Problem Set 2

This problem set is due Wednesday, September 28 at noon.

1. Prove that any undirected planar graph $G$ with non-negative edge weights can be transformed into an undirected planar graph $G'$ with maximum degree 3 such that,
   - for any $u, v \in V(G)$, $d_G(u, v) = d_{G'}(f(u), f(v))$, where $f : V(G) \to V(G')$ maps vertices between $G$ and $G'$; and
   - $|V(G')| = O(|V(G)|)$.

2. A $\rho$–clustering of $G$ is a decomposition into $O(n/\rho)$ vertex-disjoint connected pieces, each with $\Theta(\rho)$ vertices. Recall that a $\rho$–clustering, if computed efficiently, can be used to compute an $r$–division in $o(n \log n)$ time. Give a linear-time algorithm to compute a $\rho$–clustering for any graph with maximum degree three.