Subgraph Isomorphism

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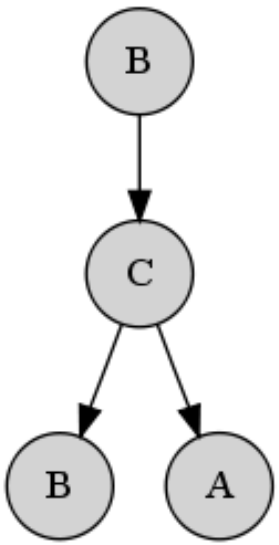
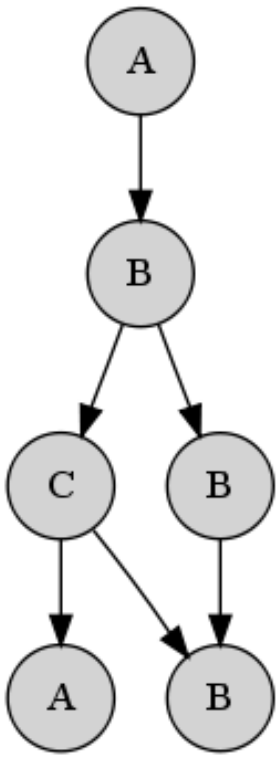
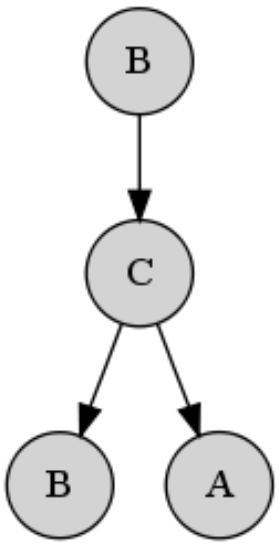
Subgraph Isomorphism

