6.869

Computer Vision

Prof. Bill Freeman

Particle Filter Tracking
- Particle filtering

Readings: F&P Extra Chapter: “Particle Filtering”

Schedule

- Tuesday, May 3:
  - Particle filters, tracking humans, Exam 2 out
- Thursday, May 5:
  - Tracking humans, and how to write conference papers & give talks, Exam 2 due
- Tuesday, May 10:
  - Motion microscopy, separating shading and paint ("fun things my group is doing")
- Thursday, May 12:
  - 5-10 min. student project presentations, projects due.

1D Kalman filter

Dynamic Model:

$$x_i \sim N(d_i x_{i-1}, \sigma_{d_i})$$

$$y_i \sim N(m_i x_i, \sigma_{m_i})$$

Start Assumptions: $\overline{x}_0^-$ and σ_0^- are known
Update Equations: Prediction

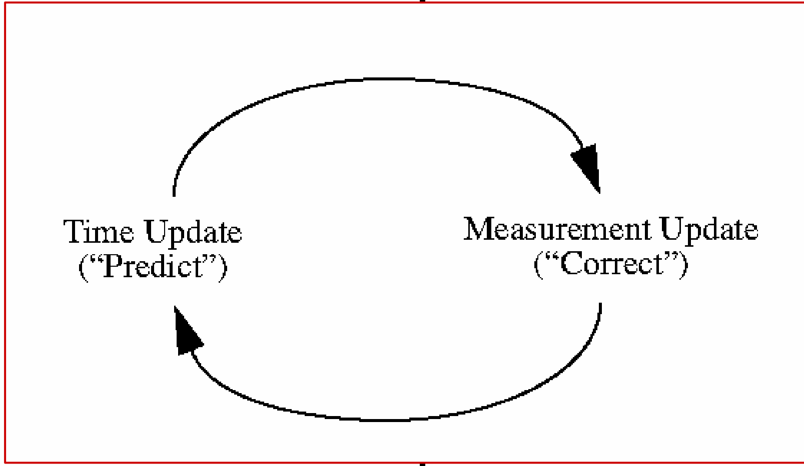
$$\overline{x}_i^- = d_i \overline{x}_{i-1}^+$$

$$\sigma_i^- = \sqrt{\sigma_{d_i}^2 + (d_i \sigma_{i-1}^+)^2}$$

Update Equations: Correction

$$x_i^+ = \left(\frac{\overline{x}_i^- \sigma_{m_i}^2 + m_i y_i (\sigma_i^-)^2}{\sigma_{m_i}^2 + m_i^2 (\sigma_i^-)^2} \right)$$

$$\sigma_i^+ = \sqrt{\left(\frac{\sigma_{m_i}^2 (\sigma_i^-)^2}{(\sigma_{m_i}^2 + m_i^2 (\sigma_i^-)^2)} \right)}$$



Kalman filter for computing an on-line average

- What Kalman filter parameters and initial conditions should we pick so that the optimal estimate for x at each iteration is just the average of all the observations seen so far?

Kalman filter model

$$d_i = 1,\ m_i = 1,\ \sigma_{d_i} = 0,\ \sigma_{m_i} = 1$$

$$x_i \sim N(d_i x_{i-1}, \sigma_{d_i})$$

$$y_i \sim N(m_i x_i, \sigma_{m_i})$$

$\overline{x}_0^-$ and σ_0^- are known

Prediction

$$\overline{x}_i^- = d_i \overline{x}_{i-1}^+$$

$$\sigma_i^- = \sqrt{\sigma_{d_i}^2 + (d_i \sigma_{i-1}^+)^2}$$

Correction

$$x_i^+ = \left(\frac{\overline{x}_i^- \sigma_{m_i}^2 + m_i y_i (\sigma_i^-)^2}{\sigma_{m_i}^2 + m_i^2 (\sigma_i^-)^2} \right)$$

$$\sigma_i^+ = \sqrt{\left(\frac{\sigma_{m_i}^2 (\sigma_i^-)^2}{(\sigma_{m_i}^2 + m_i^2 (\sigma_i^-)^2)} \right)}$$

Initial conditions

$$\overline{x}_0^- = 0 \qquad \sigma_0^- = \infty$$

Iteration	0	1	2
$\overline{x}_i^-$	0	y_0	$\frac{y_0 + y_1}{2}$
$\overline{x}_i^+$	y_0	$\frac{y_0 + y_1}{2}$	$\frac{y_0 + y_1 + y_2}{3}$
σ_i^-	∞	1	$\frac{1}{\sqrt{2}}$
σ_i^+	1	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{3}}$

What happens if the x dynamics are given a non-zero variance?

Kalman filter model

$$x_i \sim N(d_i x_{i-1}, \sigma_{d_i})$$

$$y_i \sim N(m_i x_i, \sigma_{m_i})$$

$\overline{x}_0^-$ and σ_0^- are known

Prediction

$$\overline{x}_i^- = d_i \overline{x}_{i-1}^+$$

$$\sigma_i^- = \sqrt{\sigma_{d_i}^2 + (d_i \sigma_{i-1}^+)^2}$$

Correction

$$x_i^+ = \left(\frac{\overline{x}_i^- \sigma_{m_i}^2 + m_i y_i (\sigma_i^-)^2}{\sigma_{m_i}^2 + m_i^2 (\sigma_i^-)^2} \right)$$

