Problem Set 10

This problem set is due Wednesday, November 23 at noon.

In this problem set you will develop an algorithm for canceling flow cycles in a given flow assignment. In general graphs this can be done in $O(m \log n)$ time using Sleator's and Tarjan's dynamic trees. You will use the relation between dual shortest paths and circulations to give a linear time algorithm in planar graphs.

1. Let G be a planar graph with non-negative capacities on its arcs. Let ϕ be shortest path distances from f_∞ in G^*. Let θ be the circulation induced by ϕ. That is,

$$\theta(d) = \phi(\text{head}(d)) - \phi(\text{tail}(d)).$$

   Recall that a residual path is a path whose darts all have strictly positive capacities. Show that there are no counterclockwise residual cycles in the residual graph G_θ.

2. What price function ϕ' would you use to get the same property as in (1), but with no clockwise residual cycles?

3. Use parts (1) and (2) to give a linear time algorithm that, given a flow assignment γ in G makes γ acyclic by removing all flow cycles in γ. That is, it produces another flow assignment γ' s.t.

   (a) $\gamma' - \gamma$ is a circulation

   (b) for every arc a, $\gamma'((a, 1)) \leq \gamma((a, 1))$

   (c) for any cycle C there is a dart $d \in C$ s.t. $\gamma'(d) \leq 0$