- Given graphs G, H
- Determine if H is isomorphic to a subgraph of G



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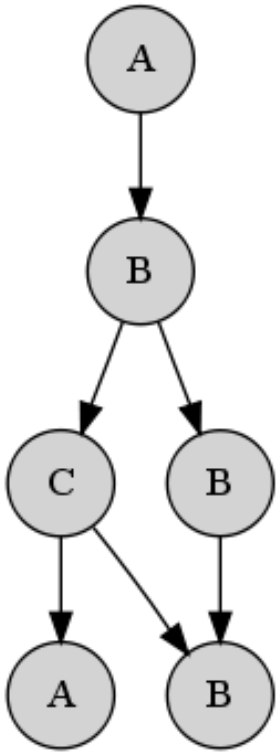
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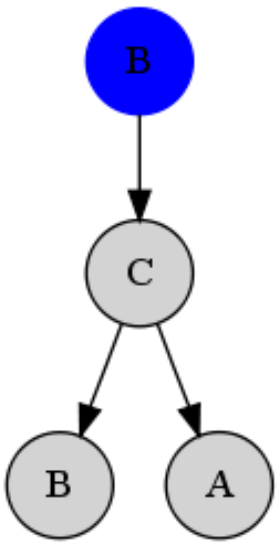
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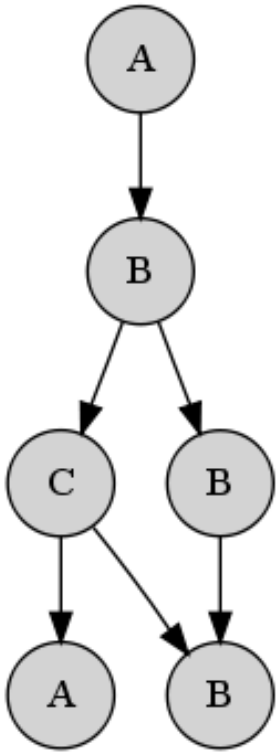
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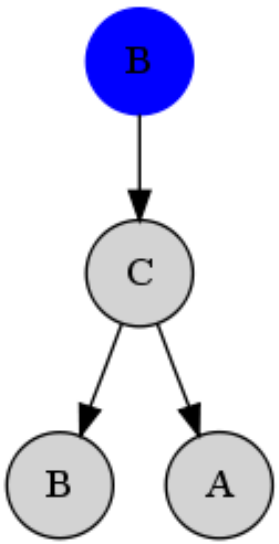
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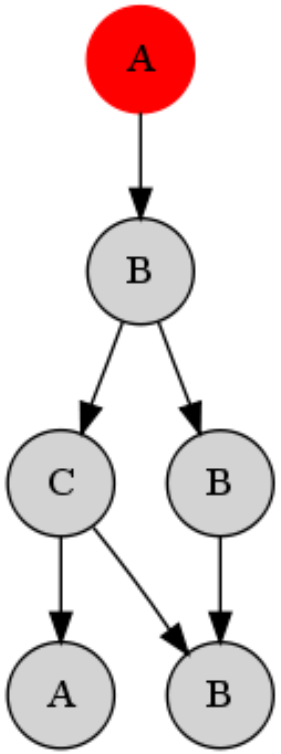
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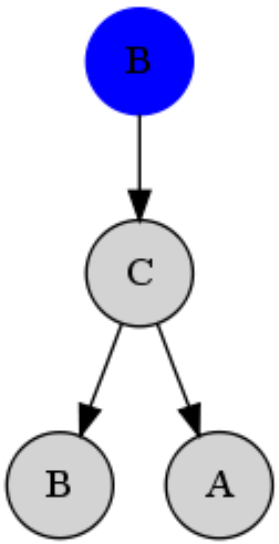
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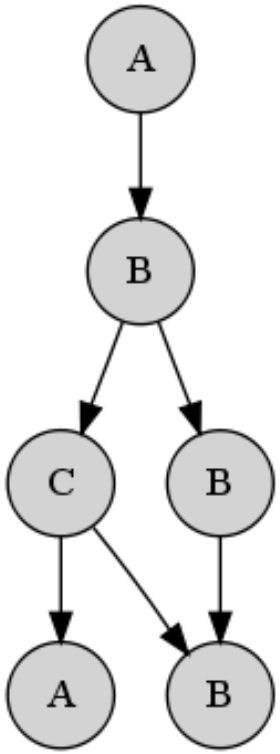
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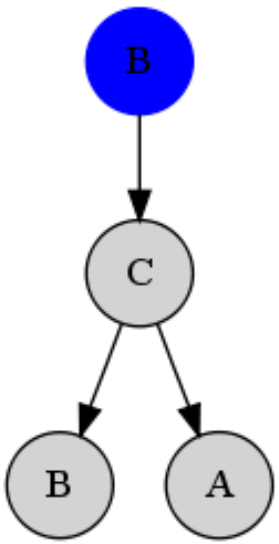