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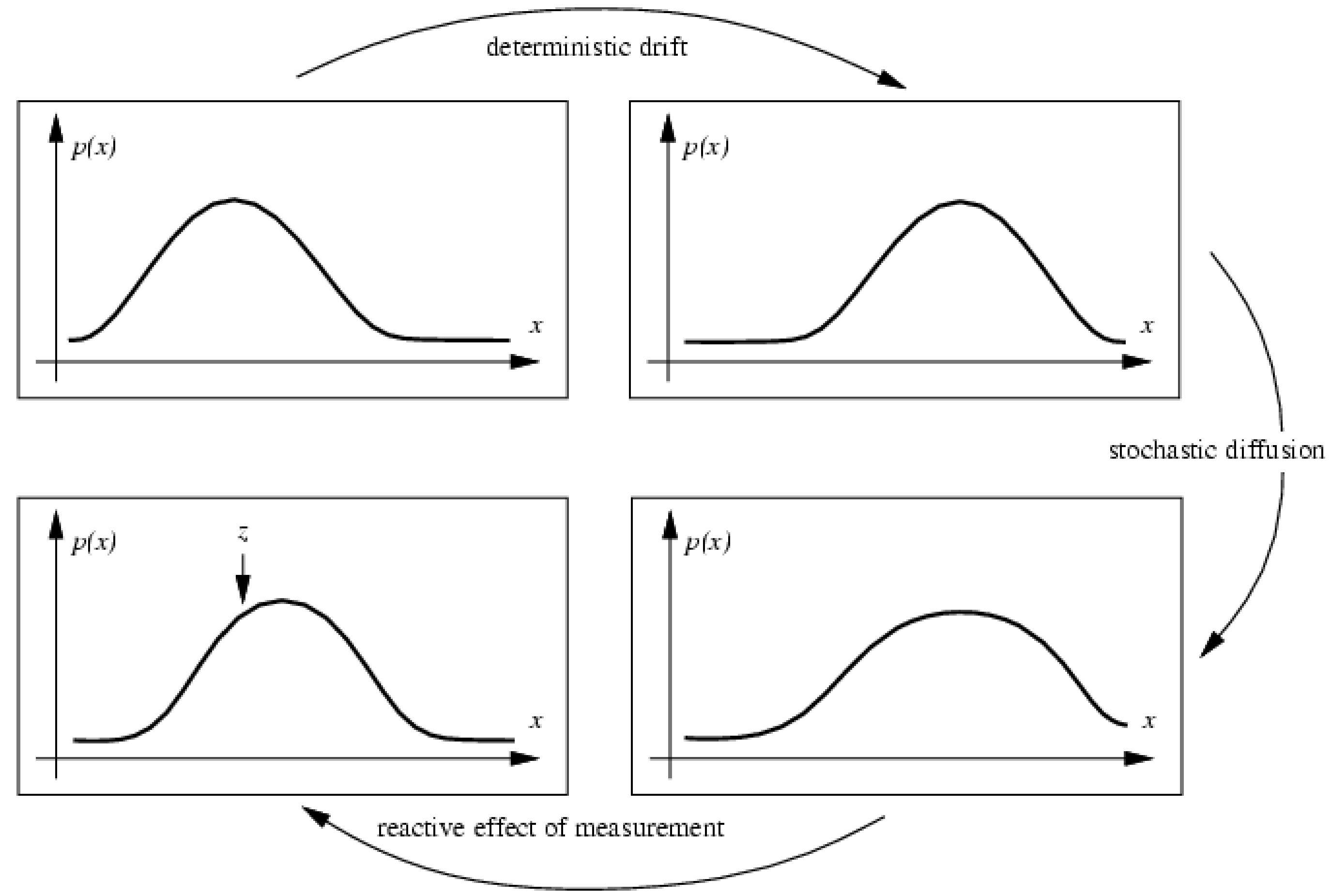
$\overline{x}_i^-$	0	y_0	$\frac{y_0 + 2y_1}{3}$
$\overline{x}_i^+$	y_0	$\frac{y_0 + 2y_1}{3}$	$\frac{y_0 + 2y_1 + 5y_2}{8}$
σ_i^-	∞	$\sqrt{2}$	$\sqrt{\frac{5}{3}}$
σ_i^+	1	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{5}{8}}$

