Problem Set 11

This problem set is due Wednesday, December 7 at noon.

1. Recall the cycle fixing procedure in the multiple-source multiple-sink maximum flow algorithm which finds a flow γ' which eliminates residual paths between nodes with positive excess and nodes with negative excess. We saw in class that after all iterations of the efficient implementation of the procedure are done we have the explicit flow η on the edges of the cycle C, and the face prices ϕ only for the faces that are endpoints of dual edges of C. In class we saw how to find a circulation θ such that $\eta + \theta$ is the desired flow γ' by using a single shortest path computation in the dual with respect to the lengths $c - \eta$ (residual capacities), which might be negative. Show how to find such a circulation θ without using an algorithm for shortest paths with negative lengths.

   Solution: Let X be the set of endpoints of dual edges of C. Let $\theta' := \phi(\text{head}(d^*) - \phi(\text{tail}(d^*))$ be the circulation induced by ϕ. Note that even though we only know ϕ on X, we can use the hybrid FR-Dijkstra to compute the distances w.r.t. $\theta' + \eta$ from some $x \in X$ to all other nodes in X (i.e., w.r.t. the reduced lengths of $c - \eta$ w.r.t. ϕ). To see this note that this is exactly what the cycle fixing procedure does at each iteration, so the arguments given in class hold.

   Next, we convert these reduced distances into unreduced distances with respect to just $c - \eta$ by subtracting, for each node $y \in X$, $\phi(x) - \phi(y)$ from the x-to-y reduced distance.

   Finally, we extend these distances to all nodes of G^* by using Dijkstra algorithm with arc lengths given by the capacities $c - \eta$, once in the subgraph enclosed by C^* and once in the subgraph not enclosed by C^*. In both these computations we initialize the nodes of X to their distances from x, which we already know. Since the original capacities are non-negative and since η is non-zero only on edges of C^*, Dijkstra's algorithm initialized in this manner will output the distances from x to all nodes w.r.t. $c - \eta$. These distances are the prices that induce the desired circulation θ.

2. Present a drawing of an embedding of $K_{3,3}$ in the projective plane, specify its embedding scheme, and list its facial walks.

   Solution: See next page.

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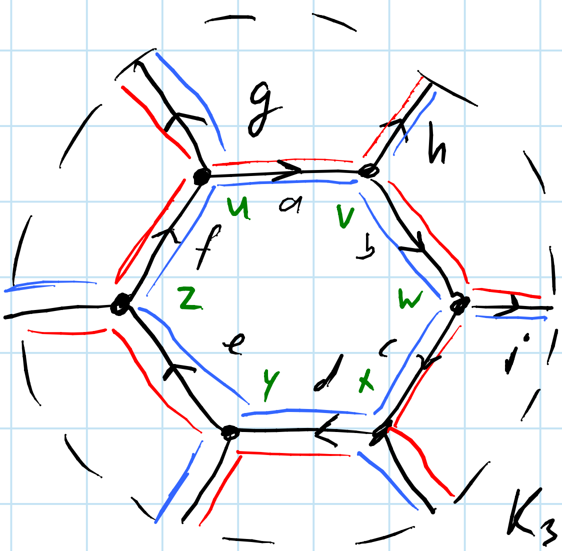
Solution to problem 2:

It's better to think of darts as being half-edges on the left and right of an edge. The direction put on an edge only helps to define what left and right mean for that edge. This way, each dart appears exactly once on the union of all facial walks:



Facial walks:
1) fedcba
2) ahdg
3) bieh
4) cgfi

$K_{3,3}$ on the projective plane.

Embedding scheme:

u: afg v: bah w: cbi
x: dcg y: edh z: fei

$$\lambda(a) = \lambda(b) = \lambda(c) = \lambda(d) = \lambda(e) = \lambda(f) = 1$$

$$\lambda(g) = \lambda(h) = \lambda(i) = -1$$