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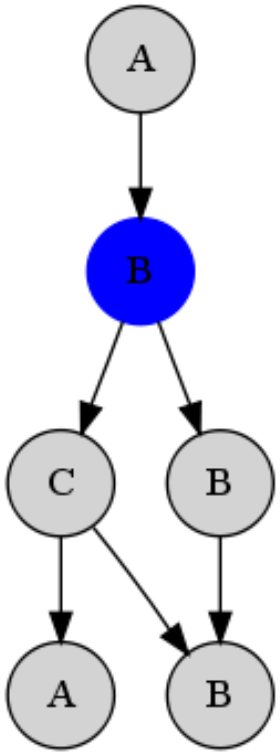
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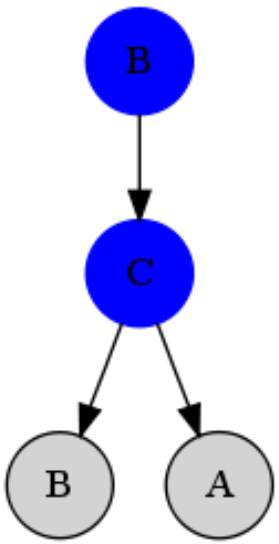
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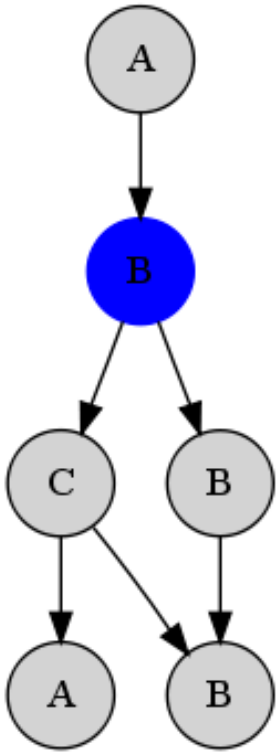
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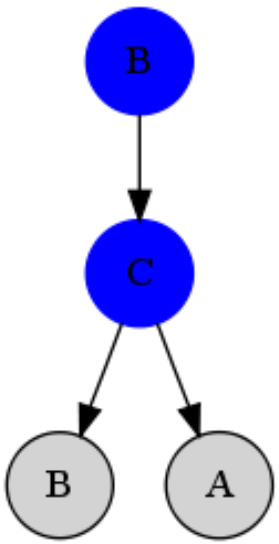
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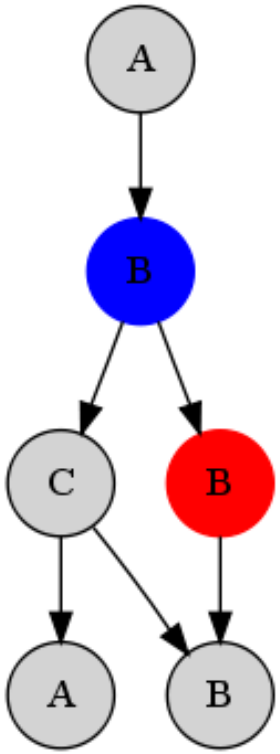
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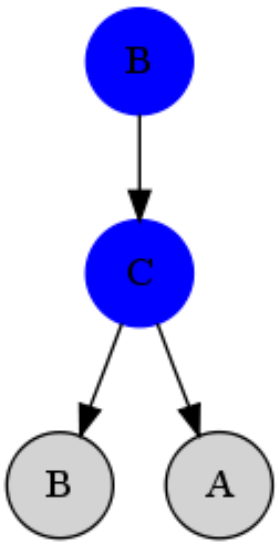
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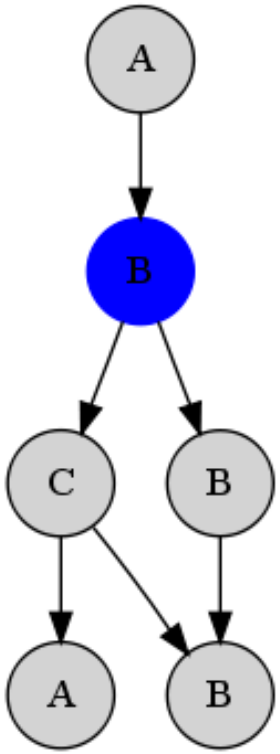
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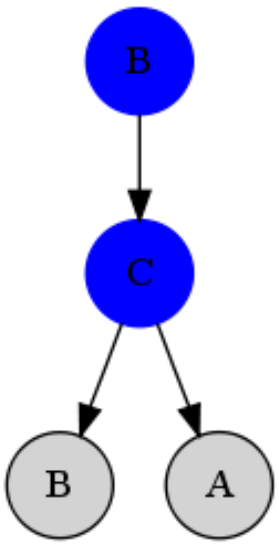
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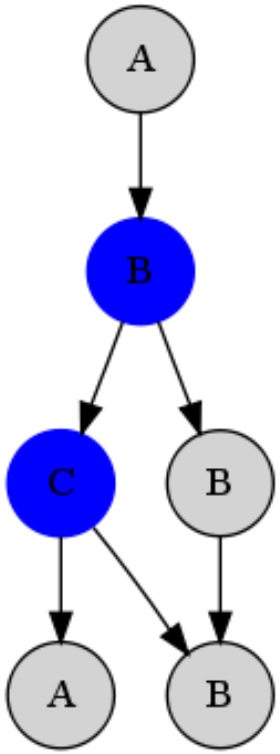
