Problem Set 2, Part b

Due: Wednesday, March 15, 2006
Problem sets will be collected in class. Please hand in each problem on a separate page, with your name on it.

Reading

Timesync
Fan, Lynch: Gradient clock sync
Attiya, Hay, Welch: Optimal clock sync paper

Topology control
Li, et. al: Cone-based topology control algorithm
Bahramgiri et. al: Fault tolerant distributed topology control algorithm

Reading for next week

Local infrastructure
Chockler et. al: Consensus and collision detector

Broadcast
Kowalski, Pelc: Deterministic broadcasting paper
Bar-Yehuda et. al: Time complexity of broadcast
Bar-Yehuda et. al: Efficient emulation of single-hop radio network

Problems

1. Consider the lower bound in the gradient clock synchronization paper.
   (a) Outline the design of a simple clock synchronization protocol, for the setting described in the paper.
   (b) Now construct a “bad execution”, in which a large clock skew is produced between two adjacent nodes. Try to make the skew as large as possible (try for \( O(D) \)). You may use the strategy described in the lower bound proof, or any other strategy you like, in trying to devise your bad execution.

2. Consider the 9-node undirected grid graph depicted below. Each label \((d, u)\) on an edge represent the average delay \(d\) and uncertainty \(u\), as in the Attiya, Hay, Welch paper.

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\(3,1\) & \(3,2\) & \\
4 & 5 & 6 \\
\(2,1\) & \(2,2\) & \(4,1\) \\
7 & 8 & 9 \\
\(1,1\) & \(0,1\) & \\
\end{array}
\]

Assume that nodes 1 and 9 are source nodes.
   (a) Give a “shallow forest” satisfying the optimality constraint given in the paper.
(b) What are the upper bounds on clock skew for the 9 nodes that arise from your shallow forest?

(c) For node 5, describe an actual execution of the algorithm of the paper (the one that precomputes the tree is fine) in which the upper bound you determined for 5 is actually attained.

3. The proof of Theorem 8 in the BHM paper is nice, simple, and short. Can you give a similar proof for Theorem 3.2 of LH?

4. The algorithms considered in LH and BHM are of a very particular kind: they use local information only, and depend on the same fixed \( \alpha \) everywhere. We can consider what might happen if these rules were loosened.

(a) Draw a graph \( G \) in the Euclidean plane with the property that the actual connectivity of the graph \( G_\alpha \), where \( \alpha = \frac{2\pi}{\theta} \), is much greater than 1.

(b) Now suppose that the processes at the nodes of your graph know the entire graph, and can use this knowledge to determine their power levels. Is there a way they can set their power levels to be much smaller than what would arise from the LH algorithm, yet still preserve connectivity?