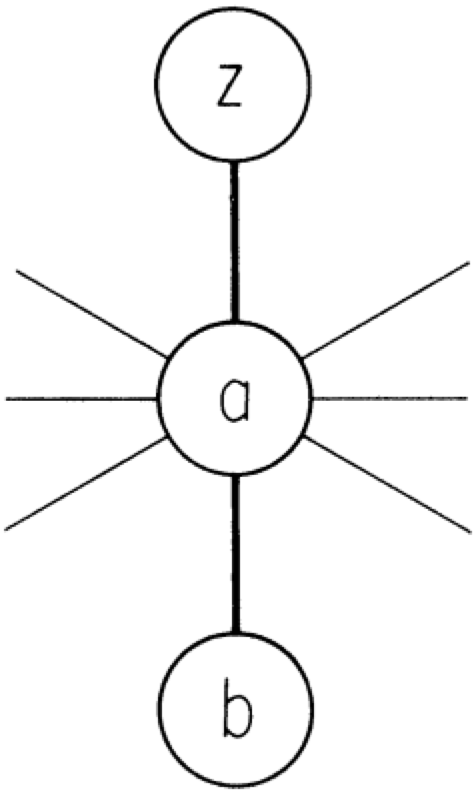


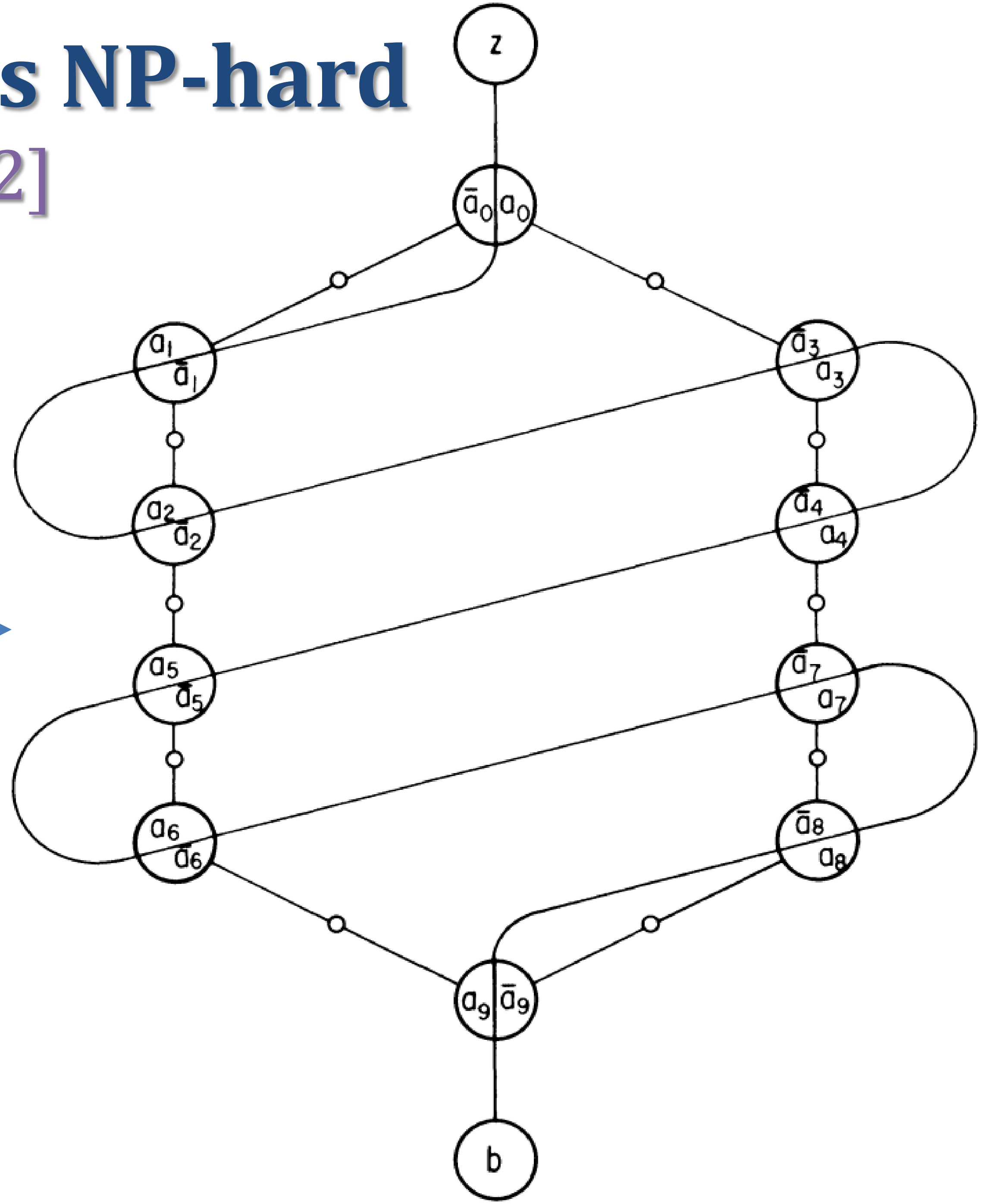




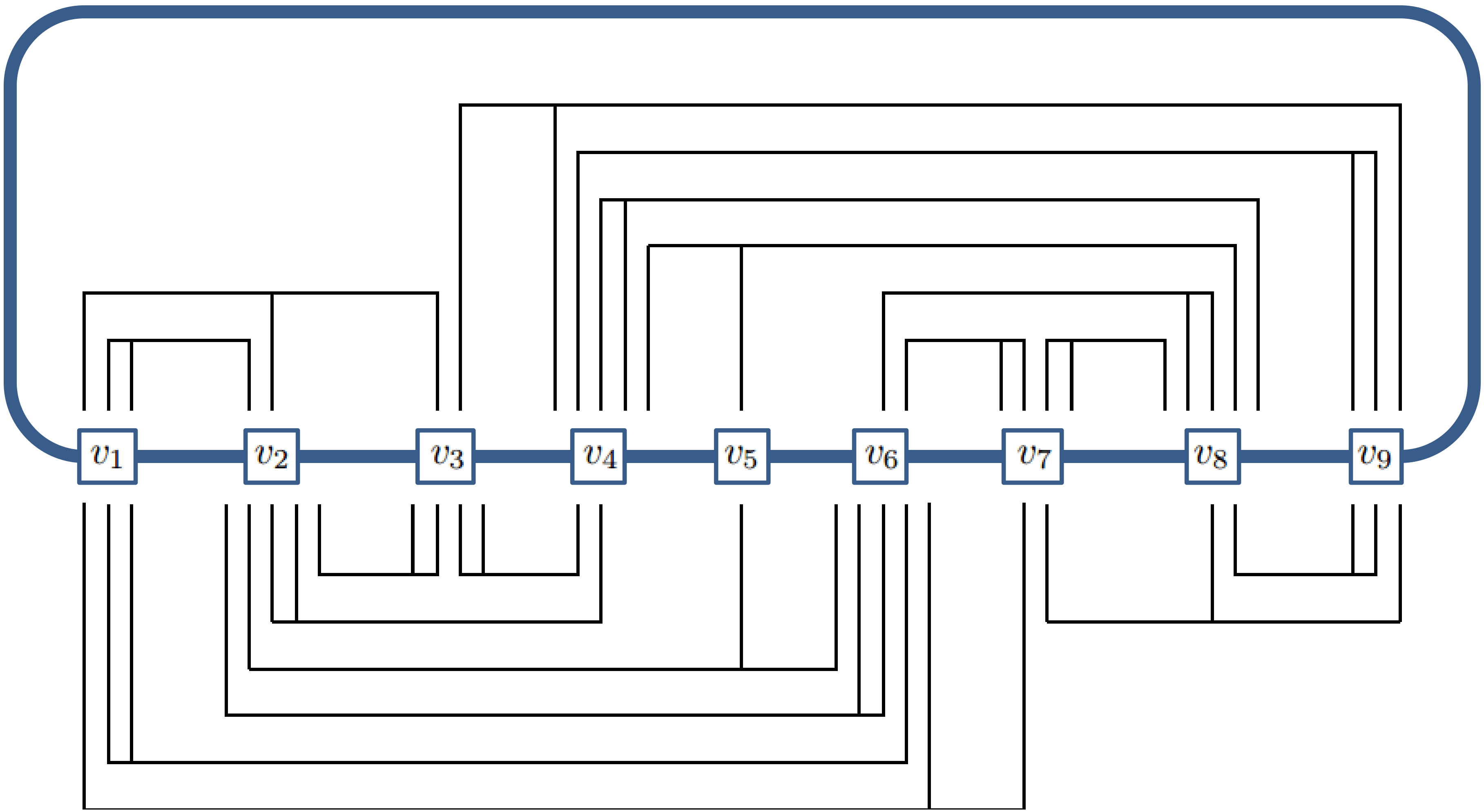
Planar 3SAT is NP-hard

[Lichtenstein 1982]





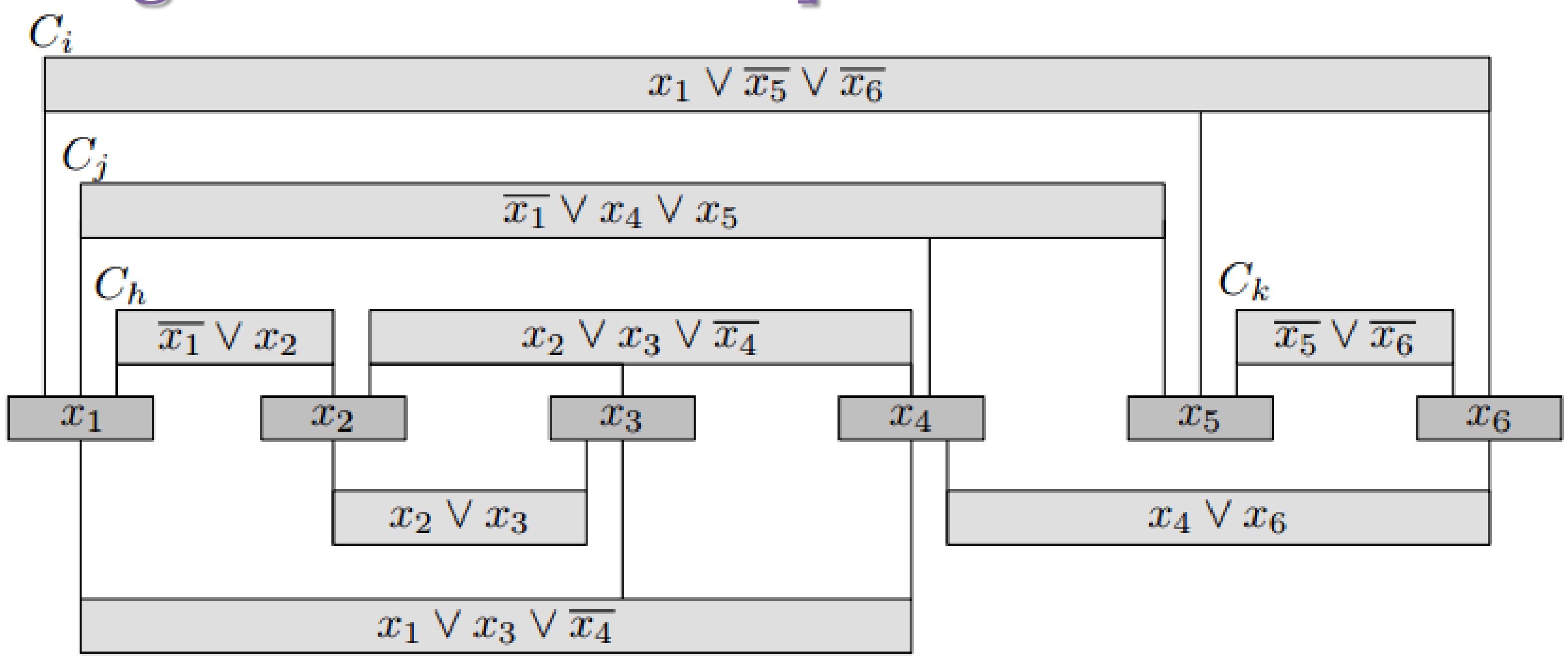
Planar Rectilinear 3SAT

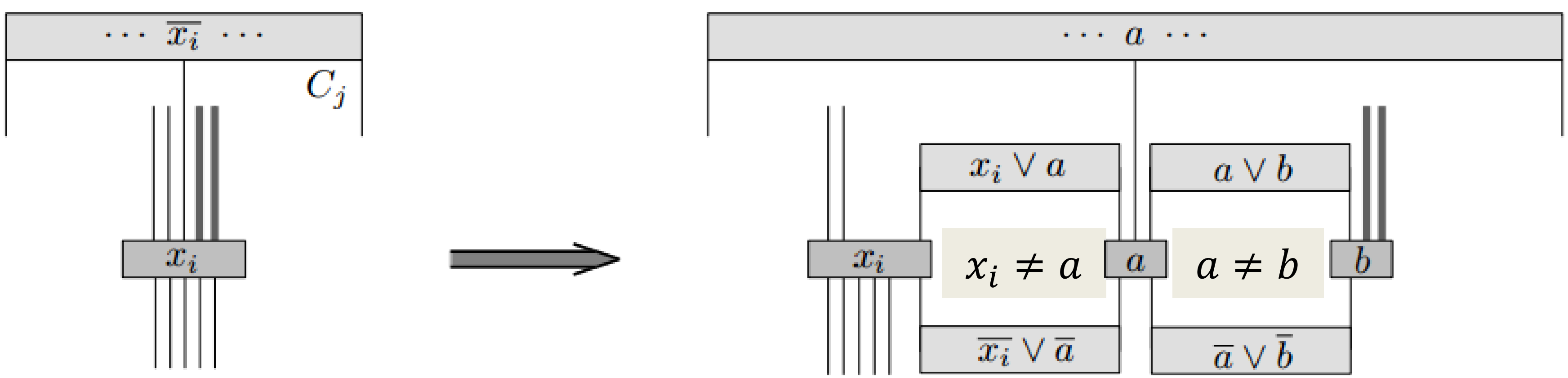


[Knuth & Raghunathan 1992]

Planar Monotone Rectilinear 3SAT

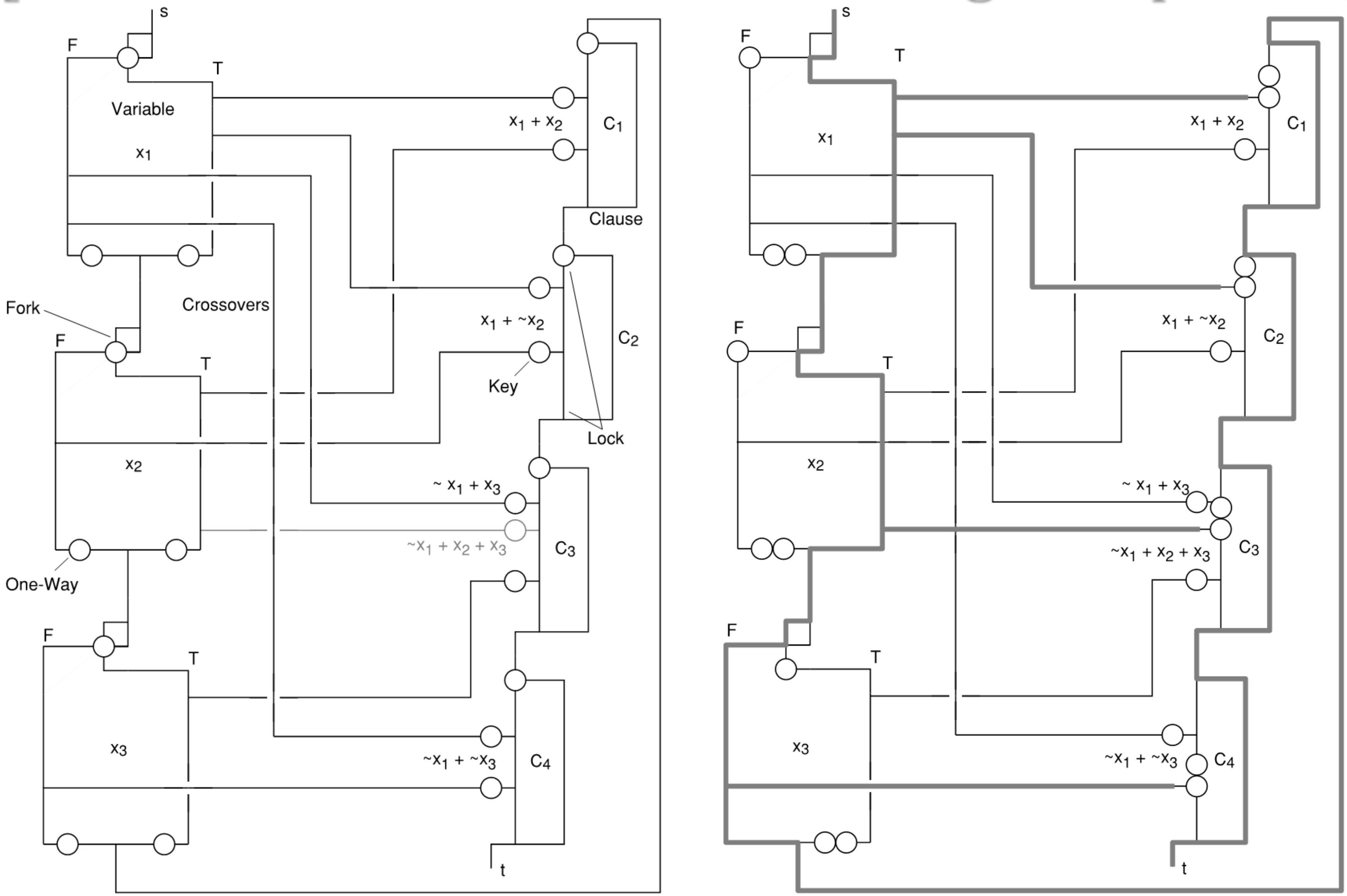
[de Berg & Khosravi 2010]