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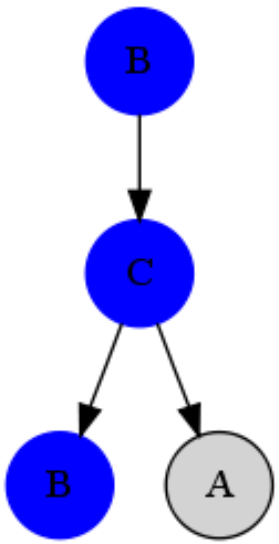
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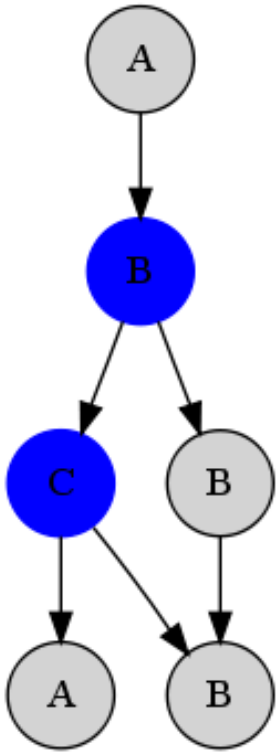
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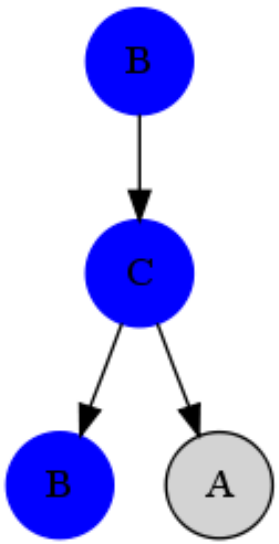
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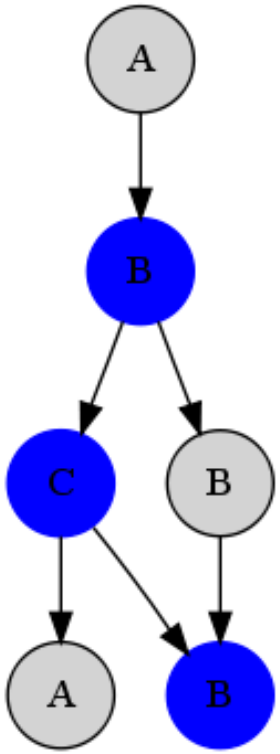
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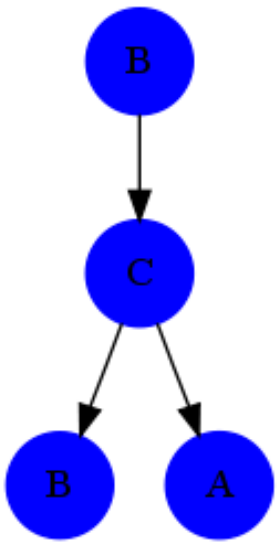
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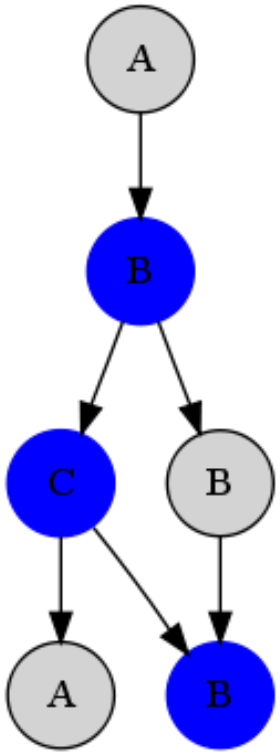
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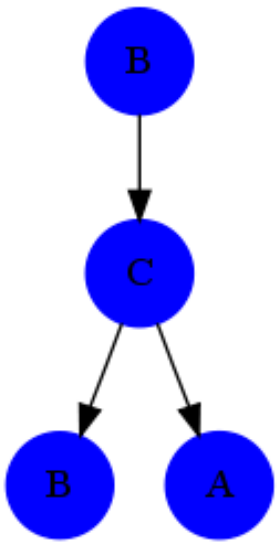
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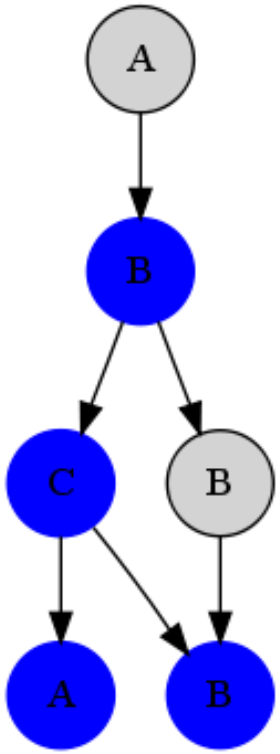
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Outline

