Packing Squares into a Square is Strongly NP-complete

\[ 3B + t \]

\[ B + a_i \]

\[ 3B + t \]

\[ B + t \]

\[ (B + t) \frac{n}{3} \]

[Leung, Tam, Wong, Young, Chin 1990]
Edge-Unfolding Polyhedra

[Biedl, Demaine, Demaine, Lubiw, Overmars, O’Rourke, Robbins, Whitesides 1998]

[Bern, Demaine, Demaine, Eppstein, Kuo, Mantler, Snoeyink 1998]
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete
[Abel & Demaine 2011]
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]
Snake Cube (Cubra)

Open:
History? (c. 1990?)
Snake Cube is NP-complete
[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl 2012]

• Reduction from 3-Partition

\( a_i \) gadget:

\[
\text{huge} \quad 8 a_i \quad \text{huge}
\]

\[
\text{zigzag is universal}
\]

\[
\text{target shape}
\]
Snake Cube is NP-complete

[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl 2012]

- Zigzag is universal
  - $2 \times 2 \times 2$ refinement makes any Hamiltonian shape
  - $4 \times 4 \times 4$ refinement makes any shape

- Parity issue: Path alternates cell parity each step

- Claim: Can start and end at any faces of cells of opposite parity
Snake Cube is NP-complete
[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl 2012]

- Reduce target box $\rightarrow$ target shape

- Reduce target cube $\rightarrow$ target box

$\Rightarrow$ NP-hard to fold snake cube into target cube
Disk packing

Viet Elser’s disk packing puzzle

Robert Lang
Disk packing is NP-hard

[Demaine, Fekete, Lang 2010]
Disk packing is NP-hard

[Demaine, Fekete, Lang 2010]
Clickomania / Same Game
[Schuessler ~2000?]

- **Move** = Remove any connected group of size > 1

**Goal:** Remove everything
Clickomania Complexity
[Biedl, Demaine, Demaine, Fleischer, Jacobsen, Munro 2000]

- **Polynomial** for one row/column via CFG
- **NP-hard** for
  - 2 columns & 5 colors
  - 5 columns & 3 colors
- **Open:**
  - 2 rows
  - 2 colors
inverse of below

\[
B \cdot t = B \cdot a_i
\]

\[
\frac{n}{3}
\]

\[
\frac{n}{3}
\]

\[
B = \frac{4}{3} n
\]

[Biedl, Demaine, Demaine, Fleischer, Jacobsen, Munro 2000]
In Honor of your Intellectual Contribution to the Art of Tetris,

for proving NP-completeness in maximization of lines,
tetrises, pieces played, or minimization of square height,

we, masters of the Harvard Tetris Society hereby confer the title of

**TETRIS MASTER**

upon

**Erik D. Demaine**

on the sixteenth day of the twelfth month in the year 17 Anno Tetri (2002)

David Bernard
HTS President

Seymour M. Lesgold
HTS Treasurer
Claim 5: When terminate, we do so on the left.

If not:
- no RG
- no LS

First LG goes into R-terminus:
- Red LG goes in here
- Otherwise, LG goes in here

Problems:
- If multiple R-termini, then have a LG-sink.
- Have a LG, LS, LG-sink, & partial LS sink.
- LS could be picked up & come early.
- Idea: Count each piece using sinks = trouble.

Without LG, LG-sink & partial LS sink, LS could be picked up & come early.
Initial Board

$\approx t$ notches
(target sum)

[T lock]

$\frac{n}{3}$ buckets
(one per sum)

(it is possible to actually get here)

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kisters, Liben-Nowell 2003]
Piece Sequence [Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

- For each input $a_i$:
Failure to Launch  [Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

"unprimed" buckets

(a)  (b)  (c)  (d)  (e)  (f)  (g)  (h)  (i)  (j)  (k)
Forced Moves

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale

Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale

Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale

Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kusters, Liben-Nowell 2003]
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale

Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Hardness of Approximation
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Tetris Open Problems
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

- Complexity of Tetris with
  - Initially empty board?
  - $O(1)$ columns?
  - $O(1)$ rows?
  - Restricted piece sets (e.g. \[\square\])?
  - No last-minute slides?

- Is two-player Tetris PSPACE-complete?
- What can we say about online (regular) Tetris?
1-planar Graph

• Each edge has at most one crossing
1-planarity is NP-complete

[Grigoriev & Bodlaender 2007]

double wheel

uncrossable edge

$K_6$
1-planarity is NP-complete

[Grigoriev & Bodlaender 2007]

\[ A = \{2, 3, 3, 3, 4, 5\} \]
GeoLoop & Ivan’s Hinge

Piano-Hinged Dissections
Time to Fold!

Greg N. Frederickson

Jan Essebaggers & Ivan Moscovich
1993

Kenneth Stevens 1993
GeoLoop & Ivan’s Hinge
[Abel, Demaine, Demaine, Horiyama, Uehara 2014]
NP-complete
universal & polynomial
2.2. Lower Bounds

Figure 2.5. Ruler folding reduction. Here $x_1 + x_3 + x_4 = x_2 + x_5 + x_6$.

[Hopcroft, Joseph, Whitesides 1985]
(Simple) Map Folding

Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000

Figure 14.4. Folding a $2 \times 4$ map via a sequence of three all-layers simple folds.
[Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000]