Super Mario Bros. is NP-Hard
[Aloupis, Demaine, Guo, Viglietta 2014]

\[(x \lor \neg y \lor z) \land (x \lor y \lor \neg y) \land (\neg x \lor \neg y \lor \neg z) \land (\neg x \lor \neg y \lor \neg z)\]
Super Mario World is NP-Hard

[Aloupis, Demaine, Guo 2012]
Super Mario Bros. is NP-Hard
[Aloupis, Demaine, Guo, Viglietta 2014]

- **Glitch:** Wall jump
Super Mario Bros. is NP-Hard
[Aloupis, Demaine, Guo, Viglietta 2014]

- **Glitch:** Wall jump
Super Mario Bros. is NP-Hard
[Aloupis, Demaine, Guo, Viglietta 2014]

- **Glitch:** Jump through walls
Super Mario Bros. is NP-Hard
[Aloupis, Demaine, Guo, Viglietta 2014]

- **Glitch**: Jump through walls
Legend of Zelda Block Pushing

The Legend of Zelda (NES) Walkthrough Part 9 - Level 7 Demon Dungeon
by Zeldajiggmin

http://youtu.be/rxjeGXhwkqI
Legend of Zelda Block Pushing

Let's Play Zelda: A Link to the Past #12 – Hookshot
by newfiebangaa  http://youtu.be/ZznLKBYcvc0
Legend of Zelda is NP-hard
[Aloupis, Demaine, Guo 2012]
Legend of Zelda Hookshot

Legend of Zelda is NP-hard
[Aloupis, Demaine, Guo 2012]

A Link to the Past

variable clause crossover
Metroid NES 100% Walkthrough Part 1 Alternate Route
by Timothy Cookson       http://youtu.be/INkHYcWvQag
Metroid is NP-complete
[Aloupis, Demaine, Guo, Viglietta 2014]
Donkey Kong Country is NP-hard
[Aloupis, Demaine, Guo 2012]

crossover

clause
Pokémon is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]
“Weak Trainers each hold a Level 100 Electrode with maximum Speed and equipped with only the Self Destruct move. Strong Trainers each hold two Snorlaxes, with Speed of 30. The player has no items, and only one Pokémon in his team. For Generation I and II games (Red/Blue/Yellow and Gold/Silver/Crystal versions respectively), the player holds a Gastly which has learned Self Destruct using TM36, and its PP for its other moves have all been expended, so it can only use Self Destruct in battle. When the player encounters a weak Trainer, the enemy Electrode will move first and use Self Destruct, which deals no damage to Gastly since Self Destruct is a Normal type attack and Gastly is Ghost type, so the weak Trainer immediately loses. When the player encounters a strong Trainer, Gastly moves first and uses Self Destruct, causing the player to lose (even if it defeats the enemy Snorlax, the opponent holds another one). This implementation only works in Generations I and II since TM36 exists only in Generation I and the Time Capsule feature in Generation II allows a Gastly with Self Destruct to be traded from Generation I to Generation II. In Generations III, IV, and V, Gastly can be replaced by Duskull, which is allowed to learn the move Memento, which serves the same purpose as Self Destruct, via breeding.”
Conway’s Phutball (Philosopher’s Football)
Conway’s Phutball (Philosopher’s Football)
Conway’s Phutball
(Philosopher’s Football)
Phutball
[Conway]

- Mate in 1 is NP-complete
  [Demaine, Demaine, Eppstein 2000]
- PSPACE-hard
  [Dereniowski 2009]
- EXPTIME-complete?
Checkers Mate-in-1 is Polynomial

[Fraenkel, Garey, Johnson, Schaefer, Yesha 1978]
Checkers Mate-in-1 is Polynomial

[Fraenkel, Garey, Johnson, Schaefer, Yesha 1978]

jumps preserve $x$ & $y$ parity
Cryptarithms / Alphametics

\[
\begin{array}{cccc}
S & E & N & D \\
+ & M & O & R & E \\
\hline
M & O & N & E & Y
\end{array}
\]

S = 9, E = 5, N = 6, D = 7, M = 1, O = 0, R = 8, Y = 2

1979, pp. 178–200
Cryptarithmetic are Strongly NP-complete

\[ [\text{Eppstein 1987}] \]

\[
\begin{align*}
0 & \quad 0 \\
0 & \quad p \\
& \quad 0 \\
1 & \quad q \\
& \quad 0
\end{align*}
\]

variable gadget

\[
\begin{align*}
d_i & \quad 0 \\
y_i & \quad 0 \\
c_i & \quad y_i \\
b_i & \quad 0 \\
y_i & \quad 0 \\
a_i & \quad 0
\end{align*}
\]

\[
\begin{align*}
e_i & \quad 0 \\
d_i & \quad y_i \\
c_i & \quad y_i \\
b_i & \quad 0 \\
y_i & \quad 0 \\
a_i & \quad 0
\end{align*}
\]

\[
\begin{align*}
\bar{v}_i & \quad 0 \\
e_i & \quad z_i \\
d_i & \quad z_i \\
v_i & \quad 0 \\
z_i & \quad 0 \\
b_i & \quad 0
\end{align*}
\]