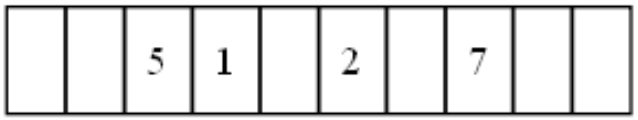
- Parallelized Subgraph Isomorphism Library
- Many failed methods
- Near-linear Speedup on Random Graphs
- Reasonable Parallelism on *Hard* Parasitic Graphs

VFLib

- General Fast Subgraph Isomorphism Library
- Memory efficient implementation of Ullman Algorithm
  - in-set, out-set
- Serial implementation only
- Works with large database of graphs

VFLib's Algorithm

- Ullman Heuristics
- Optimized Data Structures
  - Matched Set for current match
  - in-set Array
  - out-set Array
  - Indexed by node-id
  - Stores step number (basically a log)

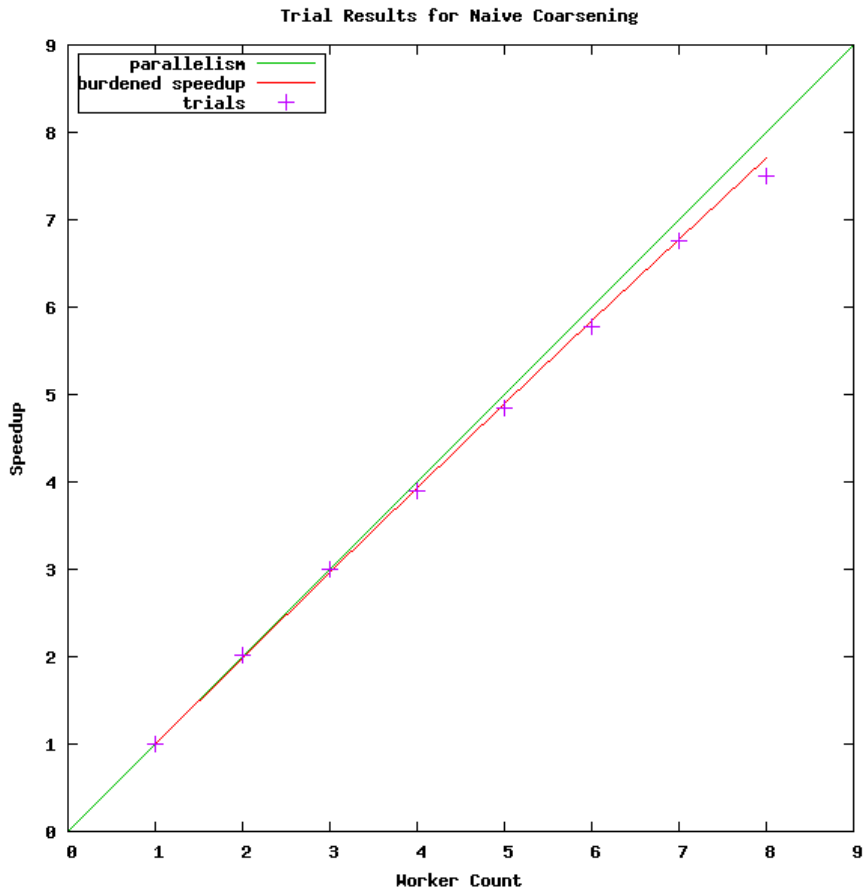


Parallel Algorithm

- Add cilk_spawn calls for each graph child
- Deep copy of data structure whenever we spawn
- Massive memory overhead: solve by “coarsening”

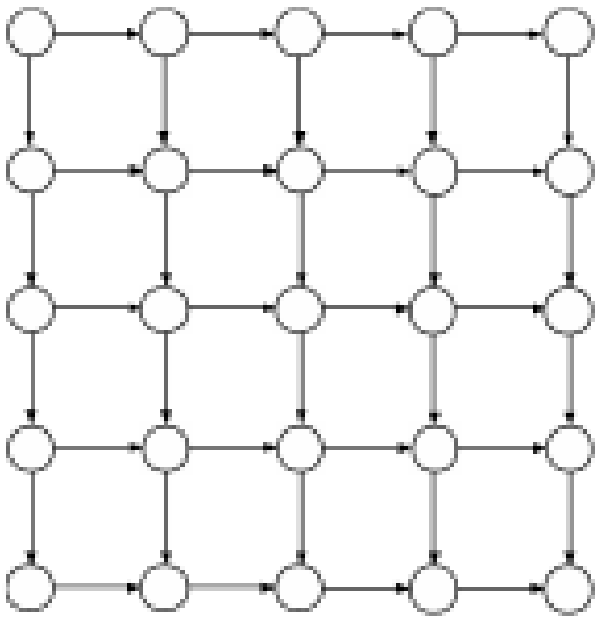
Random Graph Results

- Good parallelism on random graphs
- Low burden: 312.66 / 314.71



Trouble in Parallel Paradise

- Coarsening solves our memory issue
- Not hard to imagine parasitic graphs
- 2D Meshes - Speedup: 1.001



Initial Strategies

- 2 Basic Approaches
- Spawn more
  - Faster Deep Copying
- Spawn better
  - Develop heuristics for where and when to cilk_spawn

