Problem Set 2

This problem set is due Wednesday, September 28 at noon.

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1. Prove that any undirected planar graph G with non-negative edge weights can be transformed into an undirected planar graph G' with maximum degree 3 such that,
   - for any $u, v \in V(G)$, $d_G(u, v) = d_{G'}(f(u), f(v))$, where $f : V(G) \to V(G')$ maps vertices between G and G'; and
   - $|V(G')| = O(|V(G)|)$.
2. A ρ–*clustering* of G is a decomposition into $O(n/\rho)$ vertex-disjoint *connected pieces*, each with $\Theta(\rho)$ vertices. Recall that a ρ–clustering, if computed efficiently, can be used to compute an r–division in $o(n \log n)$ time. Give a linear-time algorithm to compute a ρ–clustering for any graph with maximum degree three.