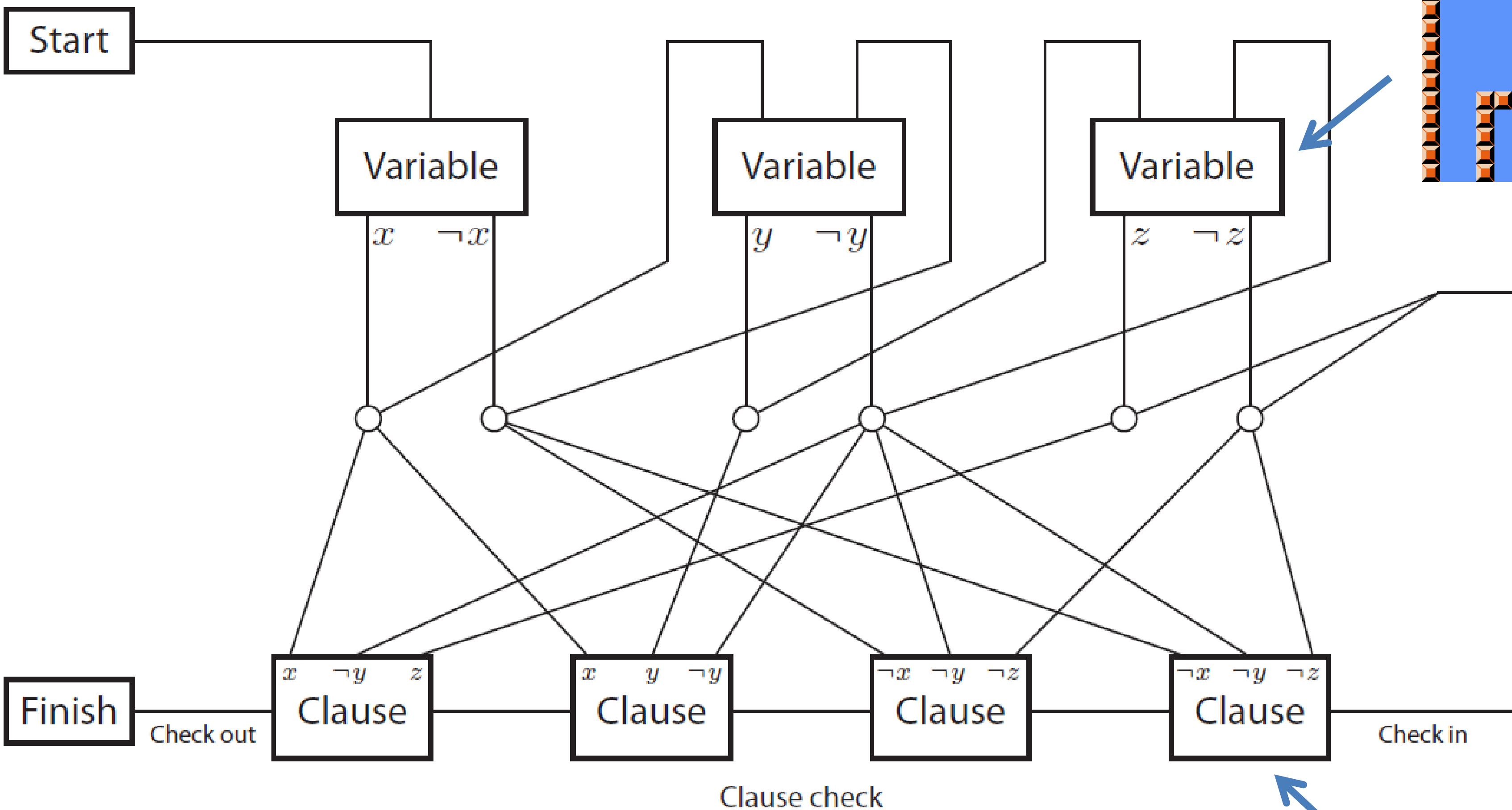
PushPush-1 is NP-hard in 3D

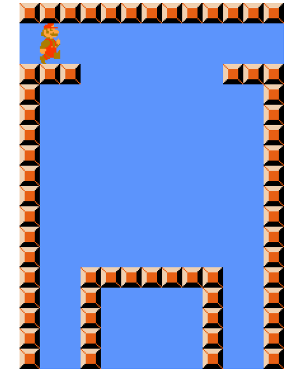
[O'Rourke & Smith Problem Solving Group 1999]

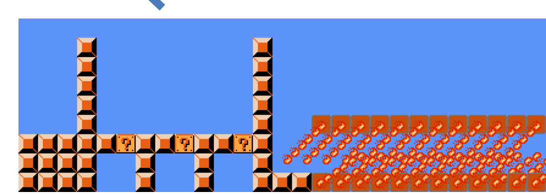


Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]



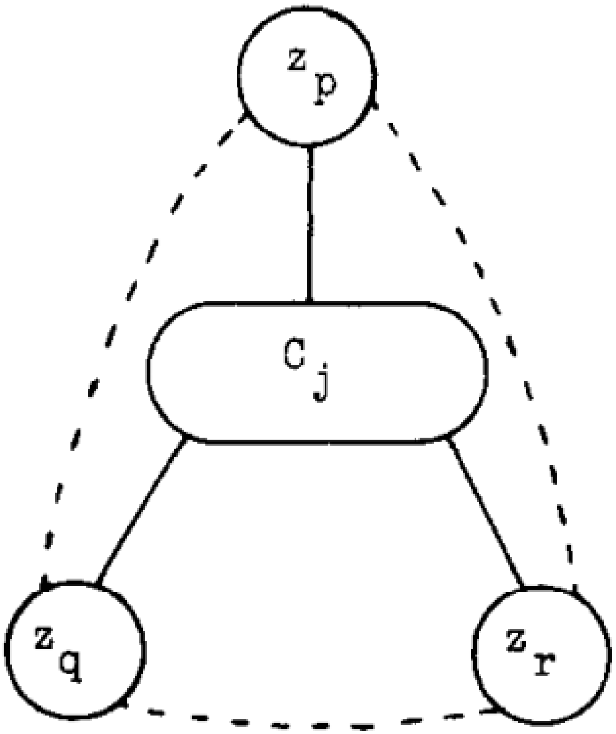


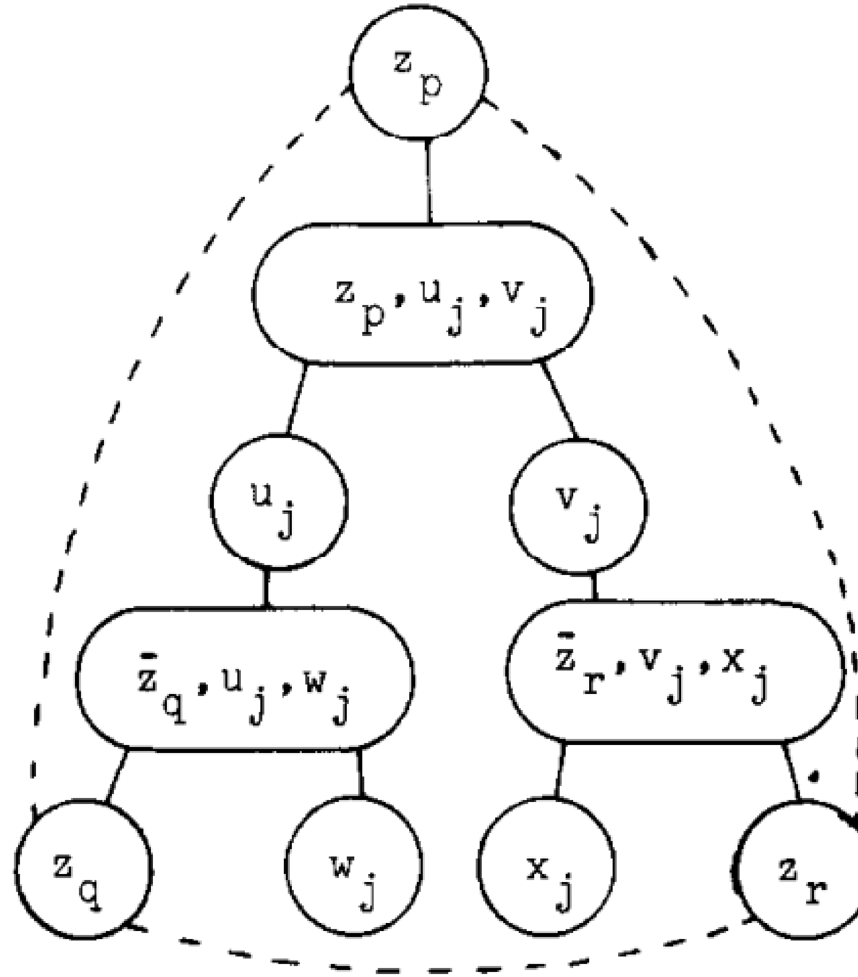


$(x \text{ OR } \neg y \text{ OR } z)$ & $(x \text{ OR } y \text{ OR } \neg y)$ &
$(\neg x \text{ OR } \neg y \text{ OR } \neg z)$ & $(\neg x \text{ OR } \neg y \text{ OR } \neg z)$

Planar 1-in-3SAT

[Dyer & Freeze 1986]

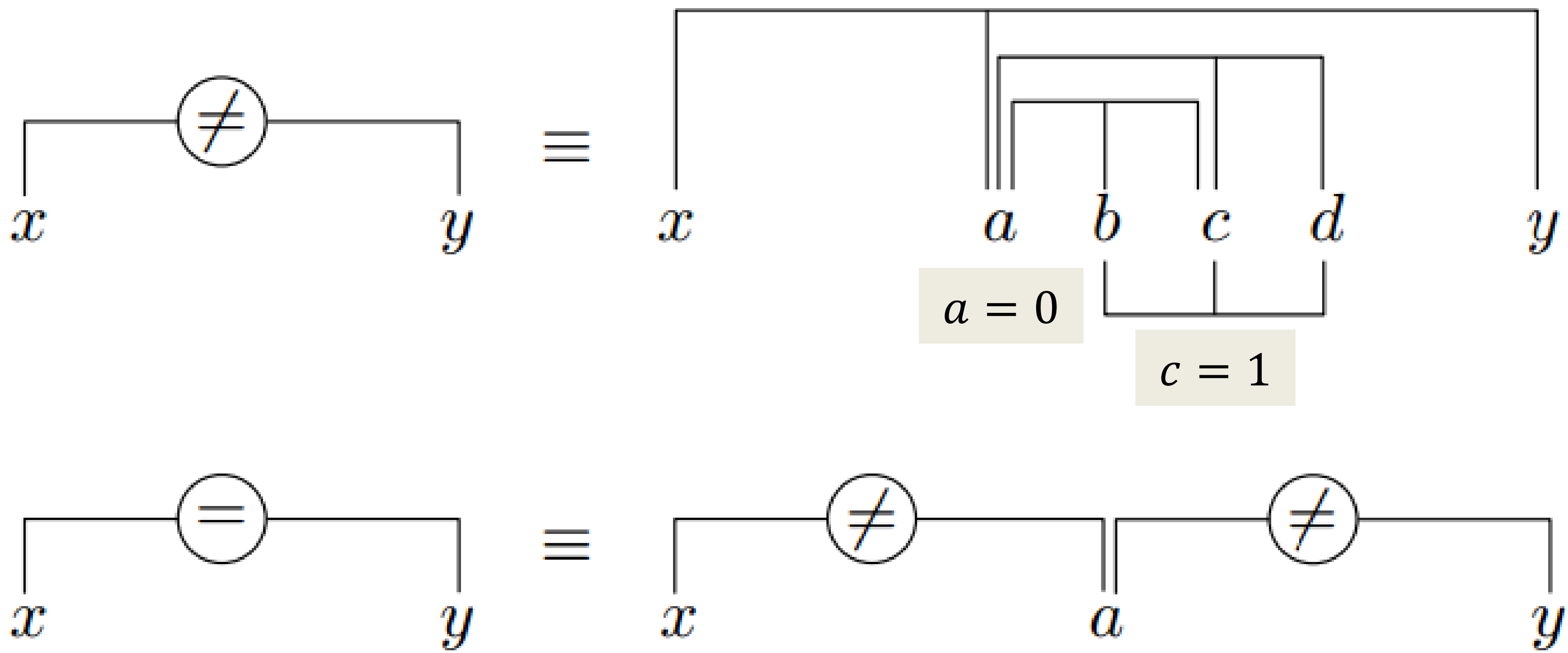




$C_j = \{z_p, z_q, z_r\}$ $\qquad$ $\{z_p, u_j, v_j\}, \{\bar{z}_q, u_j, w_j\}, \{\bar{z}_r, v_j, x_j\}$

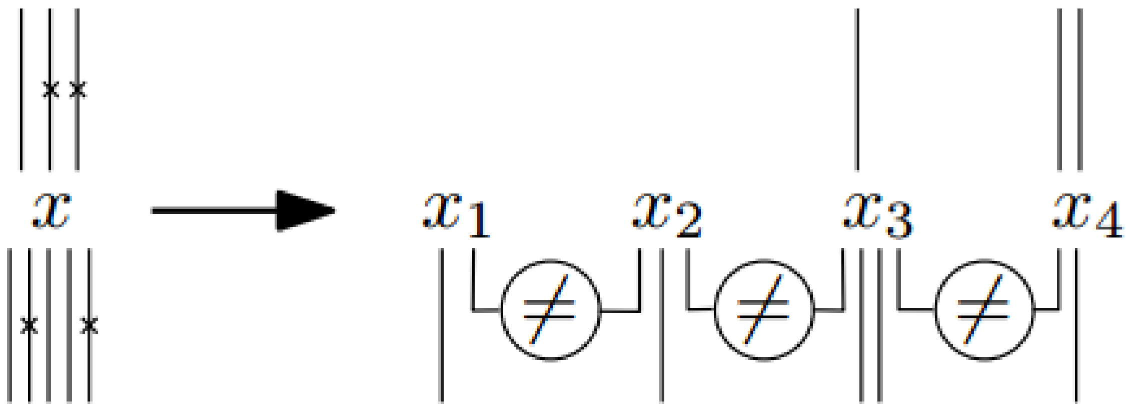
Planar Positive Rectilinear 1-in-3SAT

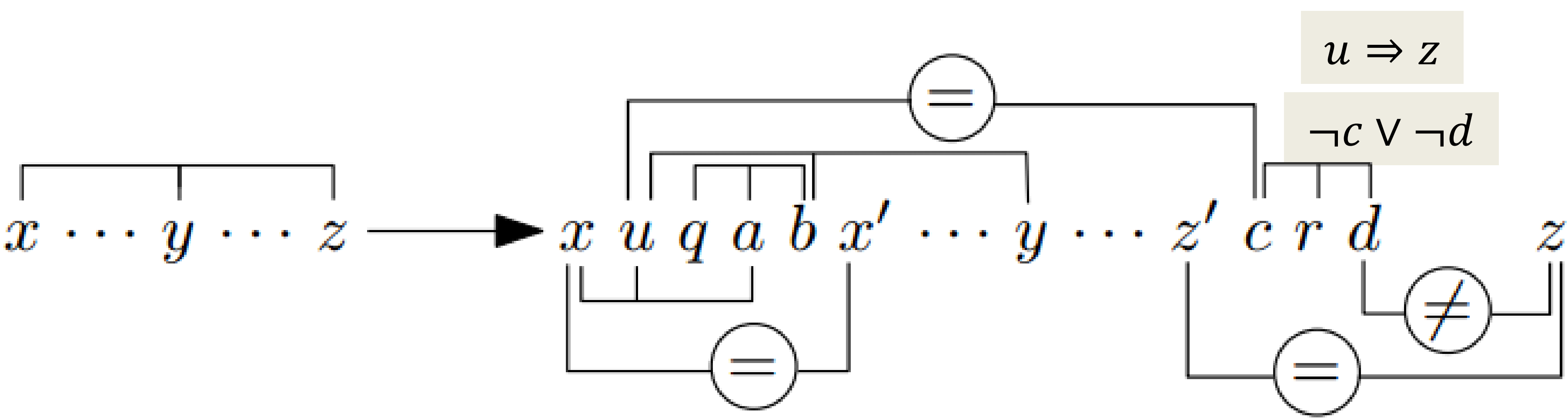
[Mulzer & Rote 2008]



Planar Positive Rectilinear 1-in-3SAT

[Mulzer & Rote 2008]