Data Structure Solutions

- Cost values are in Seconds
- Scaled by times called per deep copy – next pair is x20 and backtrack is x0.6

	Deep Copy	*Get-Next-Pair*	*BackTrack*
Arrays	$6.7 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$	$1.1 \cdot 10^{-6}$
Mapsets	$8.8 \cdot 10^{-6}$	$1.2 \cdot 10^{-4}$	$1.8 \cdot 10^{-5}$
Bitsets	$5.1 \cdot 10^{-6}$	$3.2 \cdot 10^{-5}$	$3.0 \cdot 10^{-6}$

Spawn Heuristic Solutions

- Non-linear cutoff
- Mid-tree respawn
- Spawn first child
- Really just want to spawn when there's more work

Conditional Copy

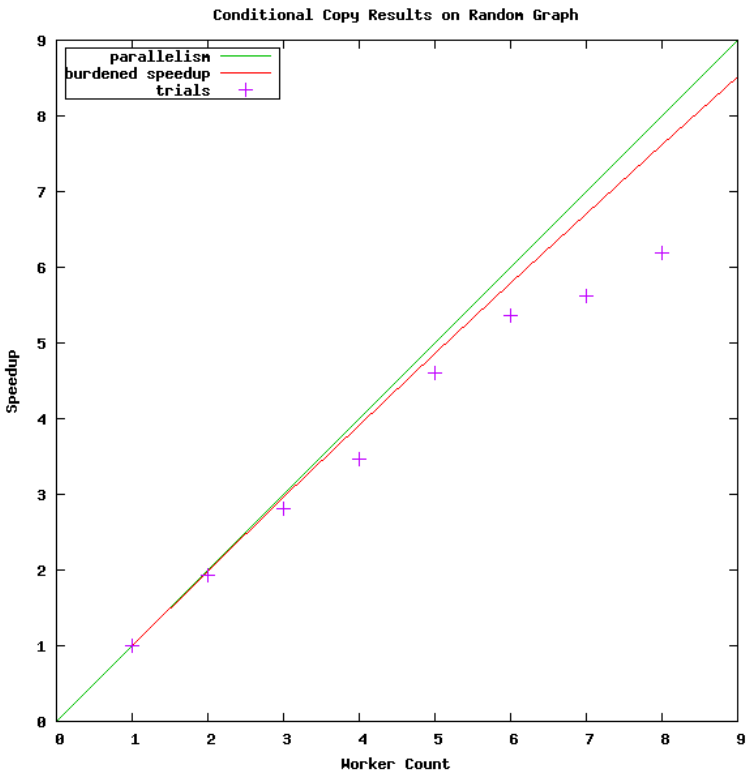
- Deep copy only when a steal occurs – requires a Snapshot

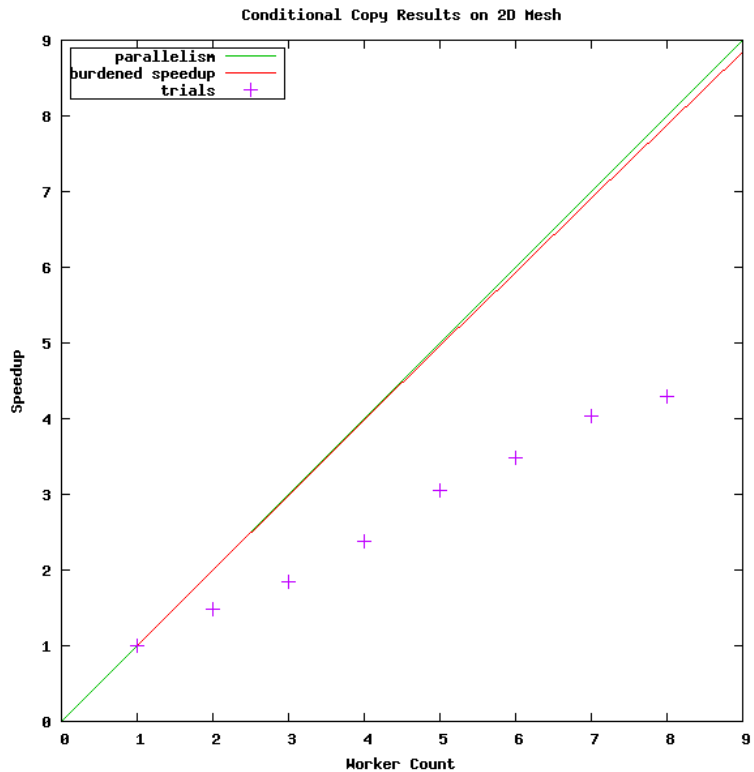
```
bool running = false
cleanCopy = state->deepClone()
for (p in state->nextPair())
    needs_clean = ! running
    if (running)
        nextState = cleanCopy->deepClone()
        nextState -> addPair()
    else
        running = true
        nextState = state
        nextState -> addPair()
    cilk_spawn match(nextState, &running, needs_clean)
....

if (needs_clean)
    nextState -> backTrack()
    * parentRunFlag = false
```