(KF) Distribution propagation

prediction from previous time frame

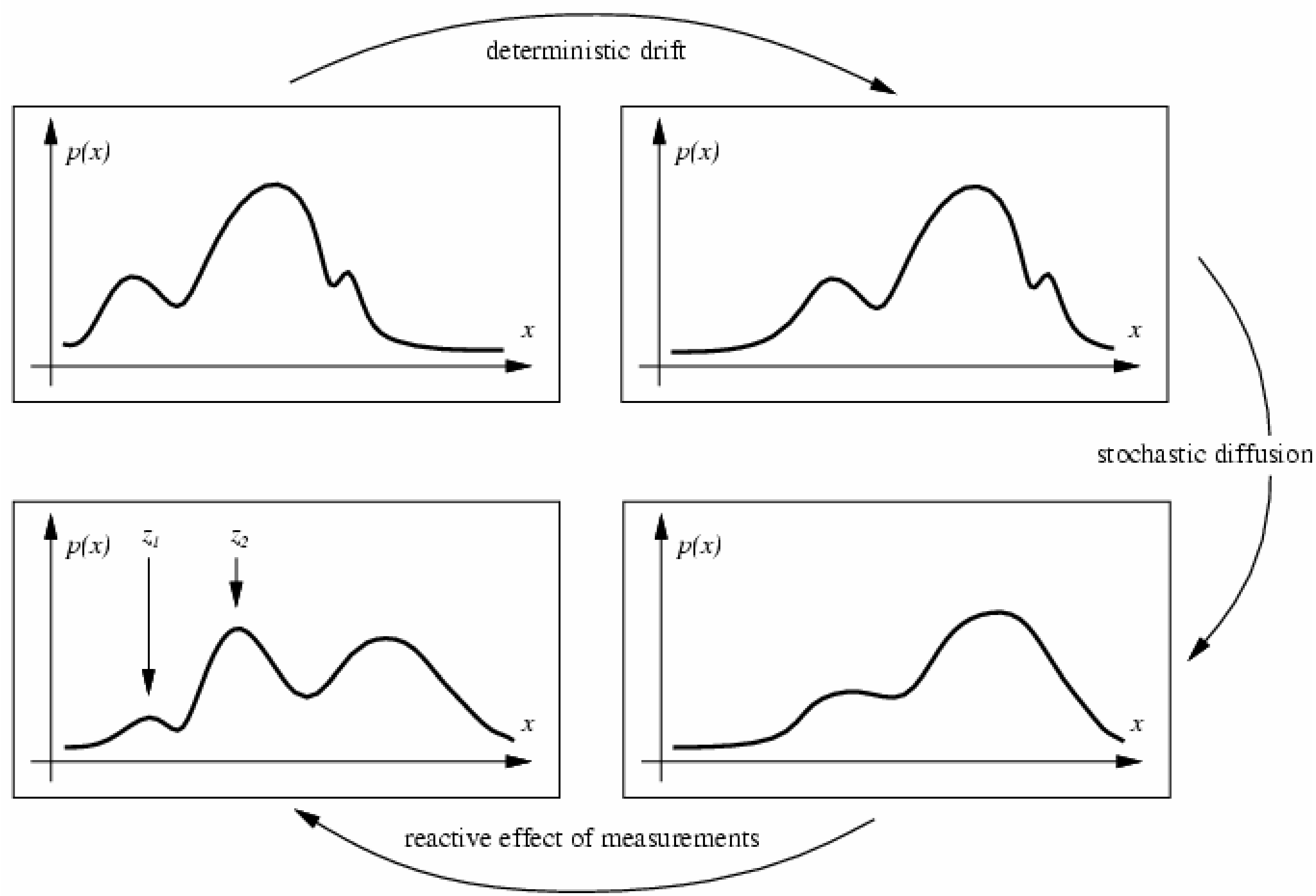


Noise added to that prediction

Make new measurement at next time frame

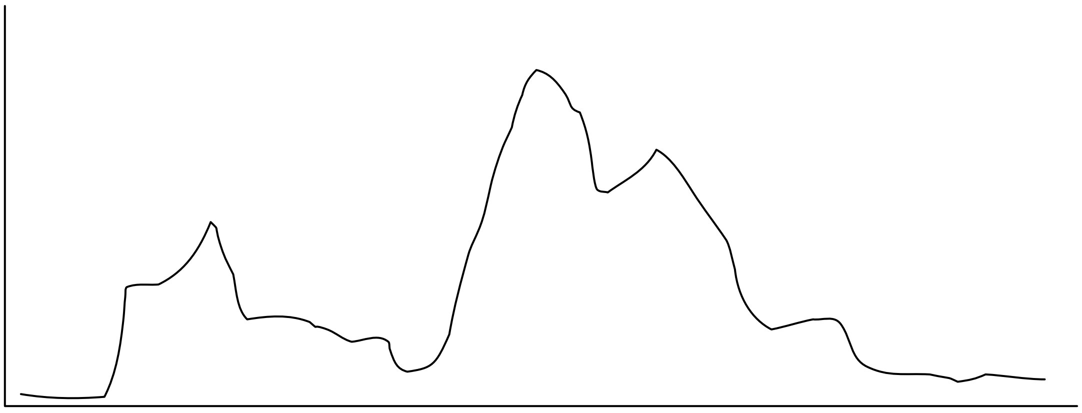
[Isard 1998]

Distribution propagation



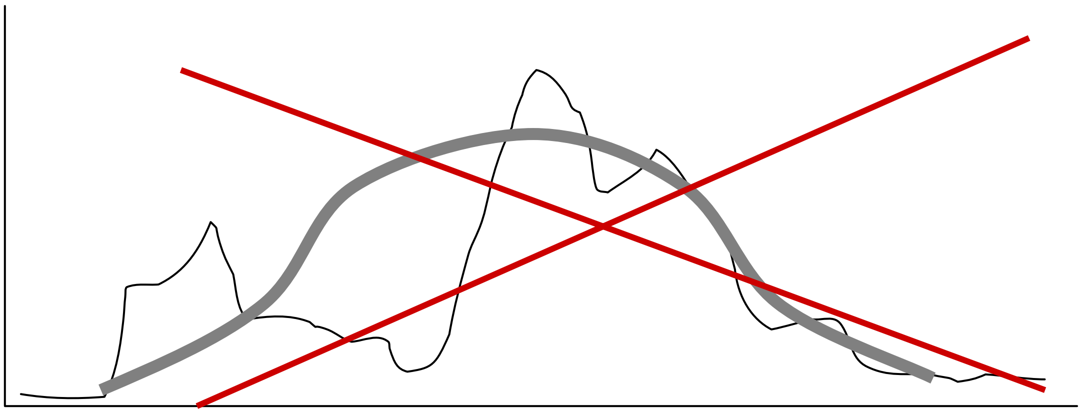
[Isard 1998]