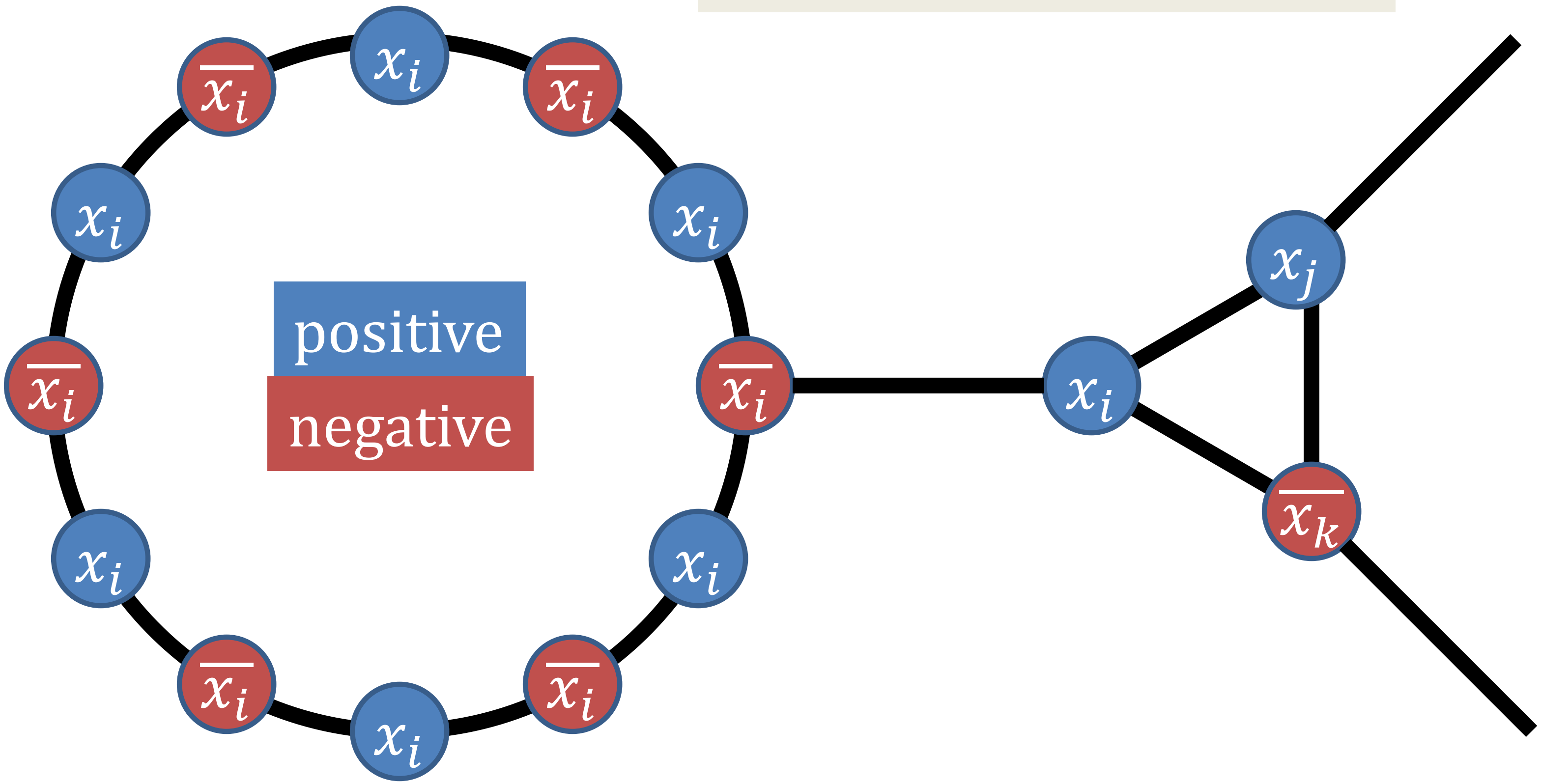




Planar NAE 3SAT is Polynomial

[Moret 1988]

reduction to Max Cut



cut size = $2n_i$

variable

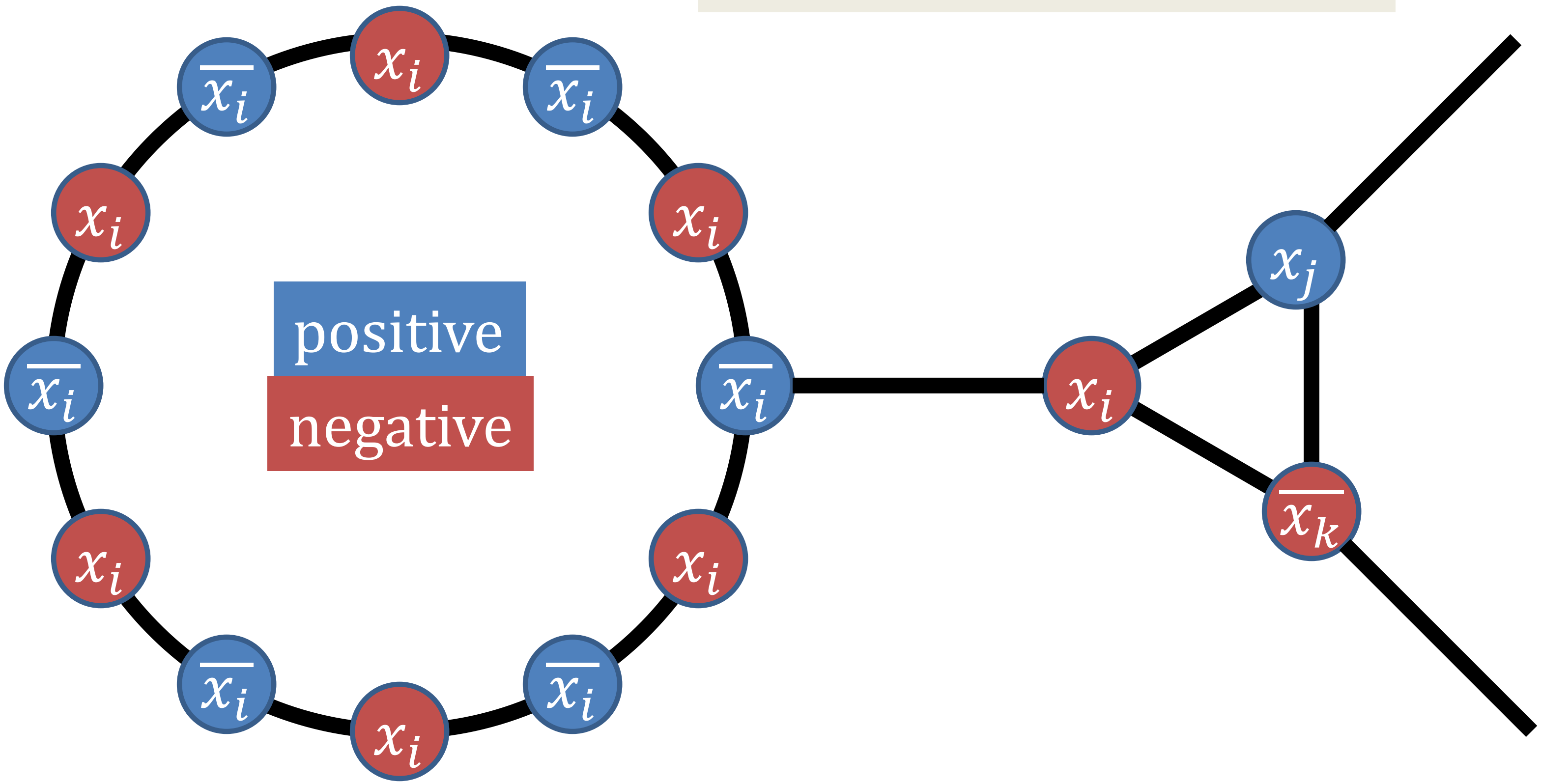
cut size = 5

NAE clause

Planar NAE 3SAT is Polynomial

[Moret 1988]

reduction to Max Cut



cut size = $2n_i$

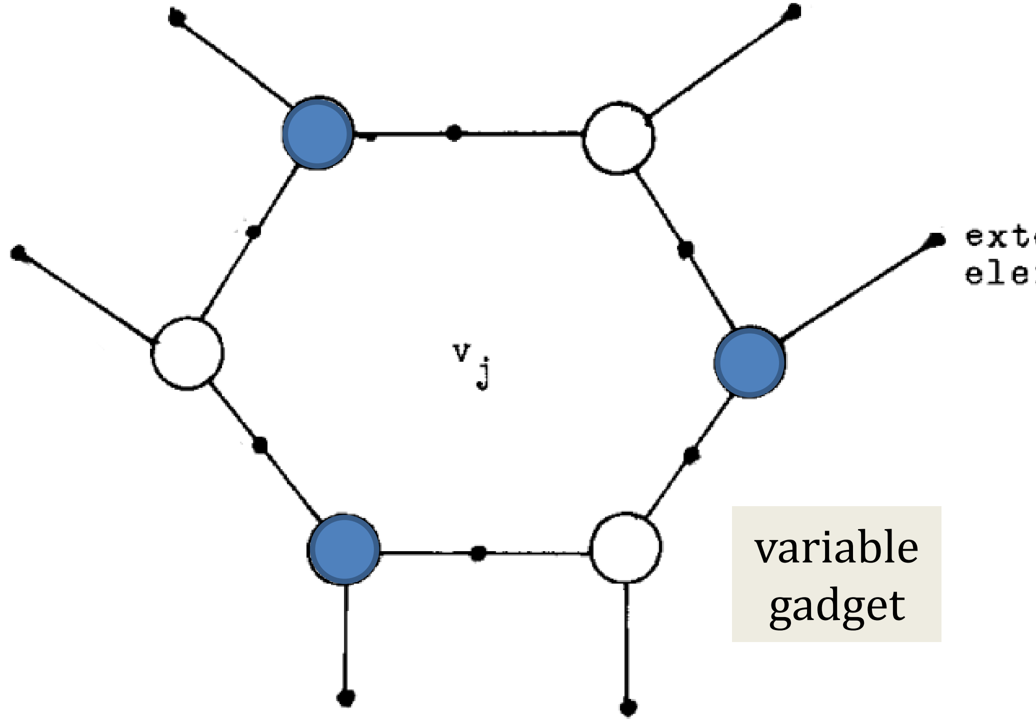
cut size = 5

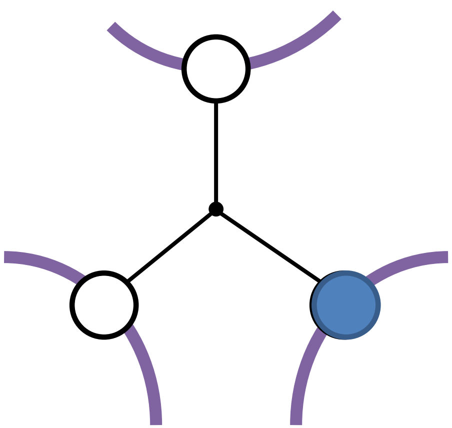
variable

NAE clause

Planar X3C

[Dyer & Freeze 1986]

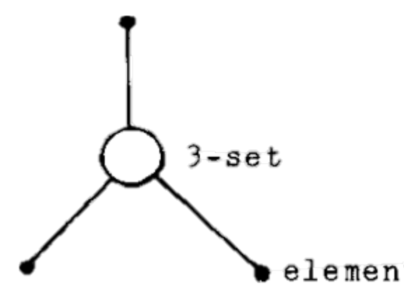




simple clause gadget

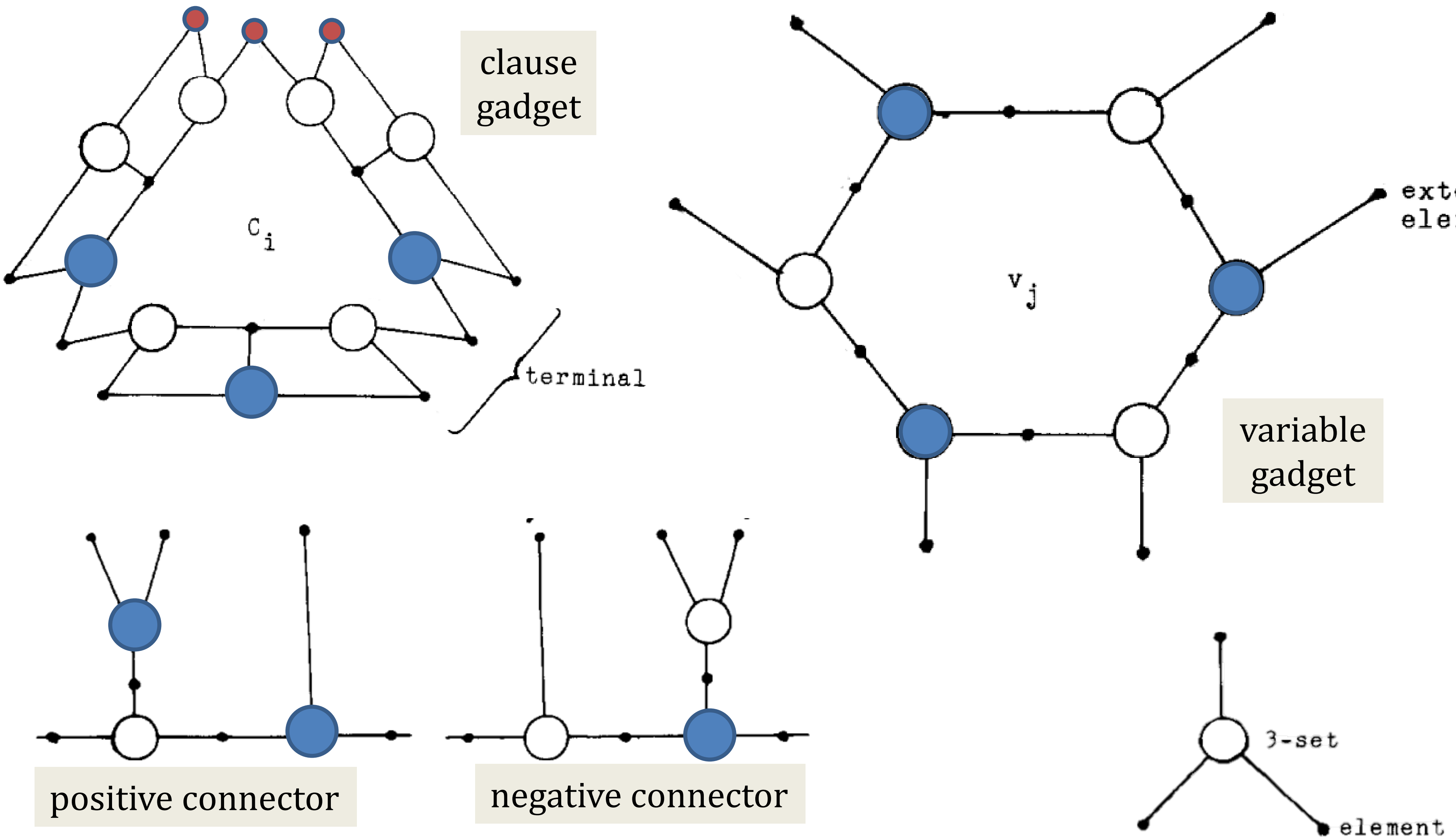
variable gadget

size of cover $= \sum_i \frac{1}{2} n_i$



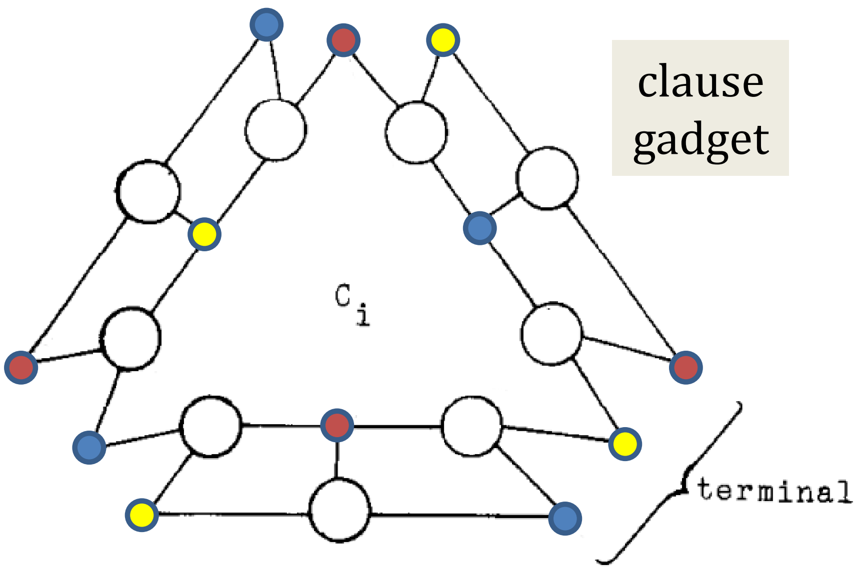
Planar X3C

[Dyer & Freeze 1986]