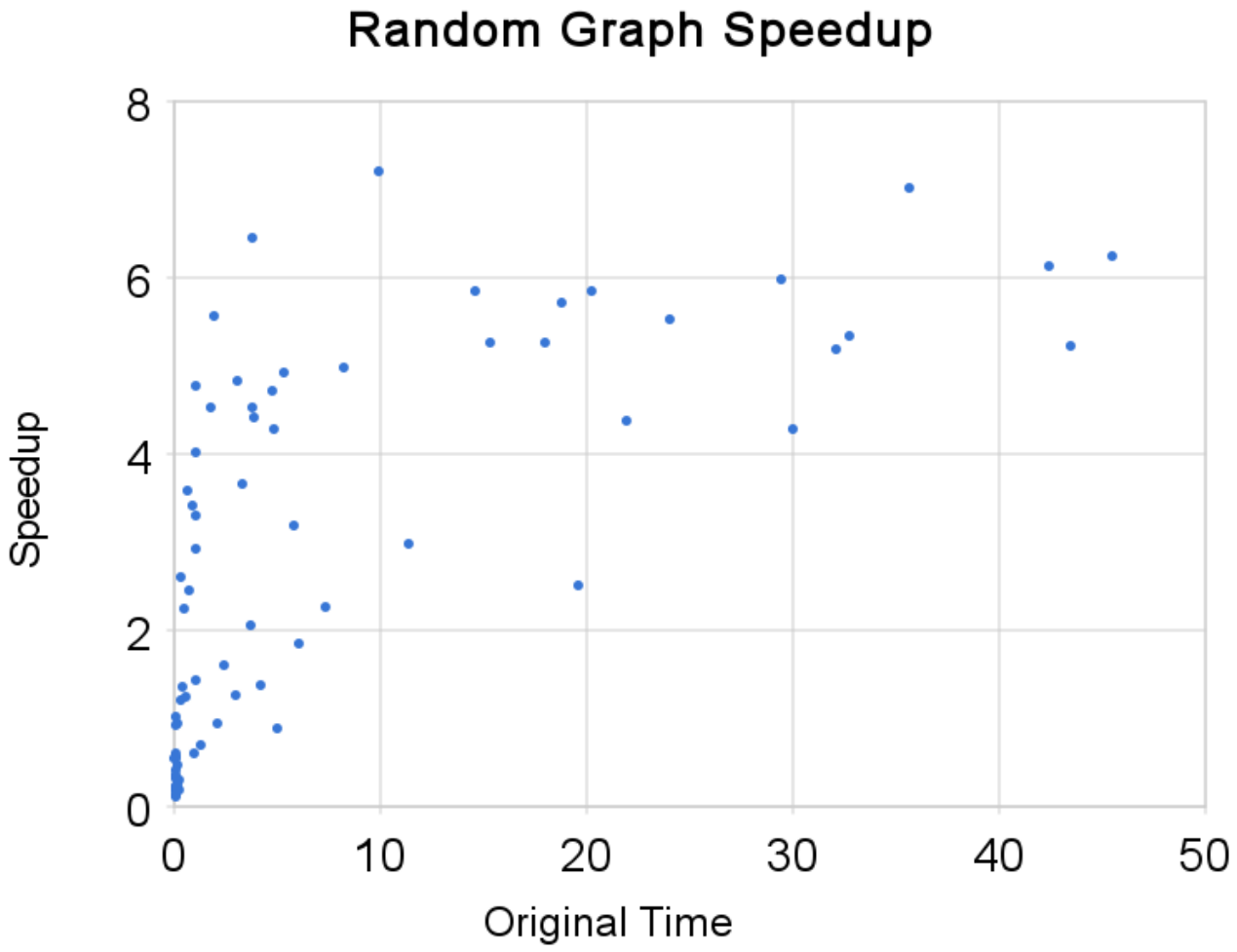
Parallelism

- Increased burden, but fixed most parasitic cases

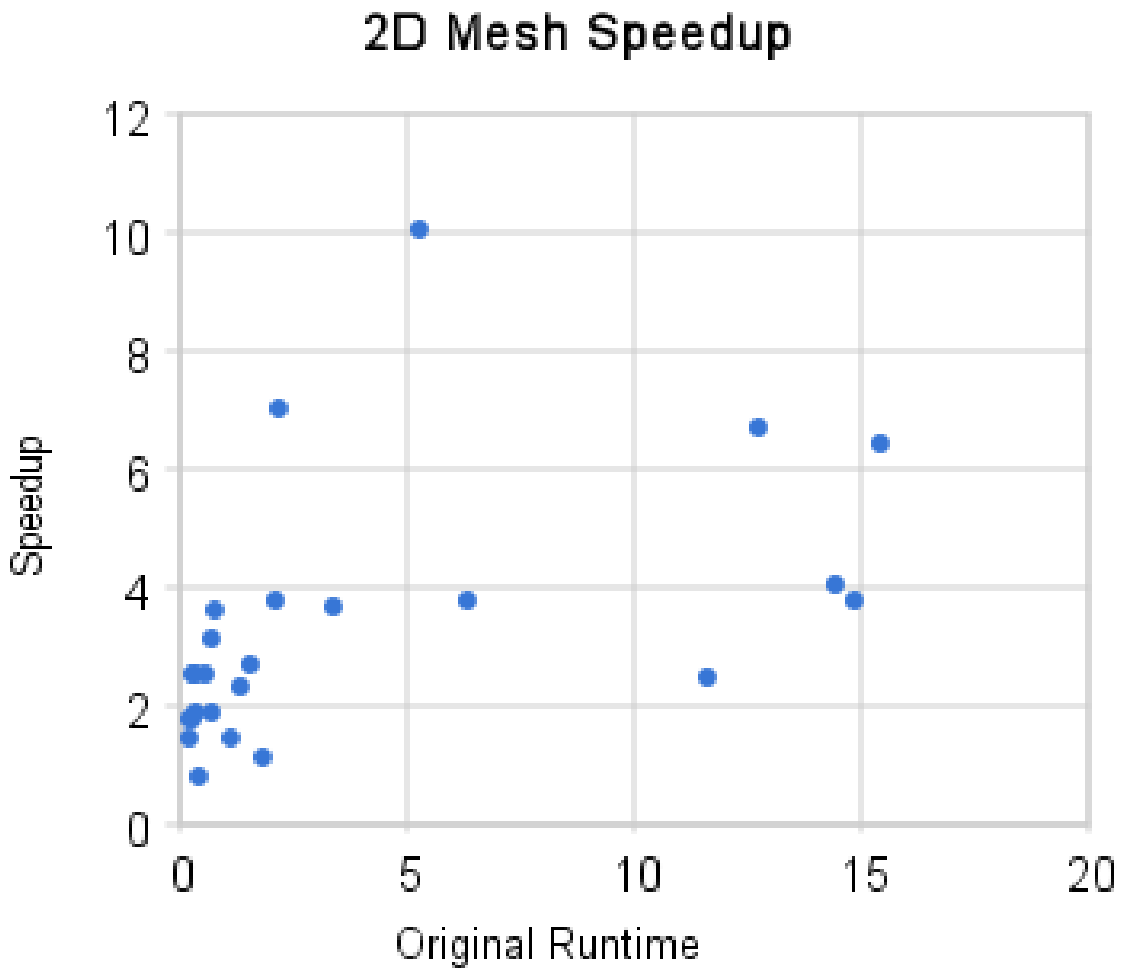




Random Graphs Speedup



2D Meshes Speedup



Parallel Programming Difficulty

- **Main issue:** spawn cost >> scheduling overhead
- Better spawn heuristics with active_workers()
- Our conditional copy is on the bleeding edge of *working* and *not working*
  - Possible language feature
  - Data Structure Wrapper - splitter

Future Work

- Optimizing for non-steal case. Mix log and snapshots.
- Design and implement a splitter hyperobject

Contributions

- Compared performance of several data structures for subgraph isomorphism
- Implemented a faster than current state-of-the-art subgraph isomorphism match algorithm.
- Detailed a general approach to dealing with large data structure copying for spawns in cilk platform
- Speculated on useful language features to enable conditional copying in cilk