\[
\begin{align*}
v_i & = 2 \cdot 2a_i + C \equiv C \pmod{4} \\
\bar{v}_i & = 2 \cdot (2c_i + C) + 1 + C \\
& = 4c_i + 3C + 1 \equiv 1 - C \pmod{4}
\end{align*}
\]

clause gadget

\[
\begin{align*}
\nu_a & \quad \lor \quad \nu_b \lor \quad \nu_c
\end{align*}
\]

\[
\begin{align*}
u_{ab} & \quad 0 \\
u_a & \quad 0 \\
1 & \quad r_i \\
\nu_i & \quad 0 \\
g_i & \quad w_i \\
0 & \quad f_i \\
0 & \quad 0
\end{align*}
\]

\[
\begin{align*}
u_c & \quad 0 \\
v_b & \quad 0 \\
h_i & \quad r_i \\
\nu_i & \quad 0 \\
g_i & \quad w_i \\
0 & \quad f_i \\
0 & \quad 0
\end{align*}
\]

\[
\begin{align*}
t_i & \quad 0 \\
u_{ab} & \quad 0 \\
t_i & \quad s_i \\
0 & \quad h_i \\
x_i & \quad 0 \\
g_i & \quad 0
\end{align*}
\]

\[
\nu_a + \nu_b + \nu_c = t_i \in \{1, 2, 3\} \pmod{4}
\]
Cryptarithms are Strongly NP-complete

[Eppstein 1987, simplified]

\[ C = \text{carry of } 2y_i \in \{0,1\} \]

\[ b_i \ y_i \ 0 \ a_i \ 0 \]

\[ b_i \ y_i \ 0 \ a_i \ 0 \]

\[ v_i \ z_i \ 0 \ b_i \ 0 \]

\[ v_i = 2 \cdot 2a_i + C \equiv C \ (\text{mod } 4) \]

\[ u_{ab} \ 0 \quad v_a \ 0 \quad 1 \ 0 \quad g_i \ 0 \quad f_i \ 0 \]

\[ v_c \ 0 \quad v_b \ 0 \quad h_i \ 0 \quad g_i \ 0 \quad f_i \ 0 \]

\[ t_i \ 0 \quad u_{ab} \ 0 \quad t_i \ 0 \quad h_i \ 0 \quad g_i \ 0 \]

\[ v_a + v_b + v_c = 4f_i + 1 \equiv 1 \ (\text{mod } 4) \]

1-in-3 clause gadget

\[ v_a \lor v_b \lor v_c \]
crease pattern

Star Tessellation
Evan Zodl, 2011
wire gadget

true signal

false signal

[Bern & Hayes 1996]
NAE clause gadget

[Bern & Hayes 1996]
splitter/negation gadget

[Bern & Hayes 1996]
Not-All-Equal ($x_1, x_2, x_3$)
Vertex-Disjoint Paths is NP-complete

[Lynch 1975]
Planar Vertex-Disjoint Paths is NP-complete [Lynch 1975]
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O’Brien, Reidl, Sánchez Villaamil, Sullivan 2014]
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O’Brien, Reidl, Sánchez Villaamil, Sullivan 2014]
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

Issue 1: Empty space parity

[Adcock, Demaine, Demaine, O’Brien, Reidl, Sánchez Villaamil, Sullivan 2014]
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O’Brien, Reidl, Sánchez Villaamil, Sullivan 2014]

Issue 2: Clause path may be absent

July 26, 2014
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O’Brien, Reidl, Sánchez Villaamil, Sullivan 2014]

Issue 3: Gadget size parity

July 28, 2014
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

[Adcock, Demaine, Demaine, O’Brien, Reidl, Sánchez Villaamil, Sullivan 2014]

July 28, 2014
Planar Vertex-Disjoint Paths is NP-complete in a Rectangle

Issue 4: Crossing versions

[Adcock, Demaine, Demaine, O’Brien, Reidl, Sánchez Villaamil, Sullivan 2014]

July 28, 2014
Sam Loyd’s “The Puzzled Neighbors”
[Brooklyn Daily Eagle, 1897]
Nikoli Numberlink Puzzle

Numberlink
Easy
Author: Ryohei Nakai

1 2 1 3
5
6
2
4 7
6
7 5 4
3

Numberlink
Easy
Author: Ryohei Nakai

1 2 1 3
5
6
7
4
3
Nikoli Numberlink Puzzle

Flow Free
Big Duck Games LLC - September 3, 2016
Puzzle
Installed
This app is compatible with all of your devices.

Number Link - Logic Puzzles
Tapps - Top Apps and Games - July 3, 2016
Puzzle
Install
This app is compatible with all of your devices.

** ON ZIGZAG MODE YOU CAN DRAW PATHS FREELY **
2/3

** ON CLASSIC MODE YOUR PATH IS AUTO-SHRUNKED **
3/3
Nikoli Numberlink Puzzle is NP-complete

[Kotsuma & Takenaga 2010]