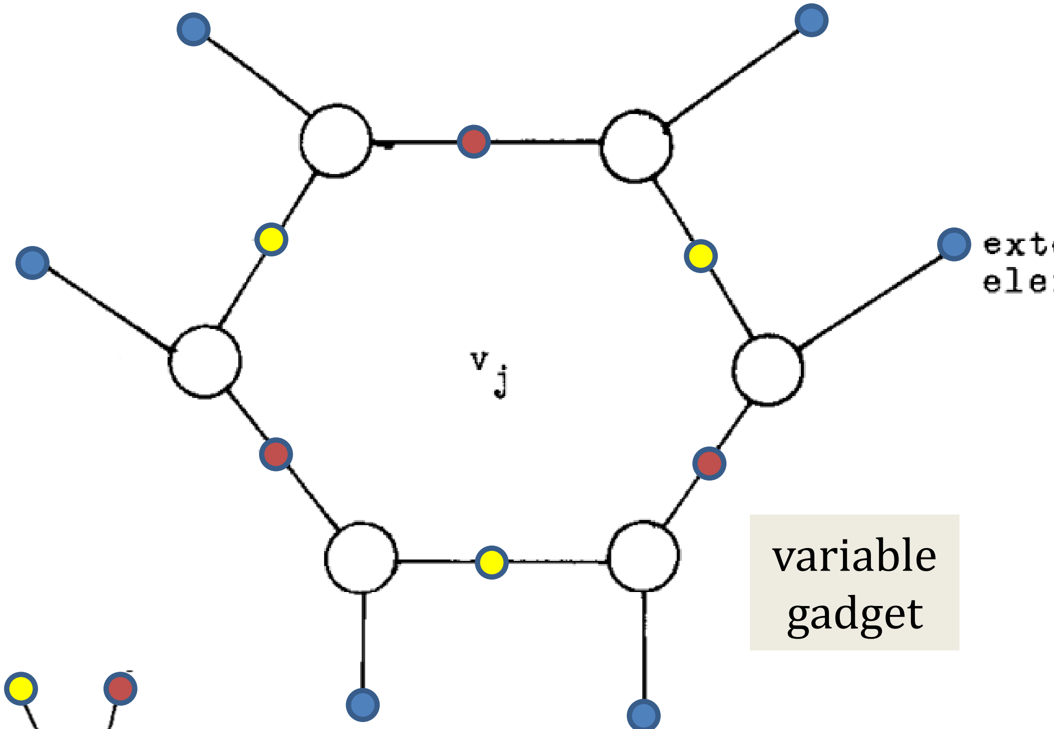


Planar 3DM

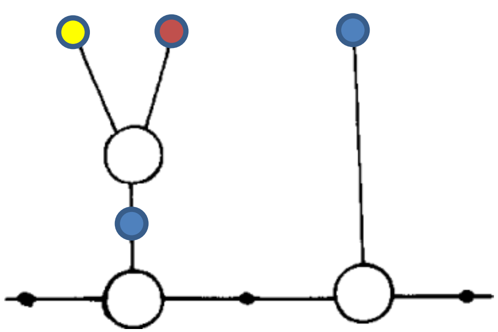
[Dyer & Freeze 1986]



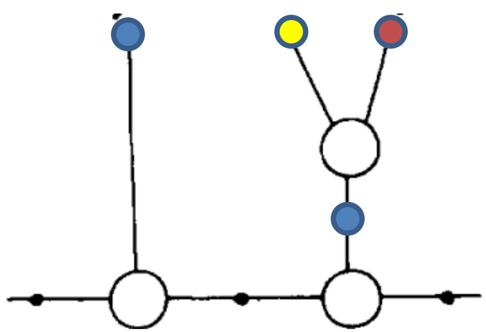
clause gadget



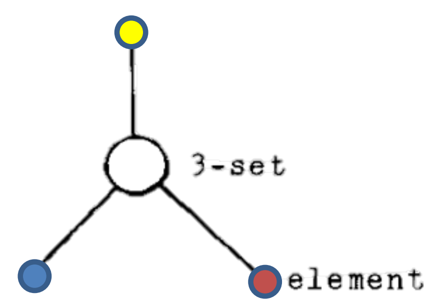
variable gadget



positive connector

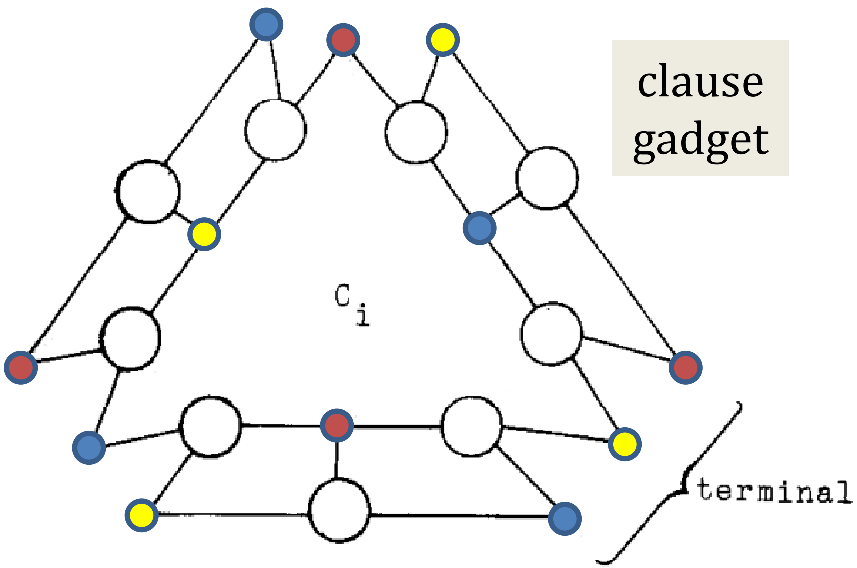


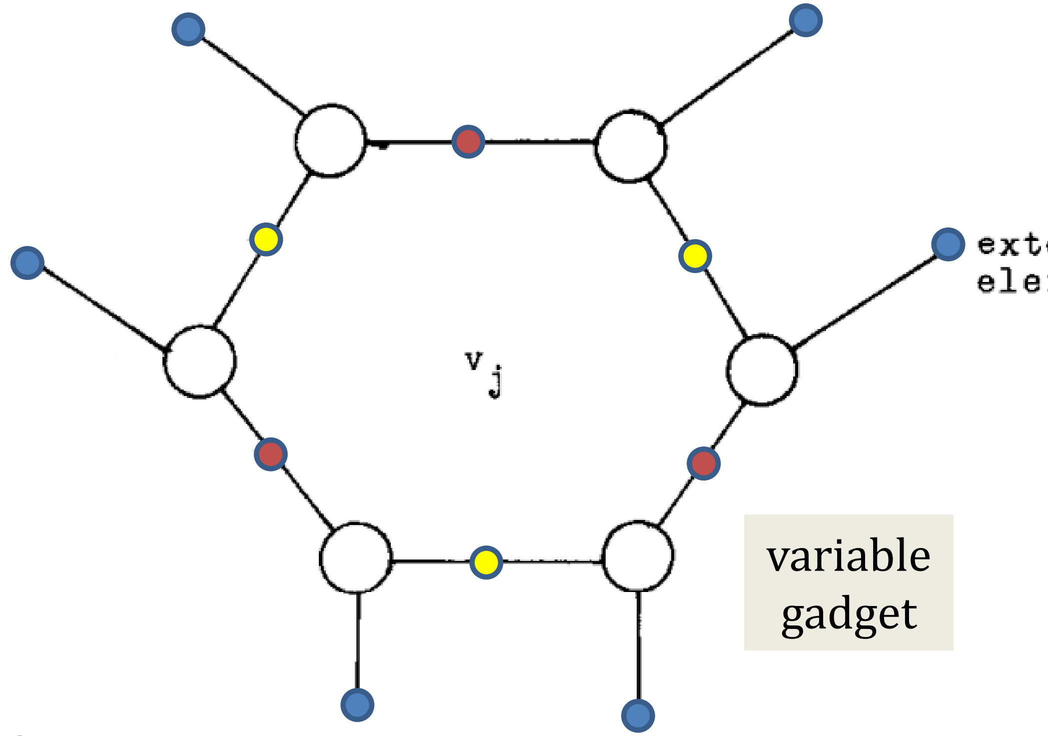
negative connector

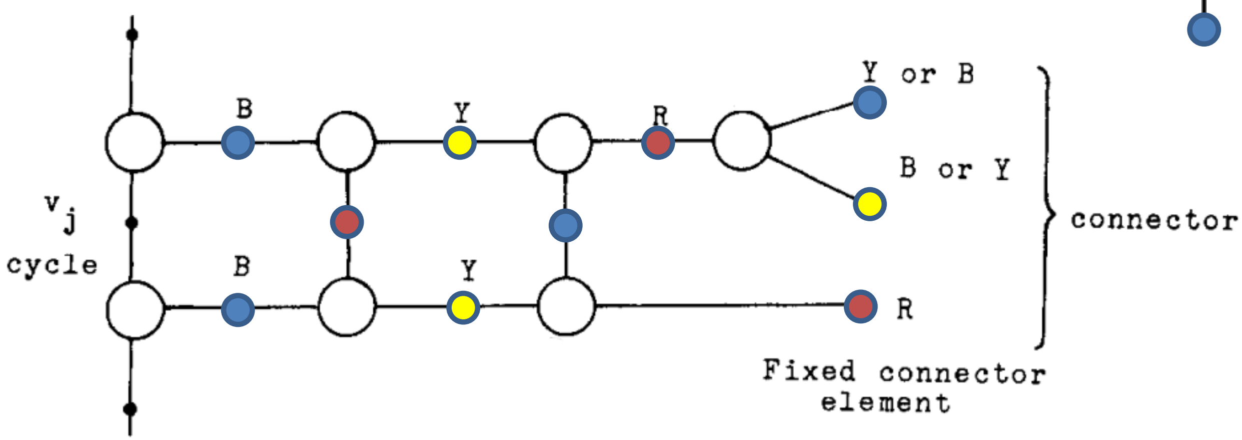


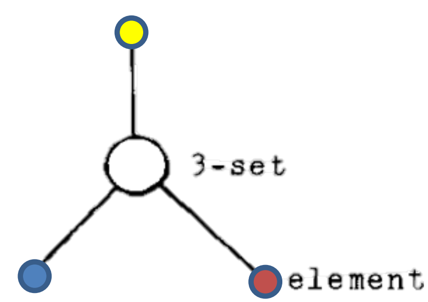
Planar 3DM

[Dyer & Freeze 1986]



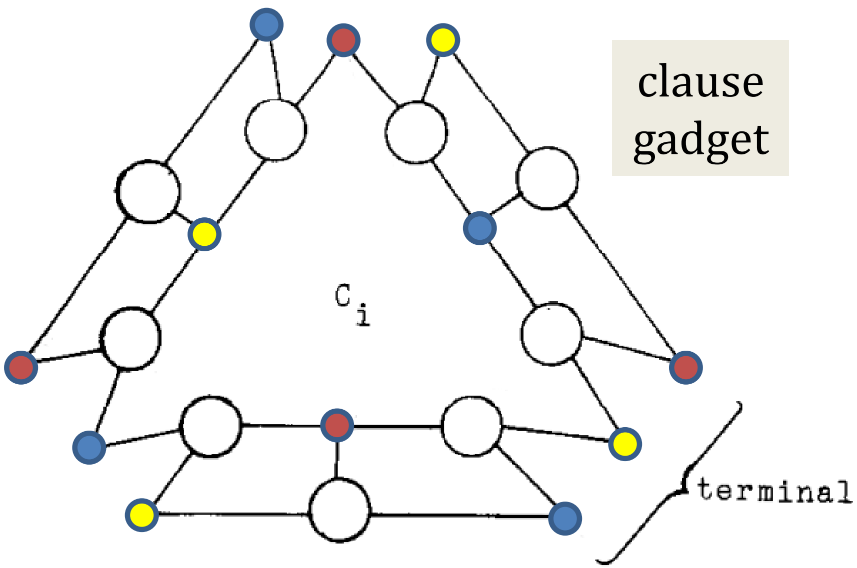


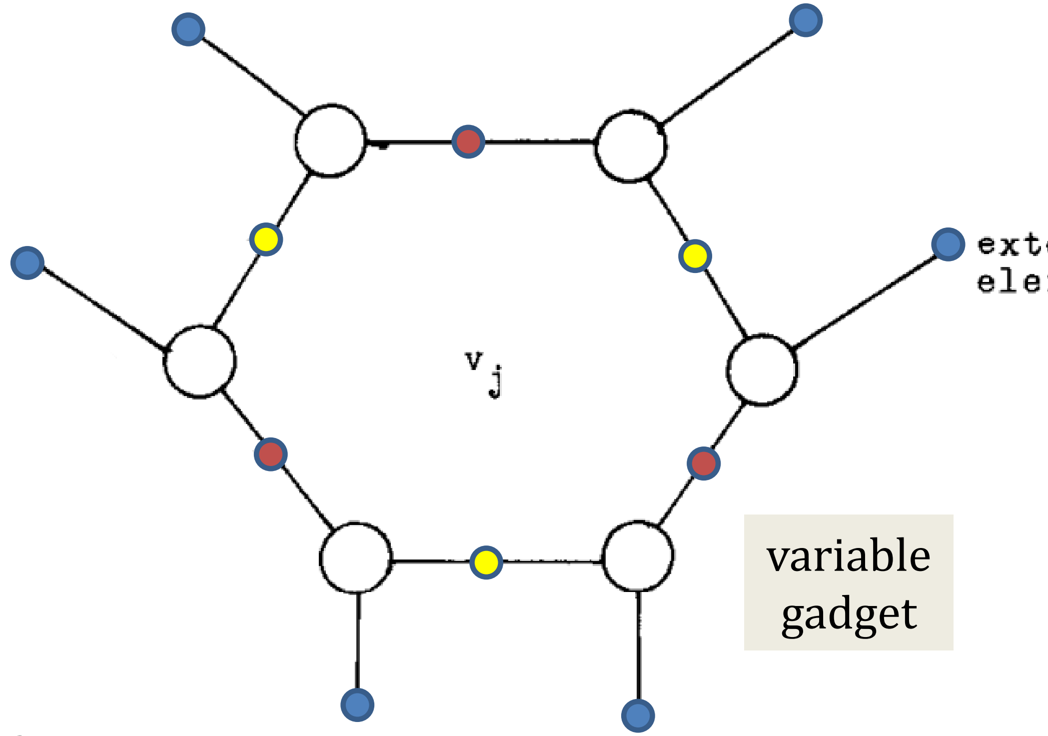


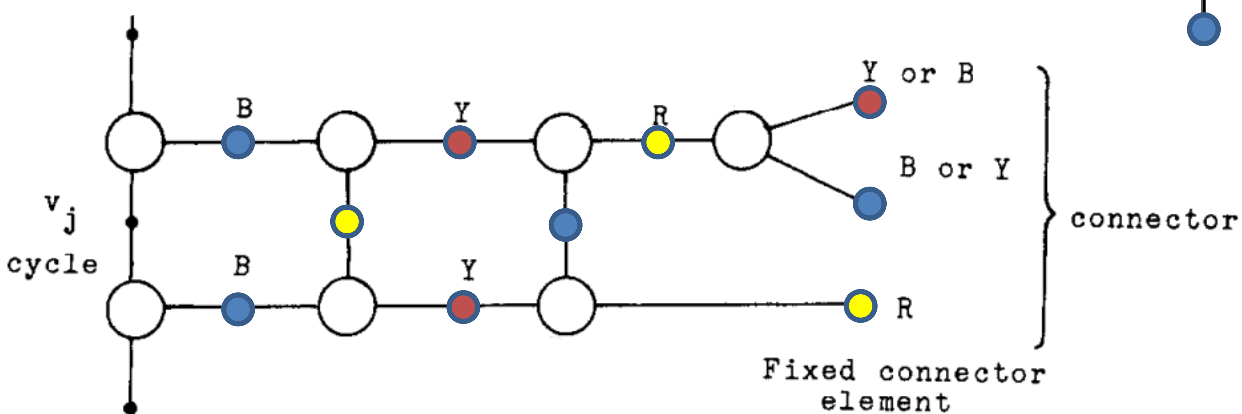


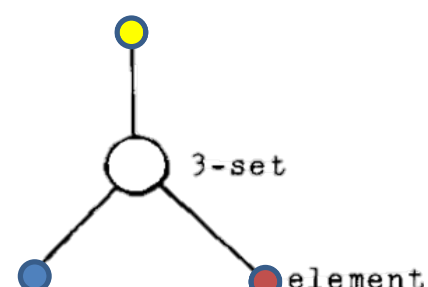
Planar 3DM

[Dyer & Freeze 1986]