Representing non-linear Distributions



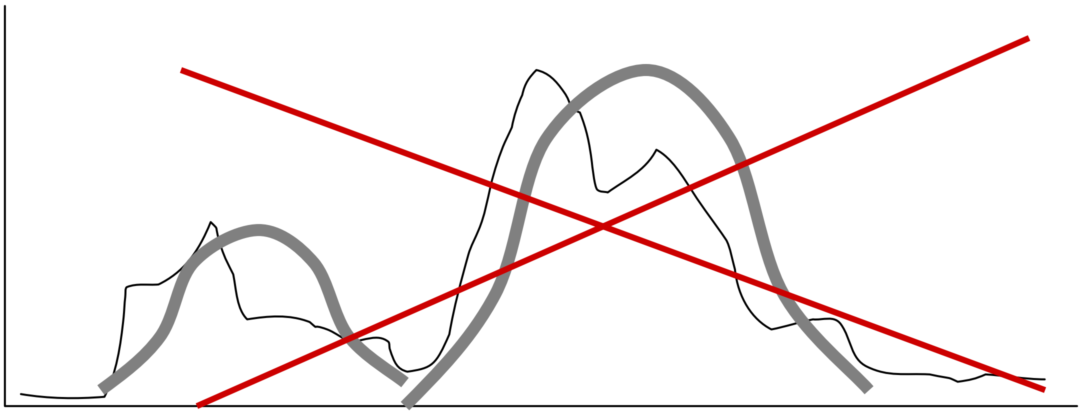
Representing non-linear Distributions

Unimodal parametric models fail to capture real-world densities…



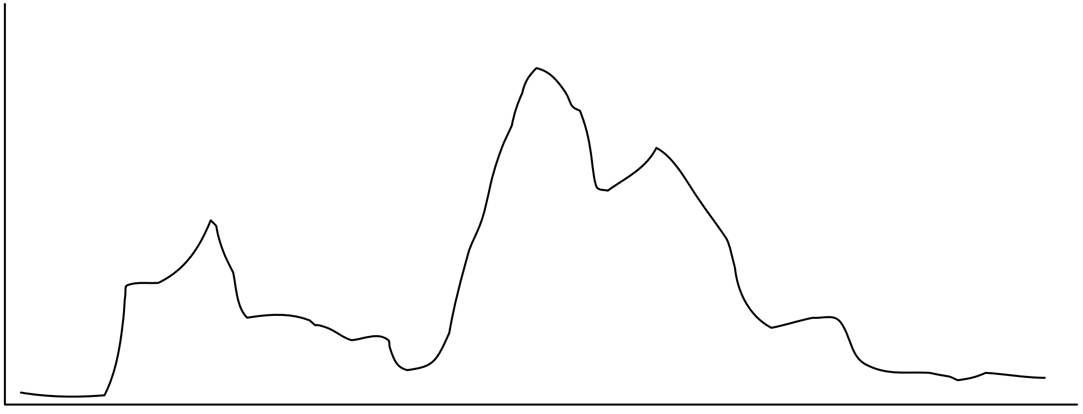
Representing non-linear Distributions

Mixture models are appealing, but very hard to propagate analytically!



Representing Distributions using Weighted Samples

Rather than a parametric form, use a set of samples to represent a density:



Representing Distributions using Weighted Samples

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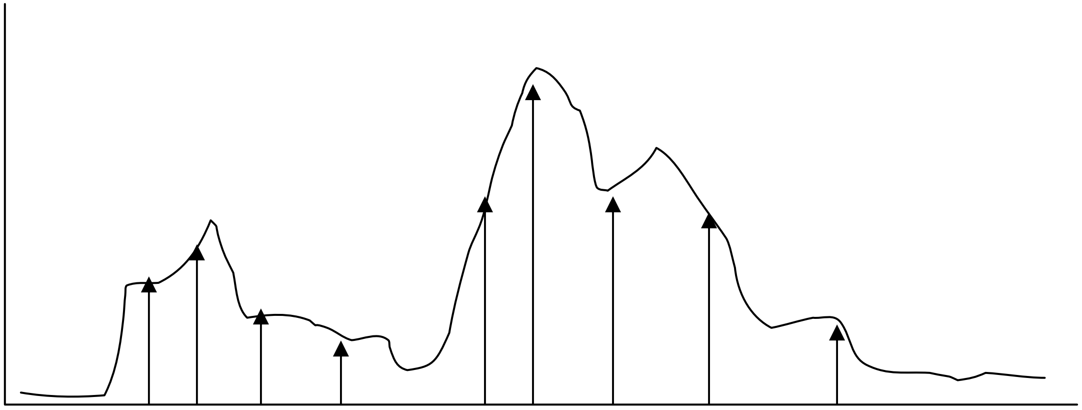
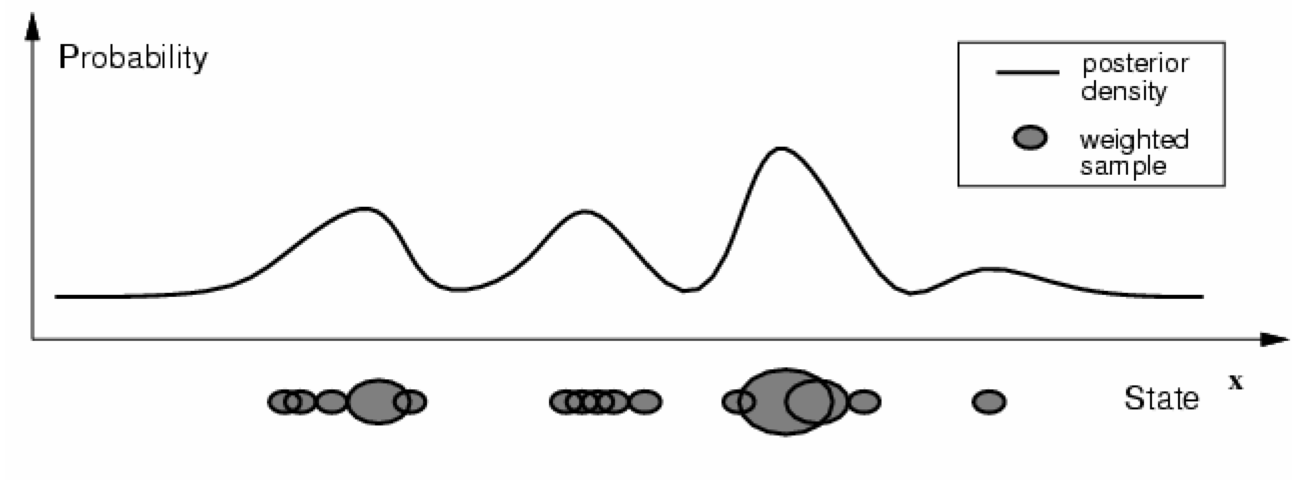


Representing distributions using weighted samples, another picture



[Isard 1998]

Sampled representation of a probability distribution

Represent a probability distribution

$$p_f(\boldsymbol{X}) = \frac{f(\boldsymbol{X})}{\int f(\boldsymbol{U}) d\boldsymbol{U}}$$

by a set of N weighted samples

$$\{(\boldsymbol{u}^i, w^i)\}$$

where $\boldsymbol{u}^i \sim s(\boldsymbol{u})$ and $w^i = f(\boldsymbol{u}^i)/s(\boldsymbol{u}^i)$.

