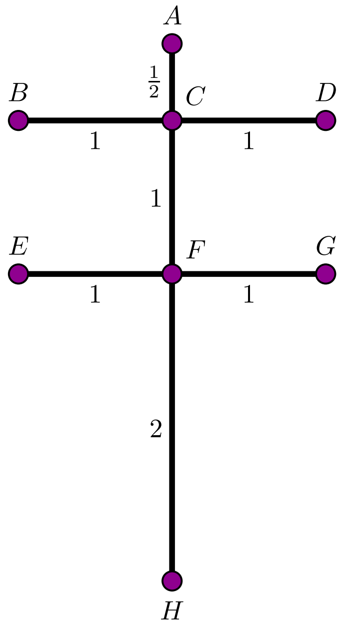
6.885: Folding and Unfolding in Computational Geometry
Fall 2004 — Prof. Erik Demaine

Problem Set 2 (optional)

Due: Friday, December 3, 2004

Problem 1. Tree Theory for Origami Design (Lecture 18).



The tree on the right describes a lizard with a short head (represented by edge AC) and a longish tail (represented by edge HF). Both the subject and the tree are bilaterally symmetric. Your crease patterns should have the same bilateral symmetry. On a square, there are two choices of symmetry axis: one parallel to the sides (called "book symmetry") and one along the diagonal ("diagonal symmetry"). In class, Robert Lang showed a crease pattern with book symmetry. In this problem, you'll be building and working with crease patterns having diagonal symmetry in a unit square.

Each leaf node of the tree (A, B, D, E, G, and H) has a corresponding leaf node vertex on the crease pattern. For simplicity, we will refer to a vertex by the same letter as its corresponding node.

The leaf node vertices must satisfy the path inequalities that were presented in class for the scaled tree:

$$\begin{aligned} & \text{distance in square}\,(\text{vertex}\,X,\ \text{vertex}\,Y) \\ \leq\ & \text{distance in tree}\,(\text{node}\,X,\ \text{node}\,Y). \end{aligned}$$

You can find a satisfying set of leaf node vertices in two ways:

(i) Set up and solve the nonlinear constrained optimization in Maple, Matlab, or Mathematica. All of this software is available on Athena; see http://web.mit.edu/acs/www/numerical.html.

(ii) Find the optimum arrangement of vertices by packing circles and rivers as described in *Origami Design Secrets*, chapters 10–11, and using a bit of algebra and geometry. No computer is needed, though a calculator will be necessary to get the numerical answers.

It's your choice which method you use; the problems can be solved both ways.

A *locally optimum* solution is an arrangement of vertices for which there is no infinitesimal change in the vertex placement that allows a larger scale factor. In the following problem parts, give all quantitative answers as numerical values to 4 decimal places.

(a) There is a locally optimum arrangement of vertices in which vertices A and H lie at diagonally opposite corners of the unit square. Draw and label the arrangement of vertices on a unit square and the active paths. What is the scale factor for this local optimum? Whichever method you used to solve for the vertex positions, draw a circle around each leaf node vertex whose radius is equal to the scaled length of the tree edge incident to the corresponding leaf node. (Hint: the convex hull of the vertices is entirely active.)

(b) There is another locally optimum arrangement in which vertex A is not at a corner of the square but vertex H is. Draw the arrangement of vertices on a unit square, the active paths, and circles as you did for problem 1. What is the scale for this local optimum? (Hint: the convex hull of the vertices is entirely active.)

(c) For the crease pattern of part (b), vertex A is actually unconstrained, meaning that there is a small region in which it can move without violating any path inequalities or changing the scale. Draw this region on the square. (Hint: it is bounded by several circular arcs.) This means that edge AC of the tree can be made longer without moving any other vertices or changing the scale. What is the longest that edge AC can be made without violating any path conditions?

(d) Lengthening edge AC as described in part (c) will create 4 new active paths. Draw all active paths, vertices, and circles on the unit square for this pattern. The active paths should decompose the convex hull of the vertices into one quadrilateral and three triangles.

(e) Use the techniques of *Origami Design Secrets*, chapters 10–11, to construct the appropriate molecules within the quadrilateral and triangles of part (d). Include all hinge creases. Draw the crease pattern.

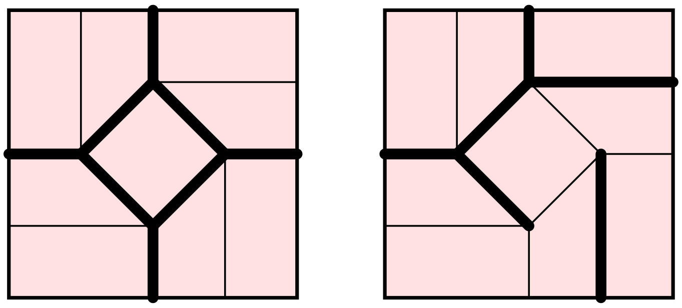
(f) Cut out the crease pattern of figure 5 along the convex hull of the vertices, fold on the creases you found so that the folded model lies flat and all active paths lie on a common line. Turn in your folded model.

(g) Extra credit: Turn the folded shape into a recognizable lizard with additional folding and shaping.

Problem 2. Rigid Origami (Lecture 19).

(a) Use the Gaussian curvature model for rigid origami to prove that no rigid single-vertex fold can have degree three.

(b) Tom Hull showed two different mountain-valley assignments for the square twist, one of which was impossible to fold rigidly, and the other of which seems possible. Enumerate the other possible mountain-valley assignments for the square twist and determine which of them are impossible to fold rigidly.



cannot fold rigidly   seems to fold rigidly