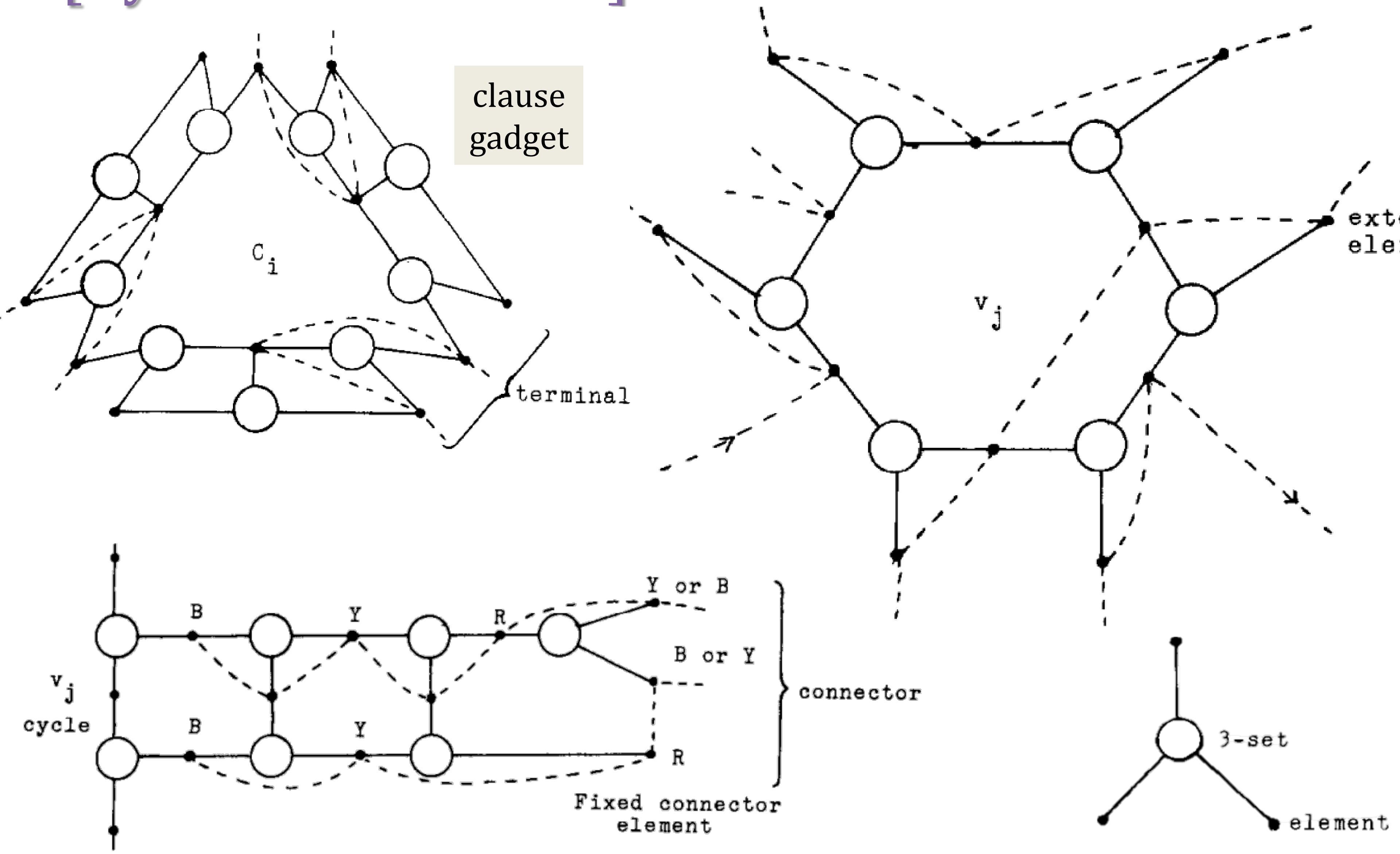






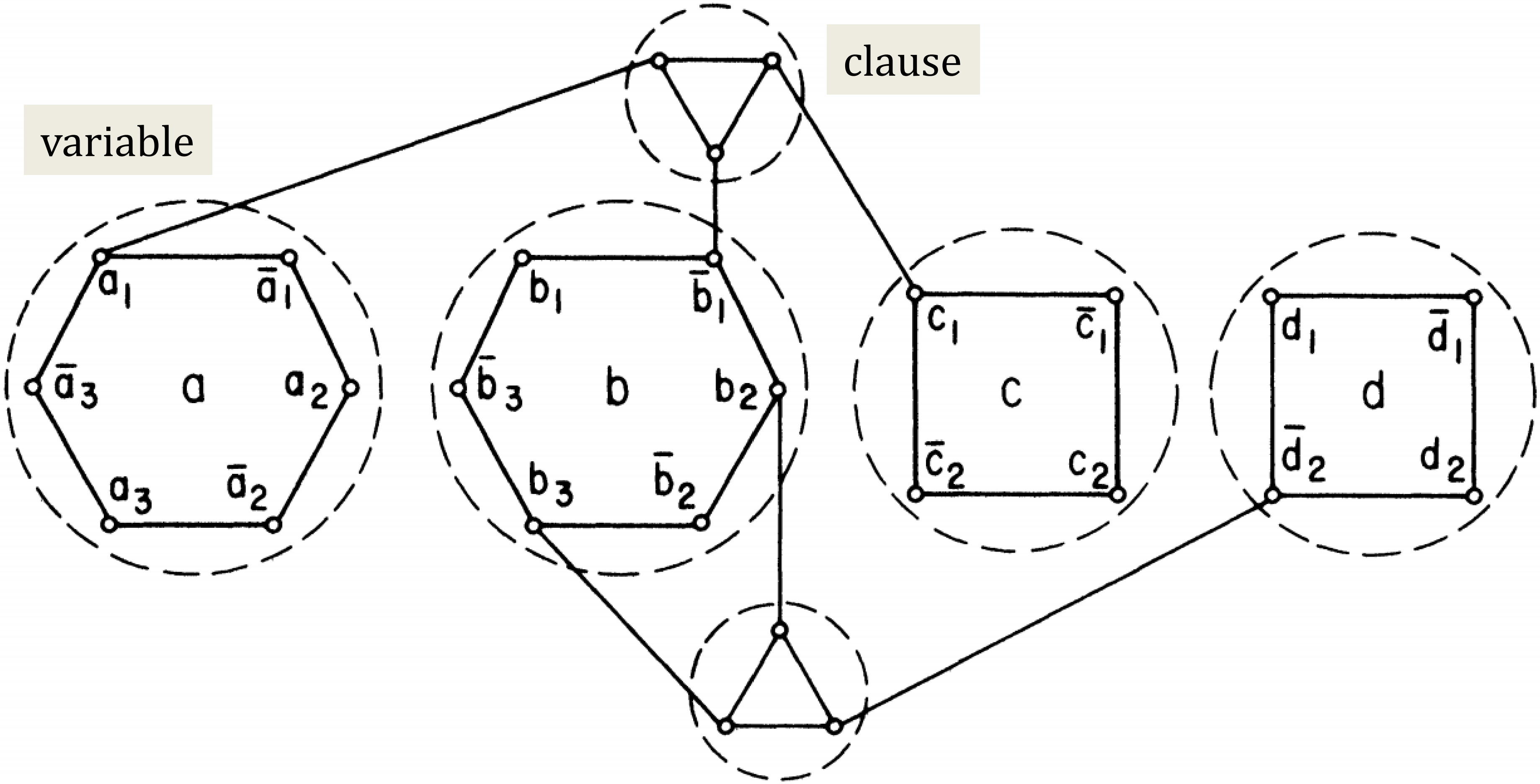
Planar 3DM with Element Cycle

[Dyer & Freeze 1986]



Planar Vertex Cover

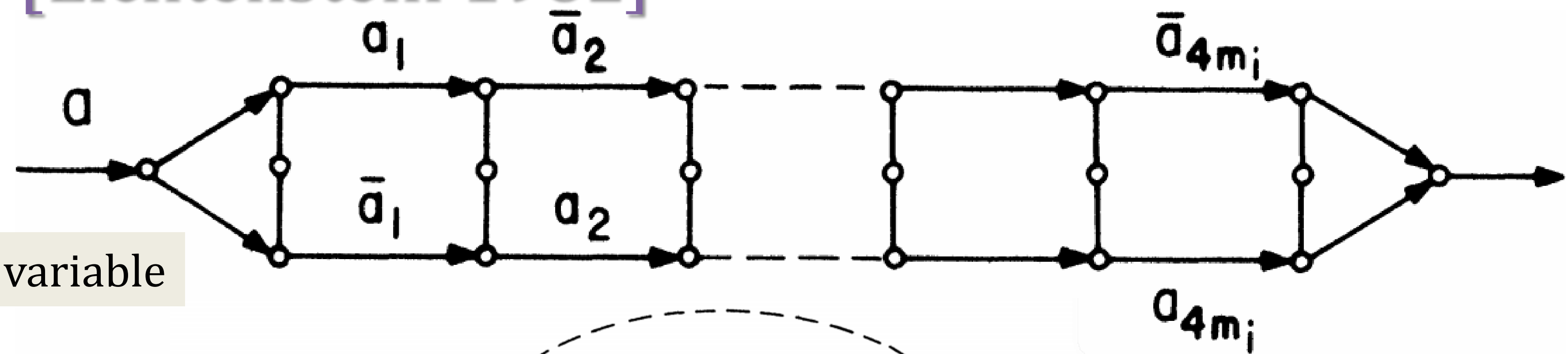
[Lichtenstein 1982]



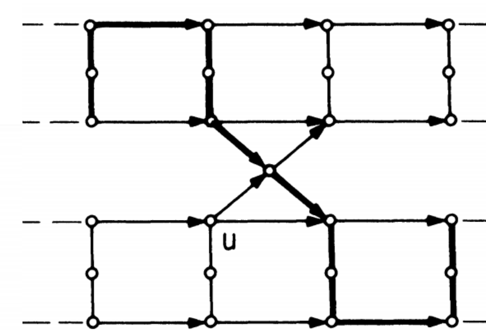
Example: $B = (a + \bar{b} + c)(b + b + \bar{d})$

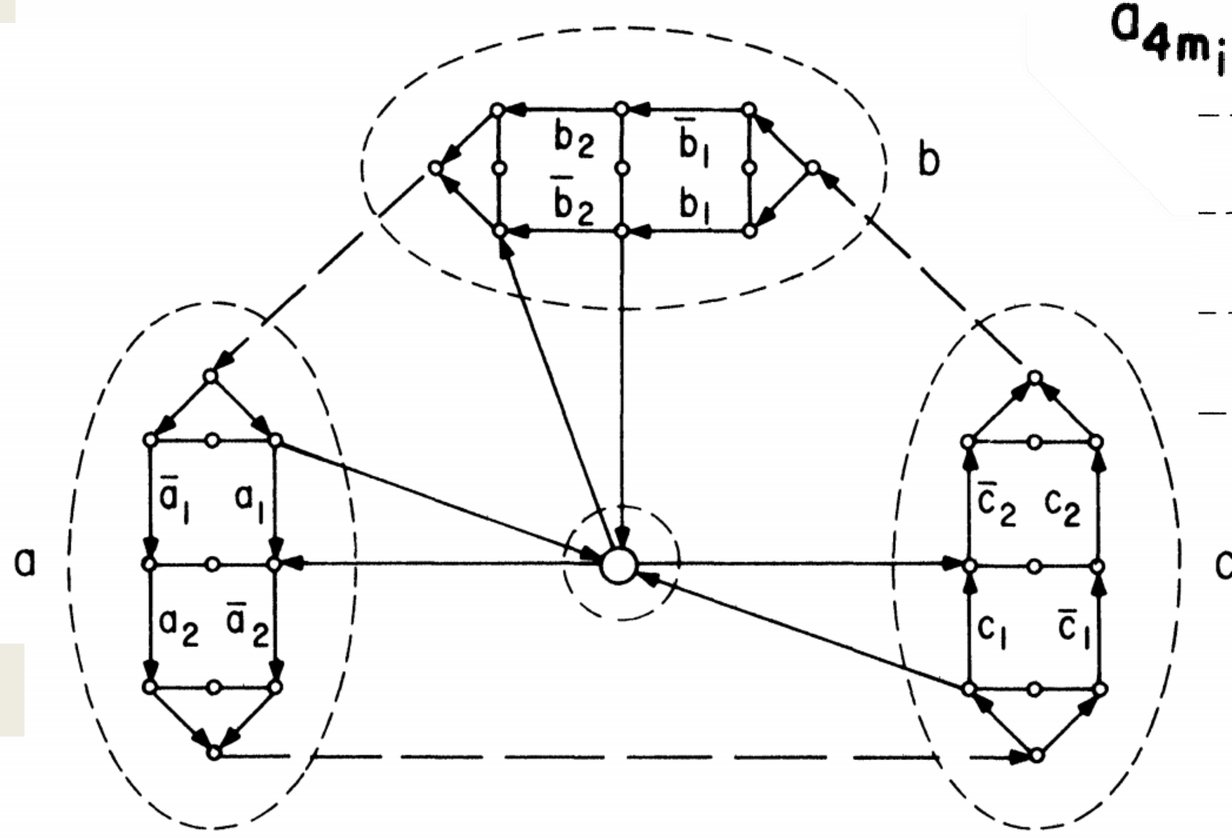
Planar (Directed) Hamiltonian Cycle

[Lichtenstein 1982]



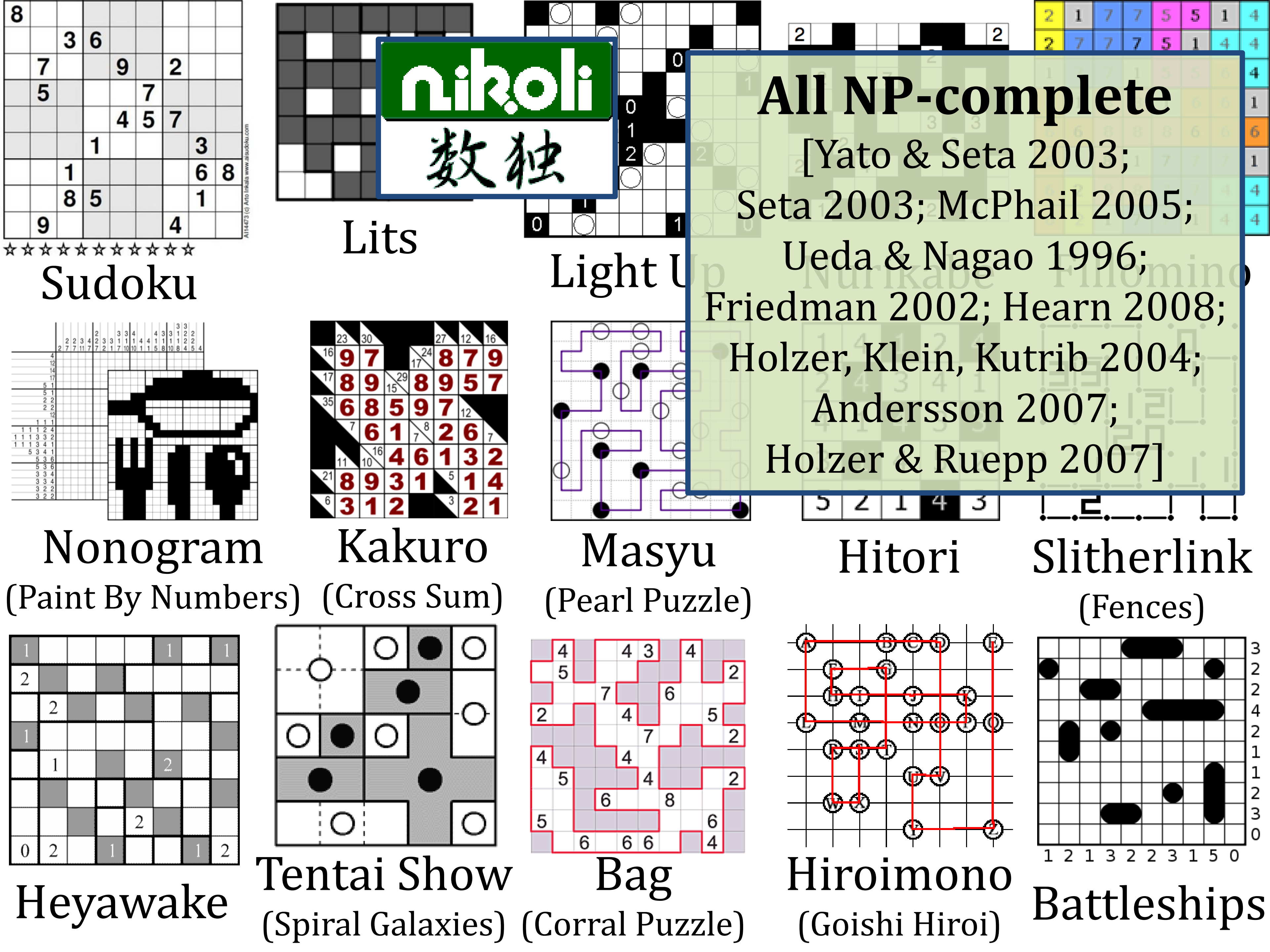
variable





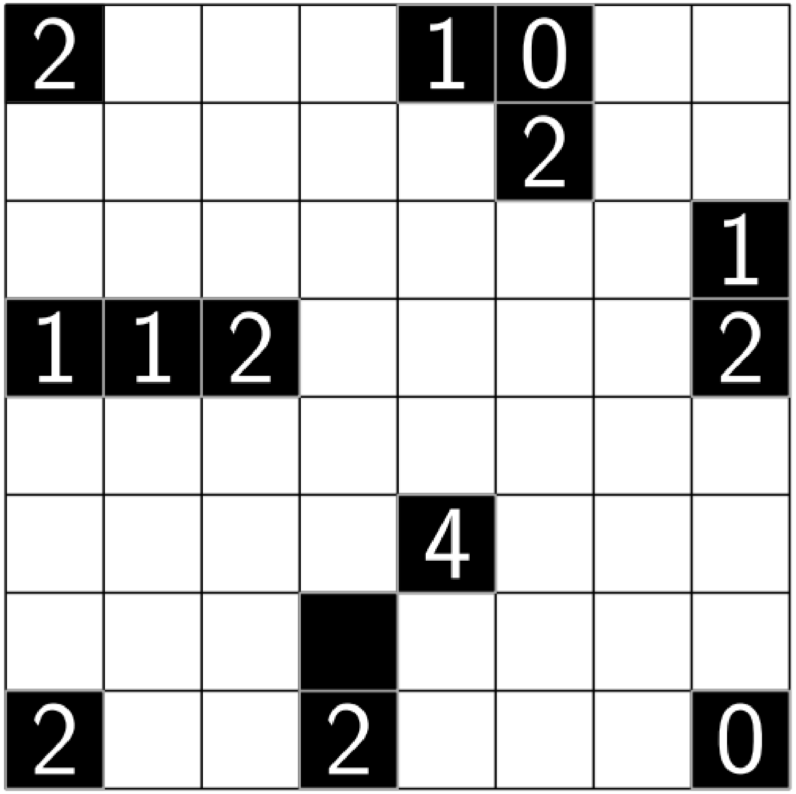
clause

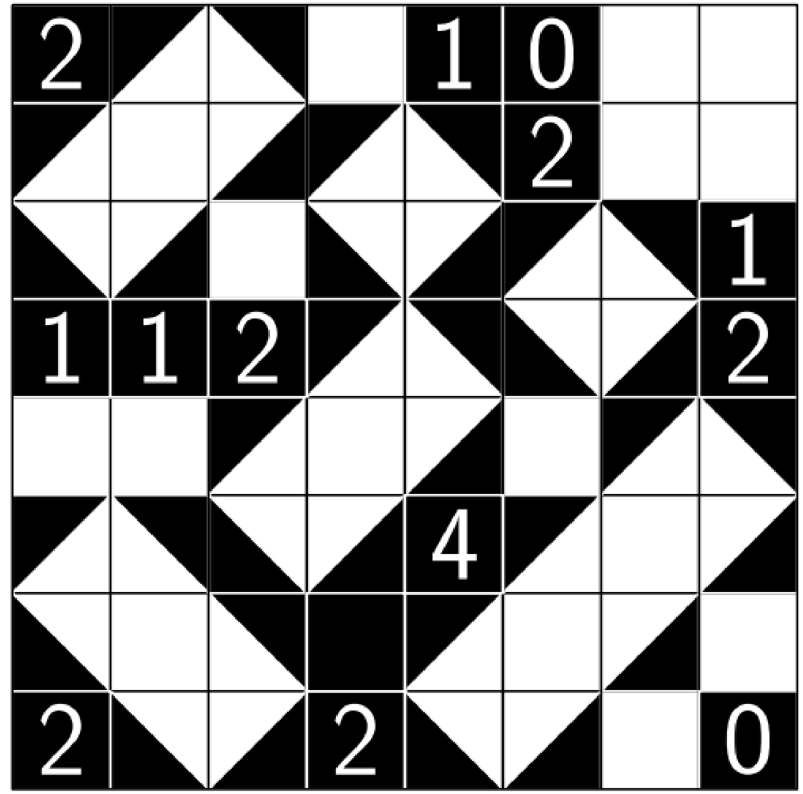
$a \vee \bar{b} \vee c$



All NP-complete
[Yato & Seta 2003;
Seta 2003; McPhail 2005;
Ueda & Nagao 1996;
Friedman 2002; Hearn 2008;
Holzer, Klein, Kutrib 2004;
Andersson 2007;
Holzer & Ruepp 2007]
niko-li
数独
Sudoku
Lits
Light Up
Nurikabe
Filomino
Nonogram
(Paint By Numbers)
Kakuro
(Cross Sum)
Masyu
(Pearl Puzzle)
Hitori
Slitherlink
(Fences)
Heyawake
Tentai Show
(Spiral Galaxies)
Bag
(Corral Puzzle)
Hiroimono
(Goishi Hiroi)
Battleships