You can also think of this as a sum of dirac delta functions, each of weight w:

$$p_f(x) = \sum_i w^i \delta(x - u^i)$$

Marginalizing a sampled density

If we have a sampled representation of a joint density

$$\{((\boldsymbol{m}^i, \boldsymbol{n}^i), w^i)\}$$

and we wish to marginalize over one variable:

$$p_f(\boldsymbol{M}) = \int p_f(\boldsymbol{M}, \boldsymbol{N}) d\boldsymbol{N}$$

we can simply ignore the corresponding components of the samples (!):

$$\begin{aligned}
\int g(\boldsymbol{M}) p_f(\boldsymbol{M}) d\boldsymbol{M} &= \int g(\boldsymbol{M}) \int p_f(\boldsymbol{M}, \boldsymbol{N}) d\boldsymbol{N} d\boldsymbol{M} \\
&= \int \int g(\boldsymbol{M}) p_f(\boldsymbol{M}, \boldsymbol{N}) d\boldsymbol{N} d\boldsymbol{M} \\
&\approx \frac{\sum_{i=1}^{N} g(\boldsymbol{m}^i) w^i}{\sum_{i=1}^{N} w^i}
\end{aligned}$$

Marginalizing a sampled density

Assume we have a sampled representation of a distribution

$$p_f(\boldsymbol{M}, \boldsymbol{N})$$

given by

$$\left\{((\boldsymbol{m}^i, \boldsymbol{n}^i), w^i)\right\}$$

Then

$$\left\{(\boldsymbol{m}^i, w^i)\right\}$$

is a representation of the marginal,

$$\int p_f(\boldsymbol{M}, \boldsymbol{N}) d\boldsymbol{N}$$

Sampled Bayes Rule

Transforming a Sampled Representation of a Prior into a Sampled Representation of a Posterior:

$$\int g(\boldsymbol{U}) \underbrace{p(\boldsymbol{U}|\boldsymbol{V}=v_0)}_{\text{posterior}} d\boldsymbol{U} = \frac{1}{K} \int g(\boldsymbol{U}) \underbrace{p(\boldsymbol{V}=\boldsymbol{v}_0|\boldsymbol{U})}_{\text{likelihood}} \underbrace{p(\boldsymbol{U})}_{\text{prior}} d\boldsymbol{U}$$

posterior

likelihood, prior

$$\approx \frac{1}{K} \frac{\sum_{i=1}^{N} g(\boldsymbol{u}^i) p(\boldsymbol{V}=\boldsymbol{v}_0|\boldsymbol{u}^i) w^i}{\sum_{i=1}^{N} w^i}$$

$$\approx \frac{\sum_{i=1}^{N} g(\boldsymbol{u}^i) p(\boldsymbol{V}=\boldsymbol{v}_0|\boldsymbol{u}^i) w^i}{\sum_{i=1}^{N} p(\boldsymbol{V}=\boldsymbol{v}_0|\boldsymbol{u}^i) w^i}$$

Sampled Bayes rule

Assume we have a representation of $p(\boldsymbol{U})$ as

$$\left\{(\boldsymbol{u}^i, w^i)\right\}$$

Assume we have an observation $\boldsymbol{V} = \boldsymbol{v}_0$,

and a likelihood model $p(\boldsymbol{V}|\boldsymbol{U})$.

The posterior, $p(\boldsymbol{U}|\boldsymbol{V} = \boldsymbol{v}_0)$ is represented by

$$\left\{(\boldsymbol{u}^i, w'^i)\right\}$$

where

$$w'^i = p(\boldsymbol{V} = \boldsymbol{v}_0|\boldsymbol{u}^i)w^i$$

Sampled Prediction

$$P(\boldsymbol{x}_i | \boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1}) \;\; = \;\; ?$$

$$p(\boldsymbol{X}_{i-1} | \boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})$$

$$\left\{ (\boldsymbol{u}_{i-1}^k, w_{i-1}^k) \right\} \longrightarrow \boldsymbol{x}_i = \boldsymbol{f}(\boldsymbol{x}_{i-1}) + \xi_i \longrightarrow$$

$$\left\{ ((f(\boldsymbol{u}_{i-1}^k) + \xi_i^l, \boldsymbol{u}_{i-1}^k), w_{i-1}^k) \right\}$$

$$p(\boldsymbol{X}_i, \boldsymbol{X}_{i-1} | \boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})$$

Drop elements to marginalize to get

$$P(\boldsymbol{x}_i | \boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1}) \;\; \sim=$$

$$\left\{ (f(\boldsymbol{u}_{i-1}^k) + \xi_i^l, w_{i-1}^k) \right\}$$

Sampled Correction (Bayes rule)

Prior → posterior

Reweight every sample with the likelihood of the observations, given that sample:

$$p(\boldsymbol{Y}_i = \boldsymbol{y}_i | \boldsymbol{X}_i = \boldsymbol{s}_i^{k,-}) w_i^{k,-}$$

yielding a set of samples describing the probability distribution after the correction (update) step:

$$\left\{ (\boldsymbol{s}_i^{k,-}, p(\boldsymbol{Y}_i = \boldsymbol{y}_i | \boldsymbol{X}_i = \boldsymbol{s}_i^{k,-}) w_i^{k,-}) \right\}$$

Naïve PF Tracking

- Start with samples from something simple (Gaussian)
- Repeat
  - Correct

$$\left\{(\boldsymbol{s}_i^{k,-}, p(\boldsymbol{Y}_i = \boldsymbol{y}_i | \boldsymbol{X}_i = \boldsymbol{s}_i^{k,-}) w_i^{k,-})\right\}$$