Shakashaka [Guten 2008; Nikoli 2012–]



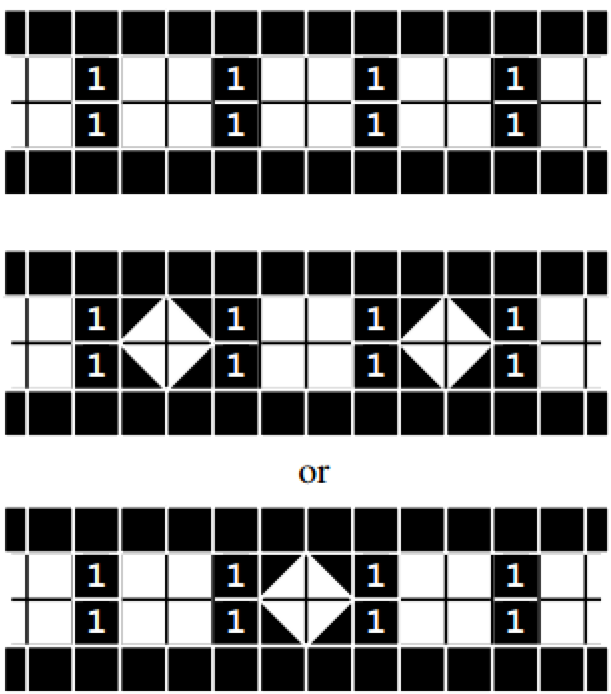


- White squares
- Black squares
  - Labeled 0, 1, 2, 3, 4, or nothing

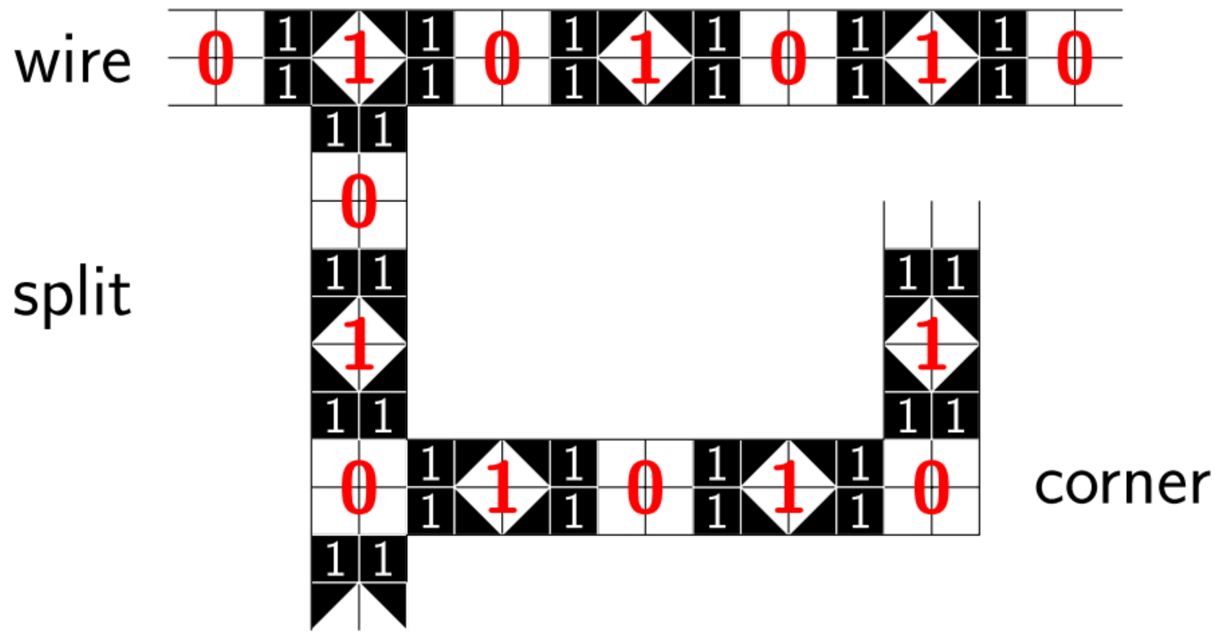
- Half-fill some white squares
- Labels specify number of half-filled neighbors
- Rectangular white regions

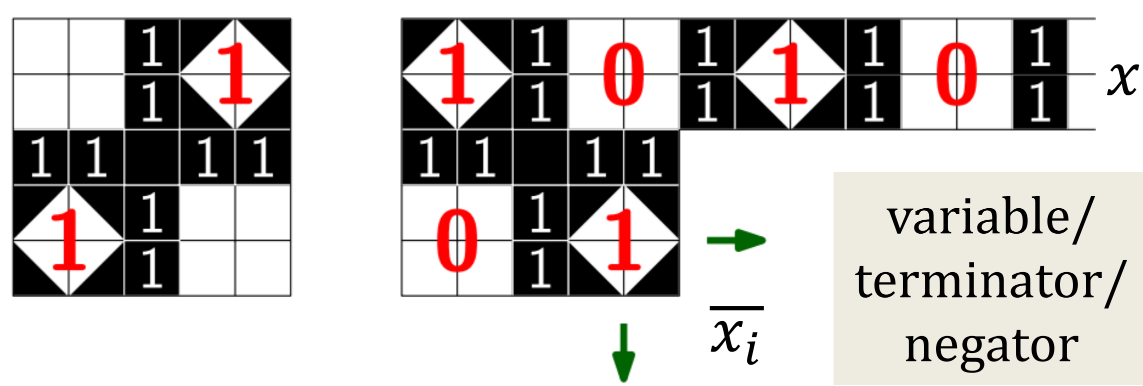
Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]



wire





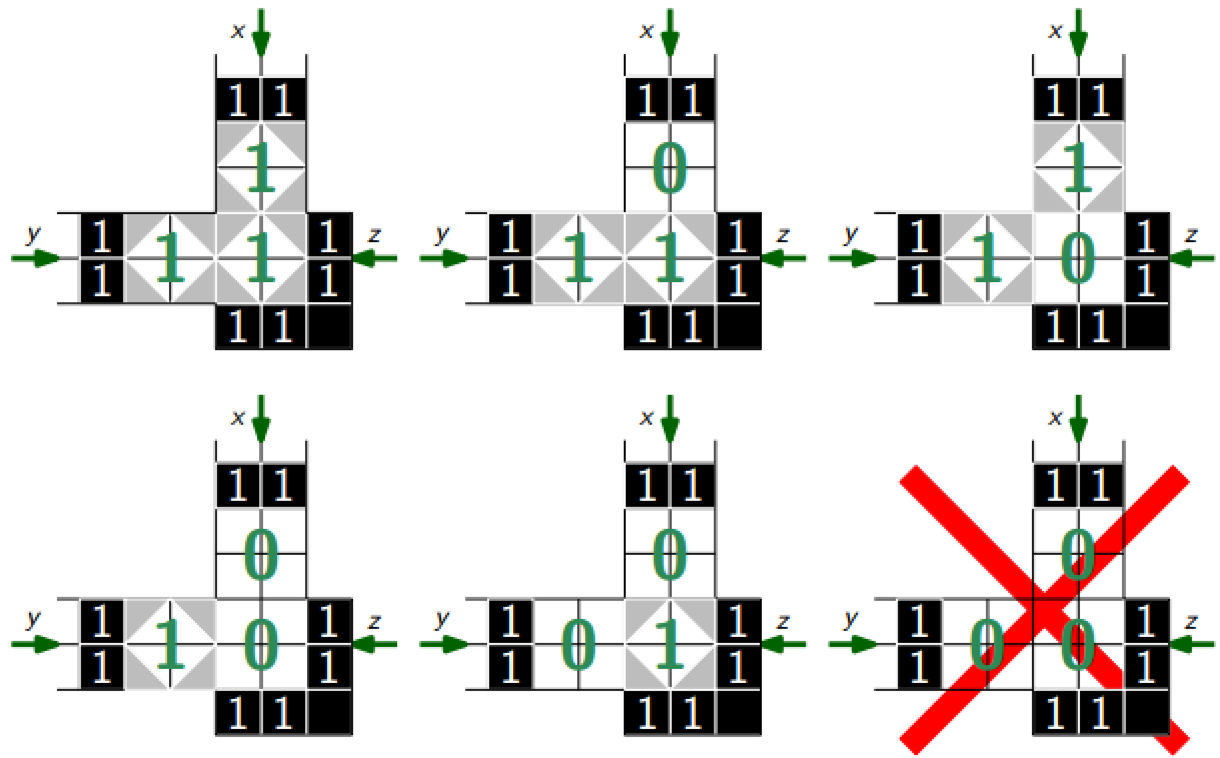
variable/
terminator/
negator

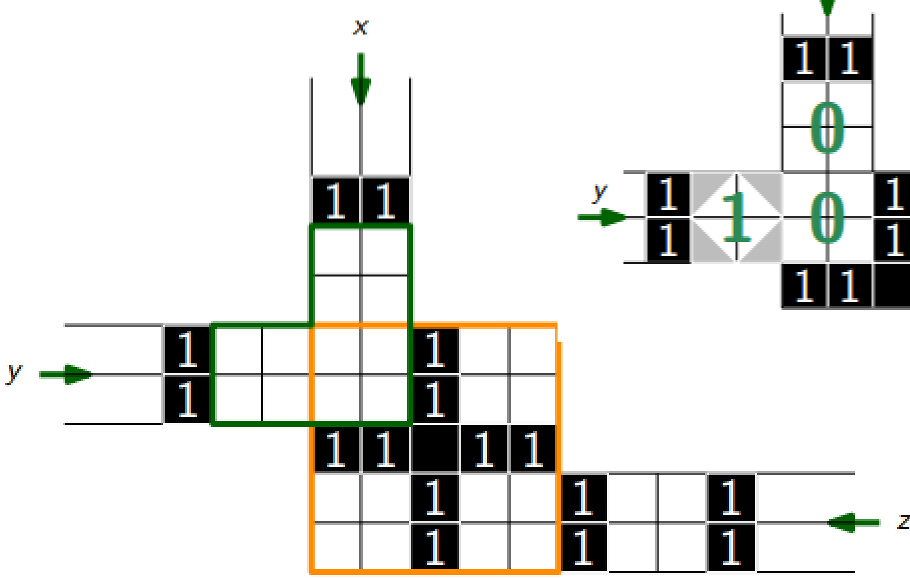
Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]

clause

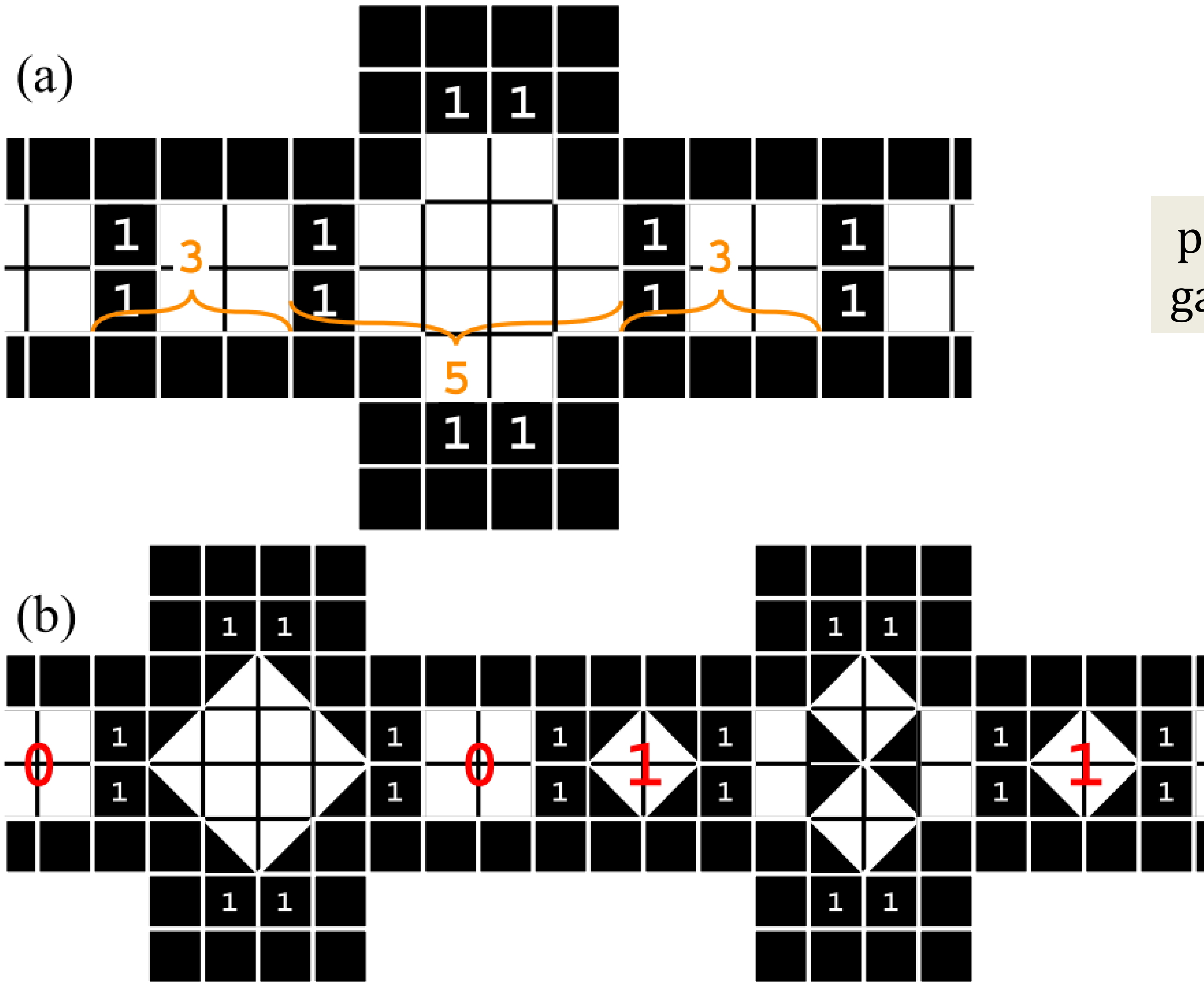
$x \vee y \vee z$





Shakashaka is NP-complete

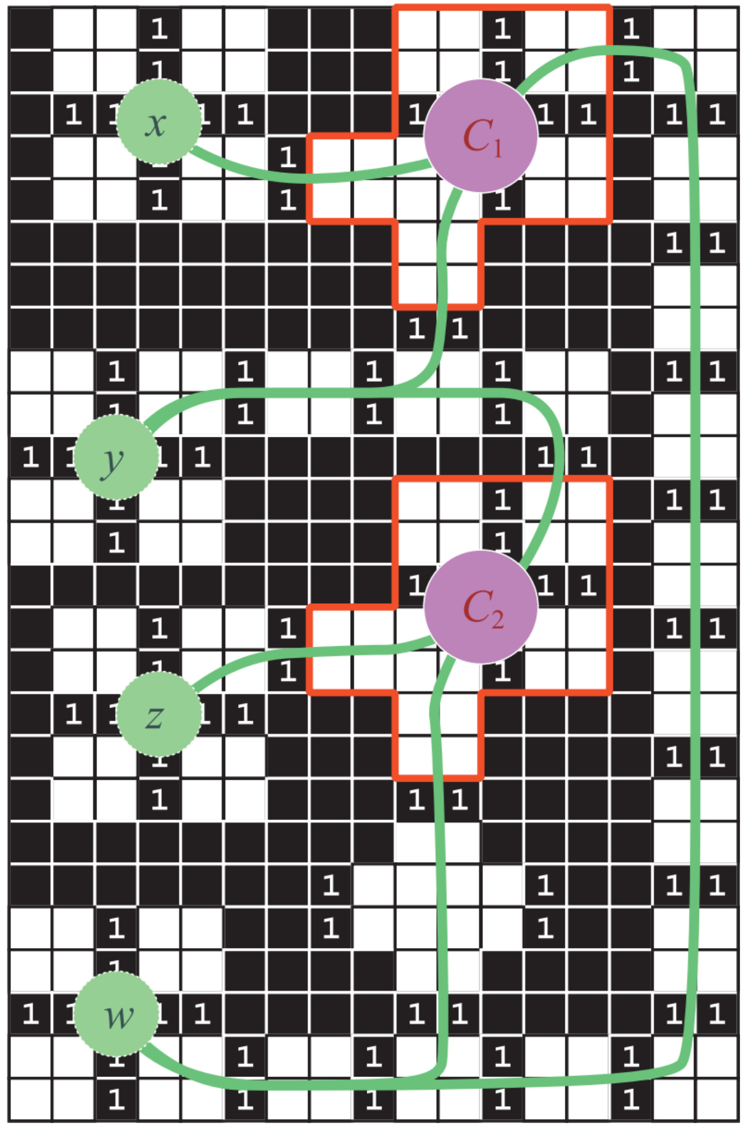
[Demaine, Okamoto, Uehara, Uno 2013]



parity gadget

Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]



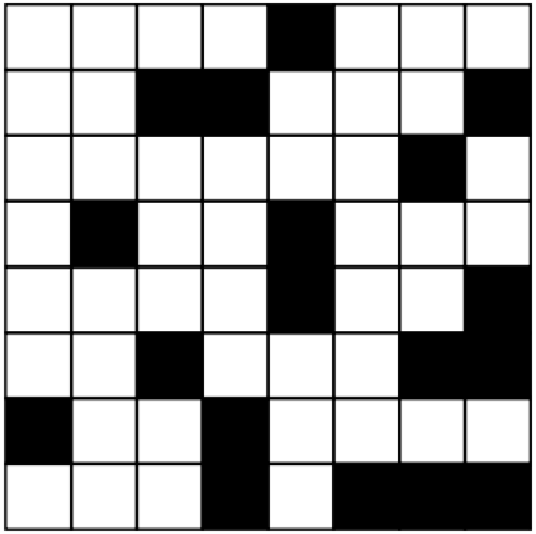
Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]

- Integer programming can solve small examples

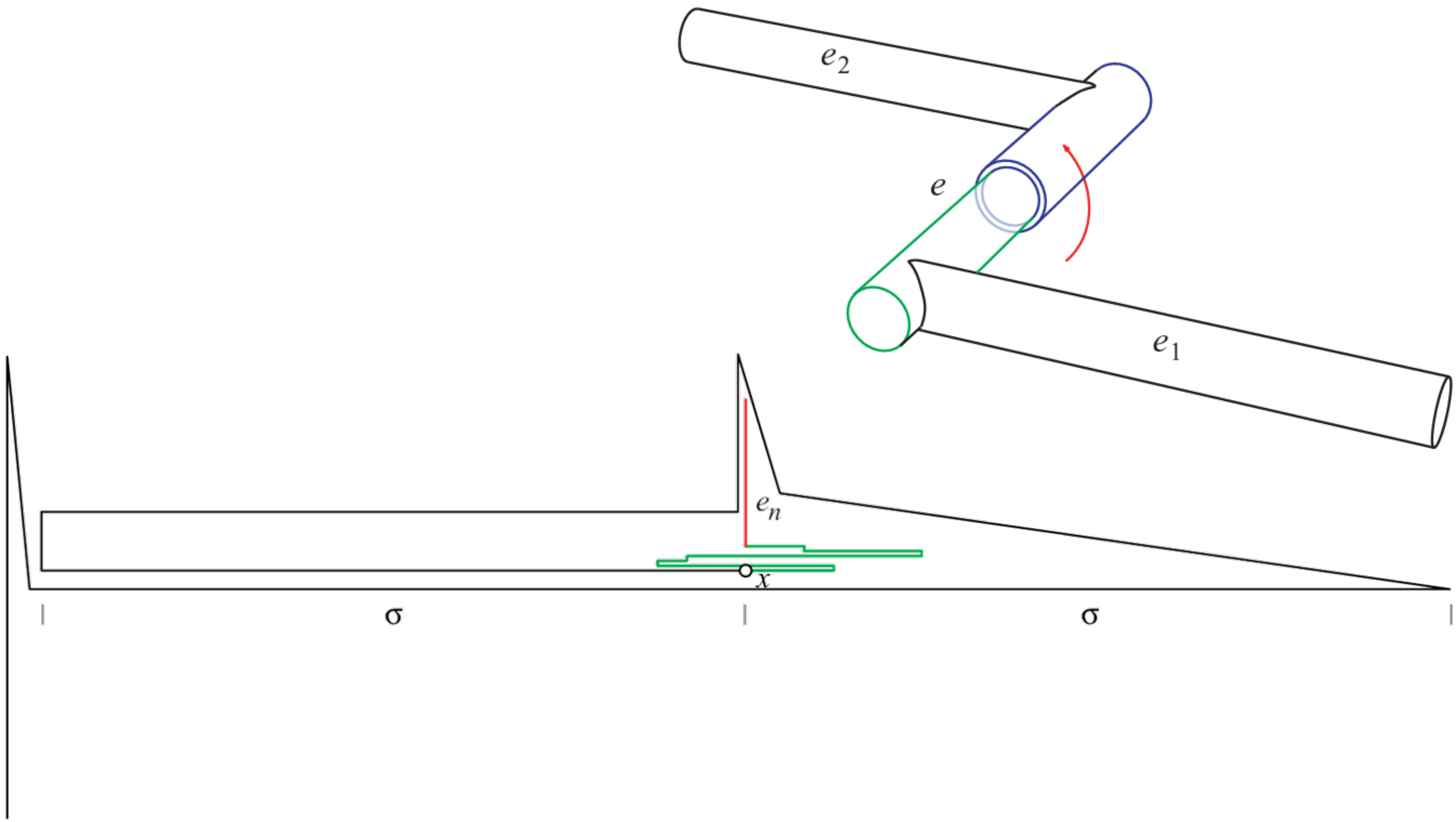
Problem	Size	Level	# of white squares	Time (sec)
1	10×10	Easy	76	0.02
2	10×10	Easy	77	0.03
3	10×10	Easy	82	0.03
4	10×18	Easy	131	0.07
5	10×18	Medium	156	0.09
6	10×18	Medium	144	0.07
7	14×24	Medium	297	0.21
8	14×24	Hard	295	0.19
9	20×36	Hard	645	0.84
10	20×36	Hard	632	0.91

- <u>Open:</u> No labels



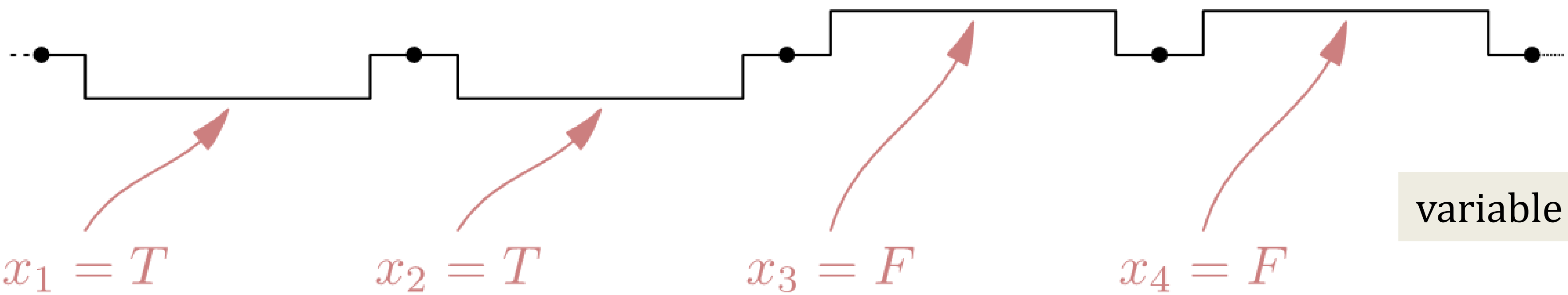
Flattening Fixed-Angle Chains

[Soss & Toussaint 2000]



Flattening Fixed-Angle Chains

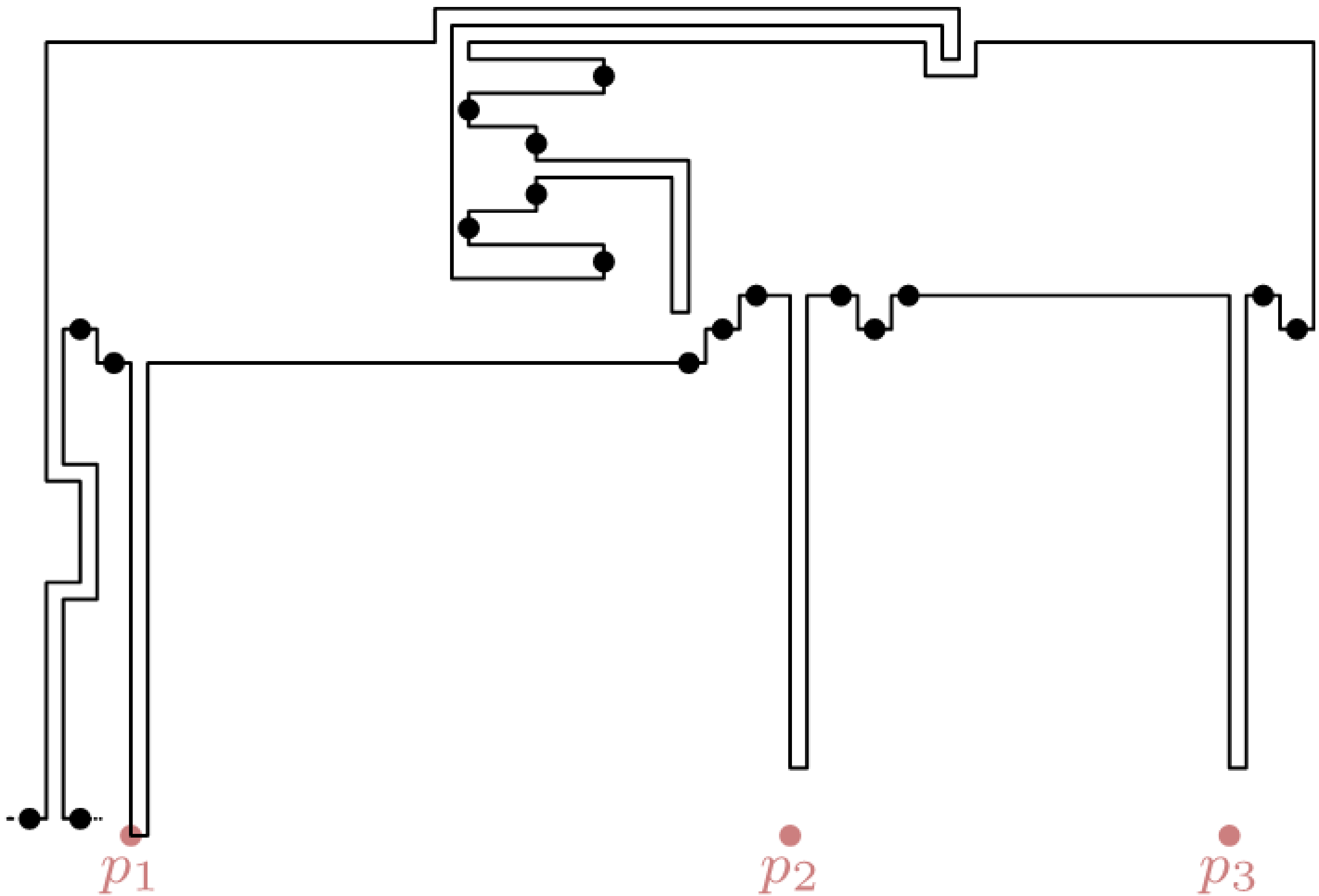
[Demaine & Eisenstat 2011]



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]

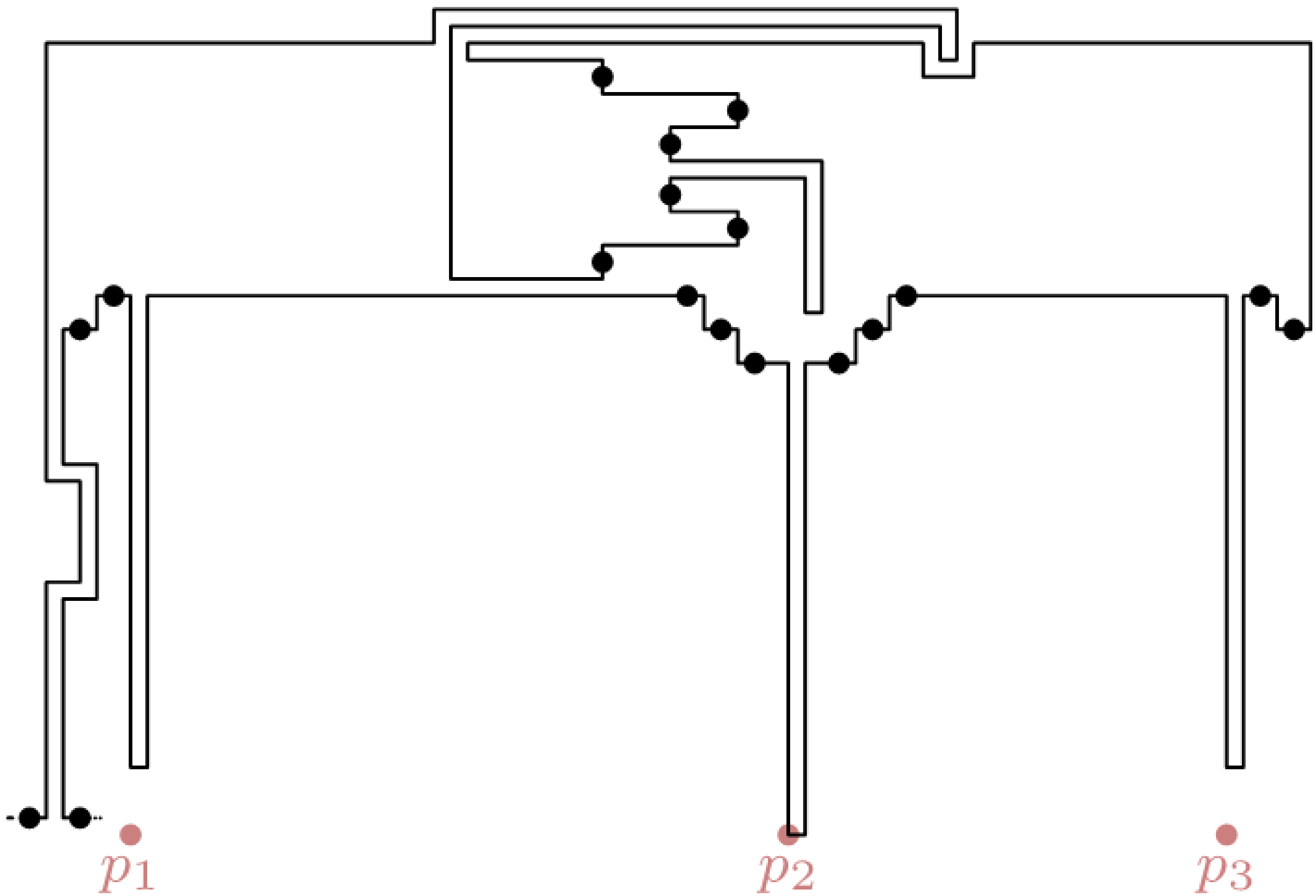
clause



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]