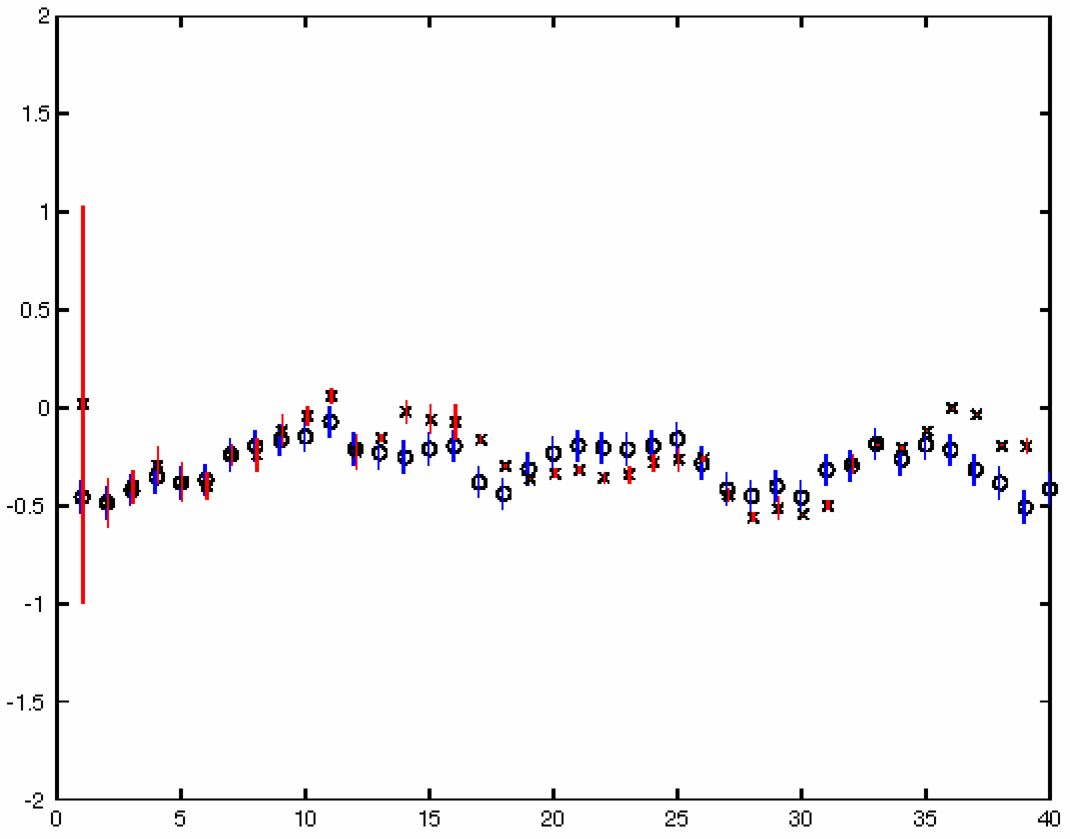
  - Predict

$$\left\{(f(\boldsymbol{u}_{i-1}^k) + \xi_i^l, w_{i-1}^k)\right\}$$

But doesn't work that well because of sample impoverishment…

Sample impoverishment

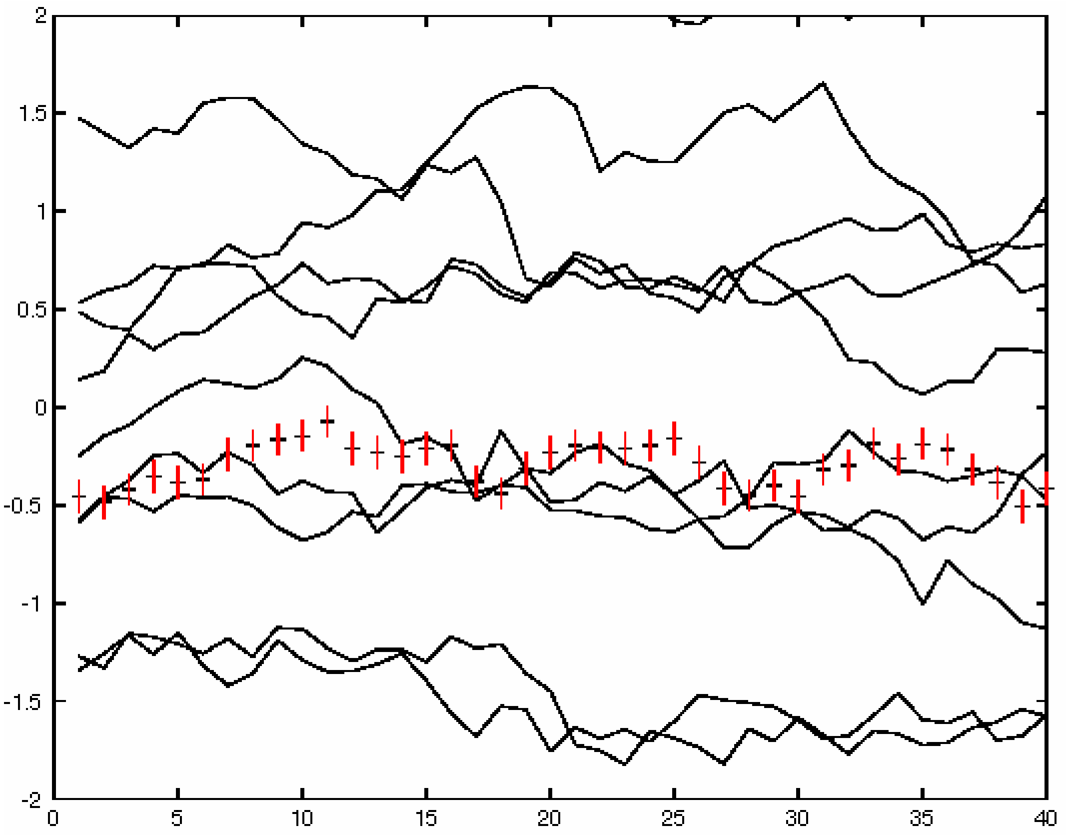
Test with linear case:



kf: x
pf: o

Sample impoverishment

10 of the 100 particles, along with the true Kalman filter track, with variance:



time

Resample the prior

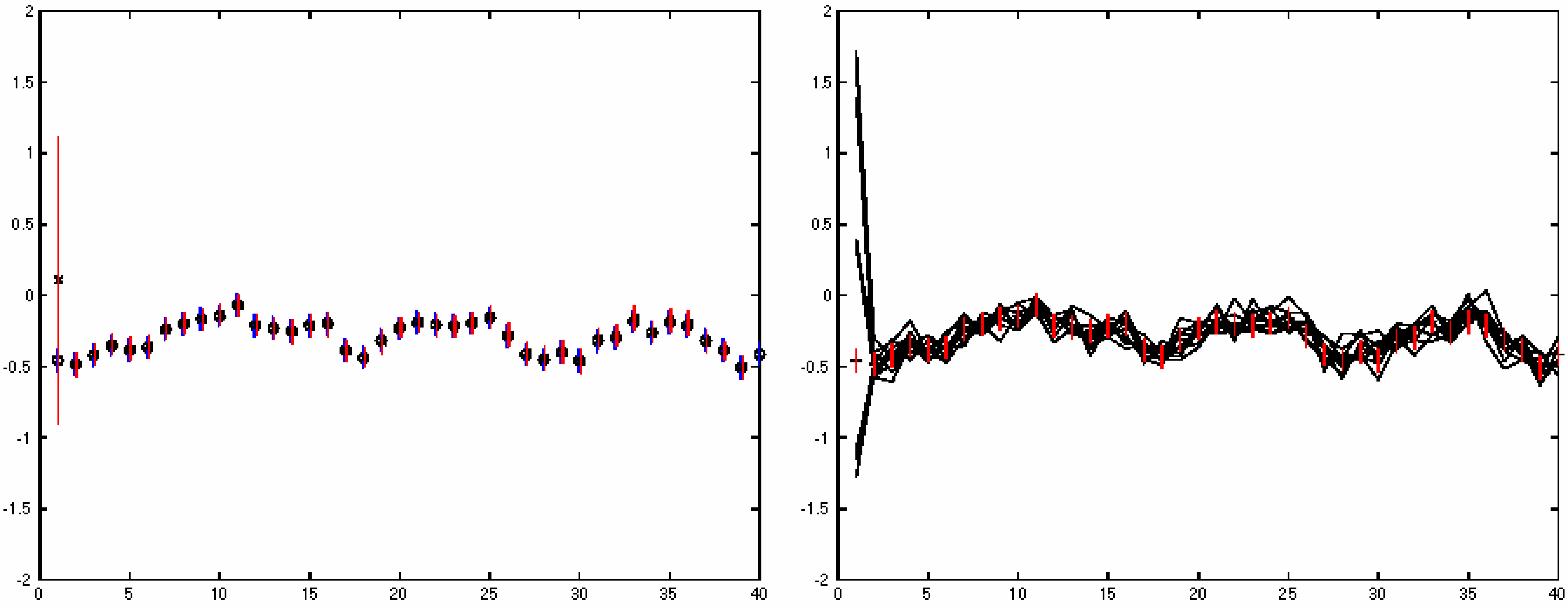
In a sampled density representation, the frequency of samples can be traded off against weight:

$$(\boldsymbol{s}_k, w_k) \longrightarrow \begin{matrix} (\boldsymbol{s}_k, 1) \\ (\boldsymbol{s}_k, 1) \\ (\boldsymbol{s}_k, 1) \\ \vdots \end{matrix} \; N_k \text{ copies} \quad \text{s.t.} \quad \frac{N_k}{\sum_k N_k} = w_k$$

These new samples are a representation of the same density.

I.e., make N draws with replacement from the original set of samples, using the weights as the probability of drawing a sample.

Resampling concentrates samples



A practical particle filter with resampling

Initialization: Represent $P(X_0)$ by a set of N samples

$$\left\{ (s_0^{k,-}, w_0^{k,-}) \right\}$$

where

$$s_0^{k,-} \sim P_s(S) \text{ and } w_0^{k,-} = P(s_0^{k,-})/P_s(S = s_0^{k,-})$$

Ideally, $P(X_0)$ has a simple form and $s_0^{k,-} \sim P(X_0)$ and $w_0^{k,-} = 1$.

Prediction: Represent $P(X_i|y_0, y_{i-1})$ by

$$\left\{ (s_i^{k,-}, w_i^{k,-}) \right\}$$

where

$$s_i^{k,-} = f(s_{i-1}^{k,+}) + \xi_i^k \text{ and } w_i^{k,-} = w_{i-1}^{k,+} \text{ and } \xi_i^k \sim N(0, \Sigma_{d_i})$$

Correction: Represent $P(X_i|y_0, y_i)$ by

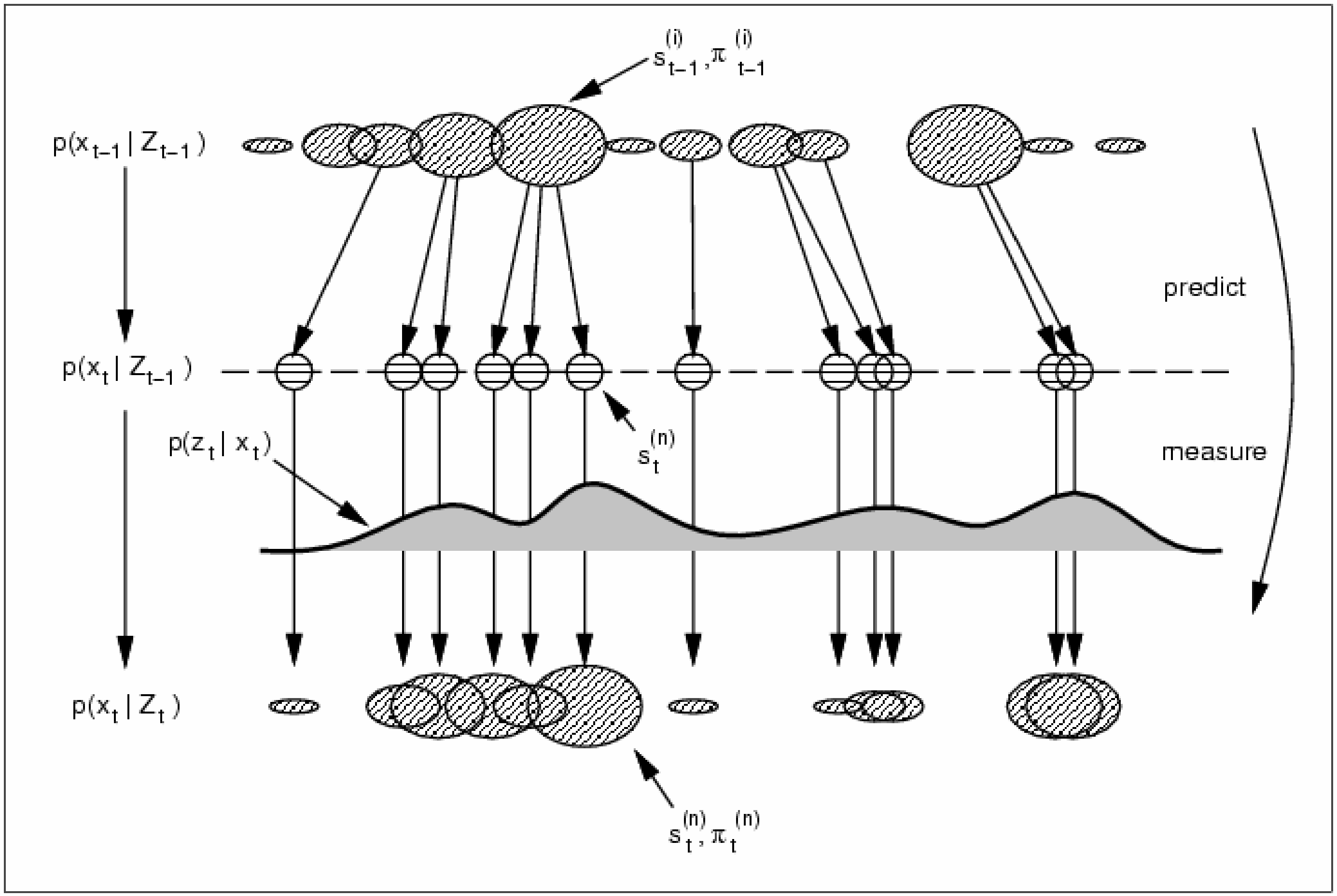
$$\left\{ (s_i^{k,+}, w_i^{k,+}) \right\}$$