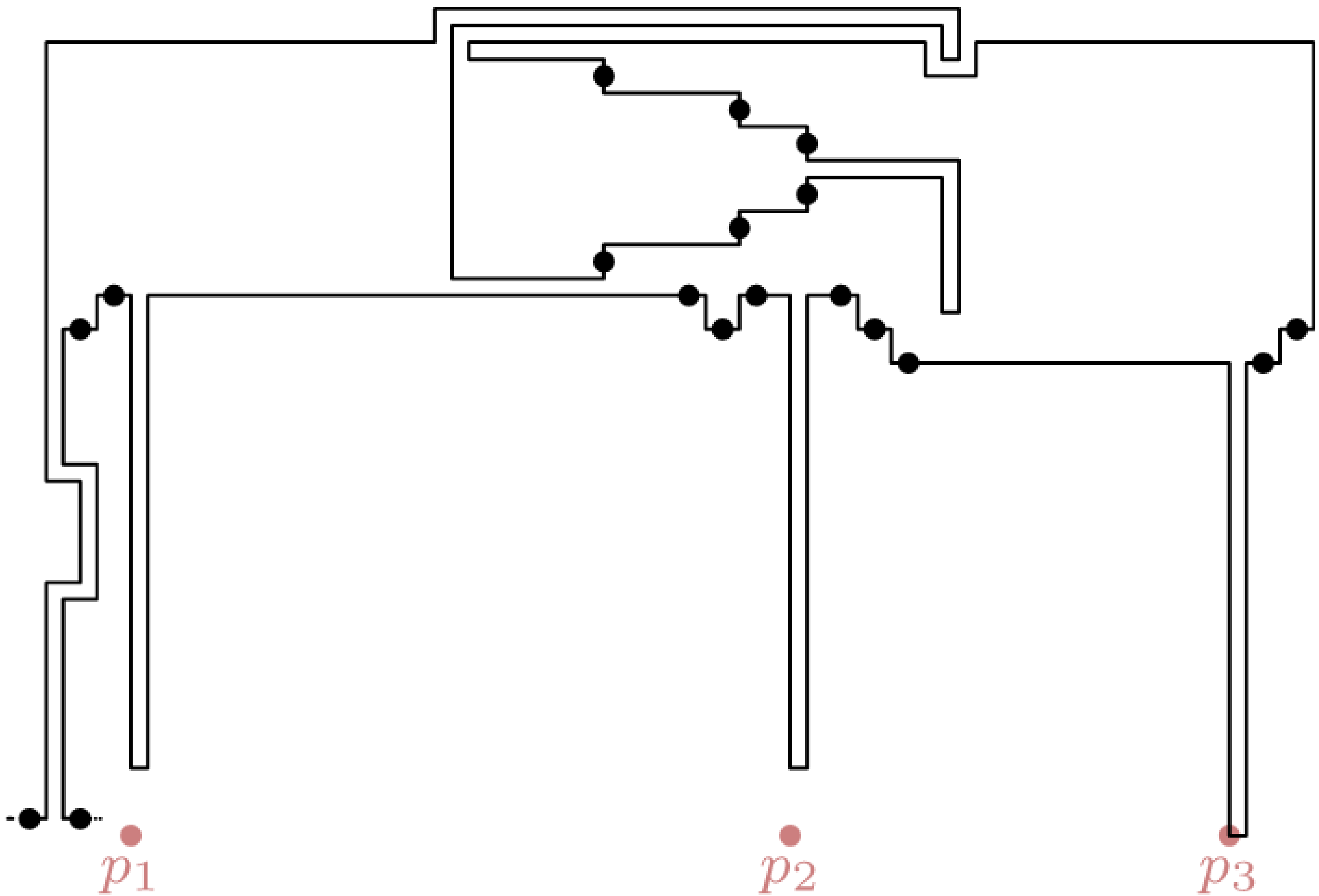
clause



Flat Folding of Fixed-Angle Chains

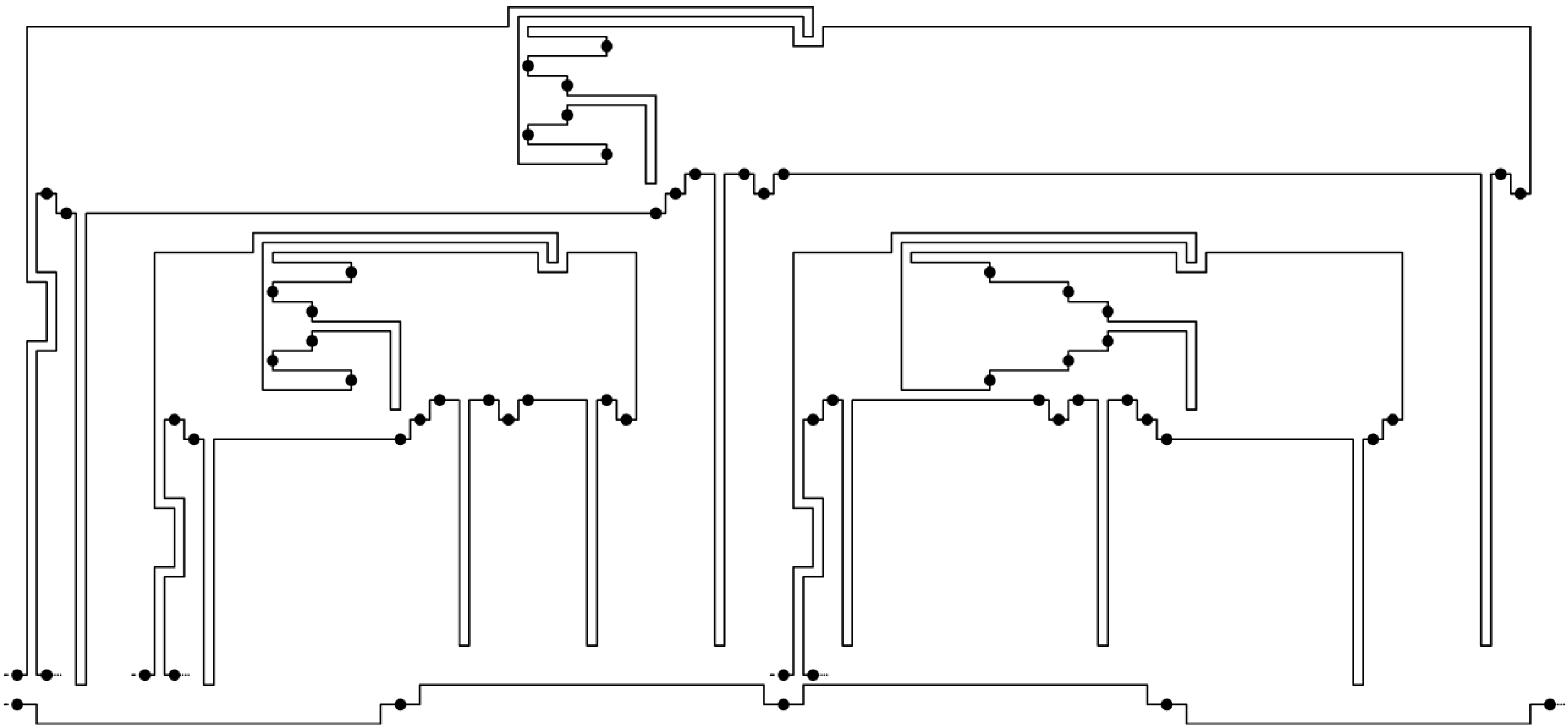
[Demaine & Eisenstat 2011]

clause



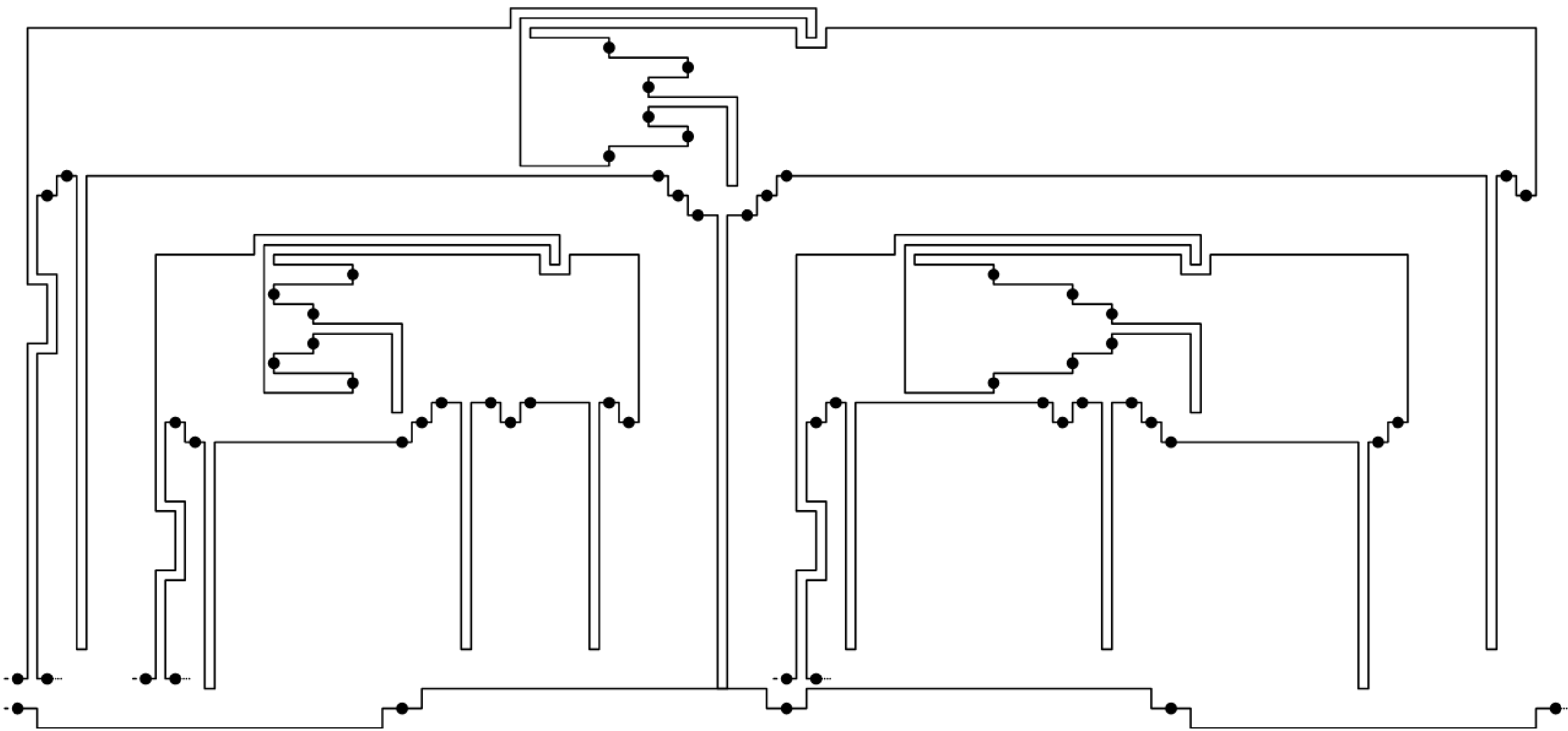
Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]



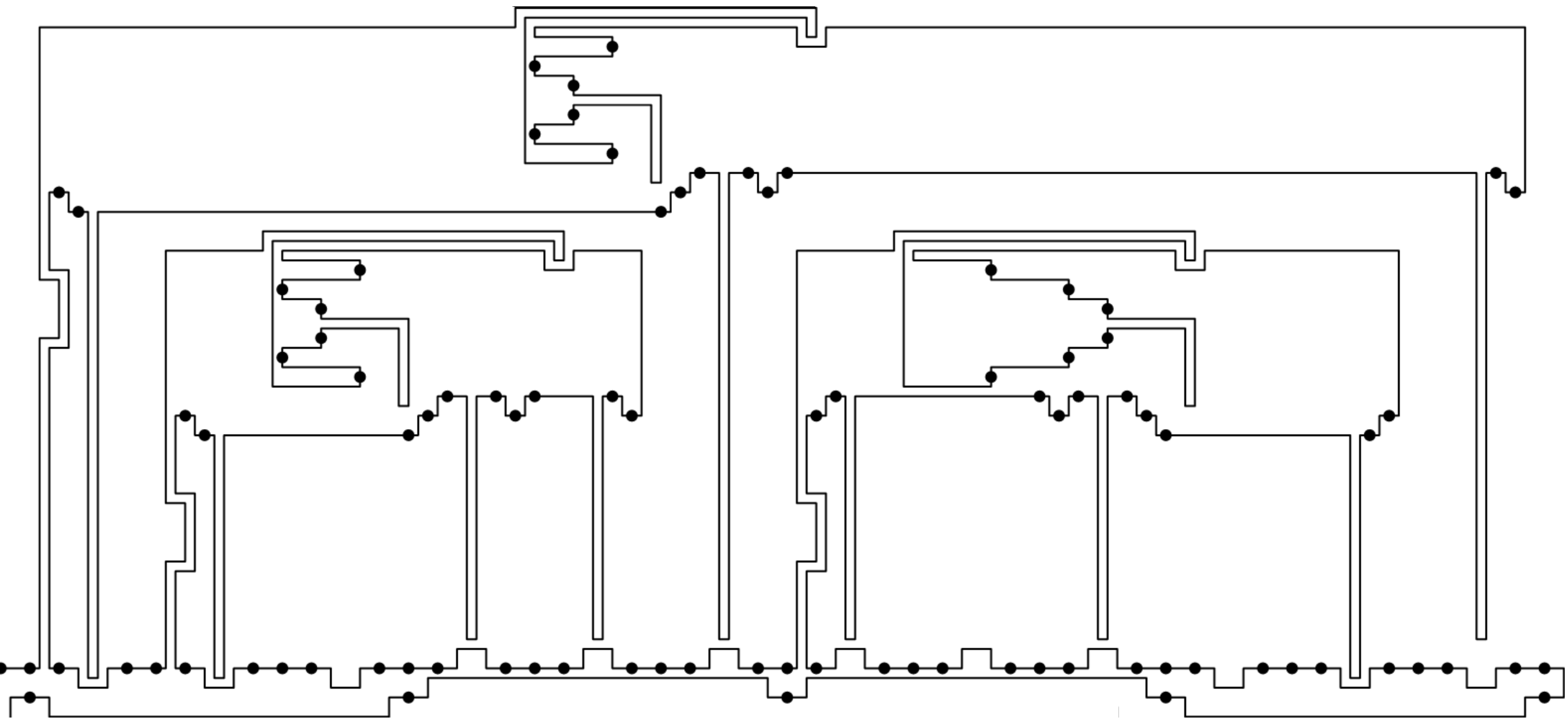
Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]



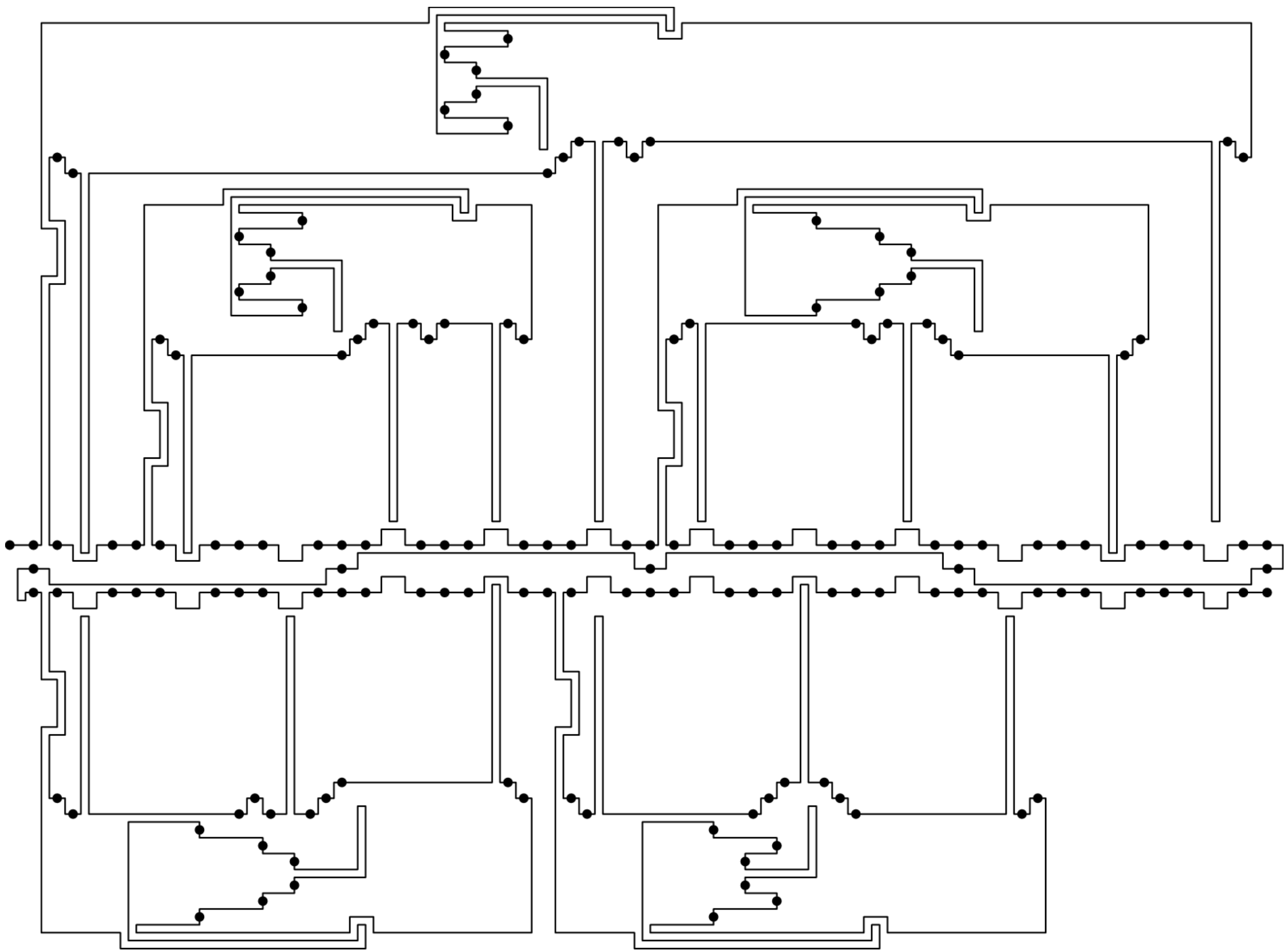
Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]

What We Just Proved

Variant Considered			
Linkage Type	**Edge Lengths**	**Angle Range**	**Result**
Fixed-angle **chain**, but **only some edges can spin**	Equilateral	$\{90°, 180°\}$	Strongly NP-hard
Fixed-angle **chain** (all edges can spin)	Equilateral	$[16.26°, 180°]$	Strongly NP-hard
Fixed-angle **chain** (all edges can spin)	$\Theta(1)$	$[60 - \varepsilon, 180°]$	Strongly NP-hard
Fixed-angle **caterpillar tree**	Equilateral	$\{90°, 180°\}$	Strongly NP-hard