where

$$s_i^{k,+} = s_i^{k,-} \text{ and } w_i^{k,+} = P(Y_i = y_i | X_i = s_i^{k,-}) w_i^{k,-}$$

Resampling: Normalise the weights so that $\sum_i w_i^{k,+} = 1$ and compute the variance of the normalised weights. If this variance exceeds some threshold, then construct a new set of samples by drawing, with replacement, N samples from the old set, using the weights as the probability that a sample will be drawn. The weight of each sample is now $1/N$.

A variant (predict, then resample, then correct)



[Isard 1998]

Contour tracking by stochastic propagation of conditional density - Mozilla Firefox

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Contour tracking by stochastic propagation of conditional density

Michael Isard and Andrew Blake

Proc. European Conference on Computer Vision, vol. 1, pp. 343--356, Cambridge UK, (1996).

Abstract

The problem of tracking curves in dense visual clutter is a challenging one. Trackers based on Kalman filters are of limited use; because they are based on Gaussian densities which are unimodal, they cannot represent simultaneous alternative hypotheses. Extensions to the Kalman filter to handle multiple data associations work satisfactorily in the simple case of point targets, but do not extend naturally to continuous curves. A new, stochastic algorithm is proposed here, the **Condensation** algorithm --- Conditional Density Propagation over time. It uses `factored sampling', a method previously applied to interpretation of static images, in which the distribution of possible interpretations is represented by a randomly generated set of representatives. The **Condensation** algorithm combines factored sampling with learned dynamical models to propagate an entire probability distribution for object position and shape, over time. The result is highly robust tracking of agile motion in clutter, markedly superior to what has previously been attainable from Kalman filtering. Notwithstanding the use of stochastic methods, the algorithm runs in near real-time.

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A variant (animation)

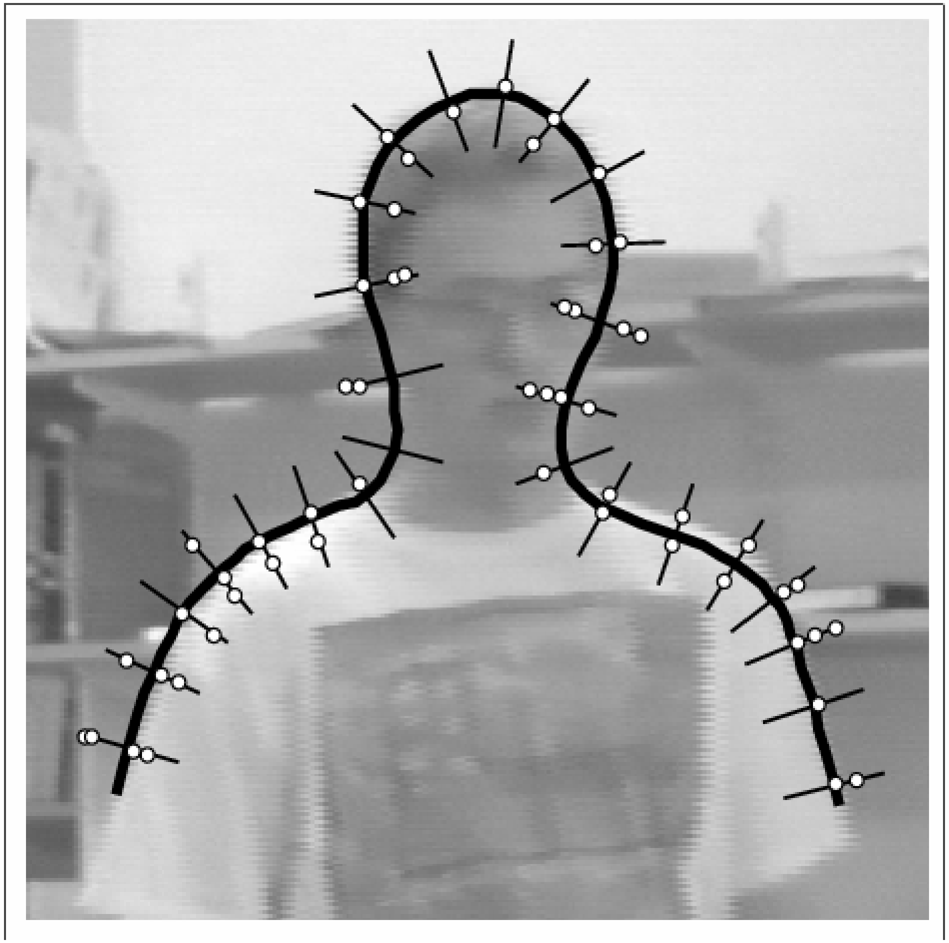
[Isard 1998]

Applications

Tracking

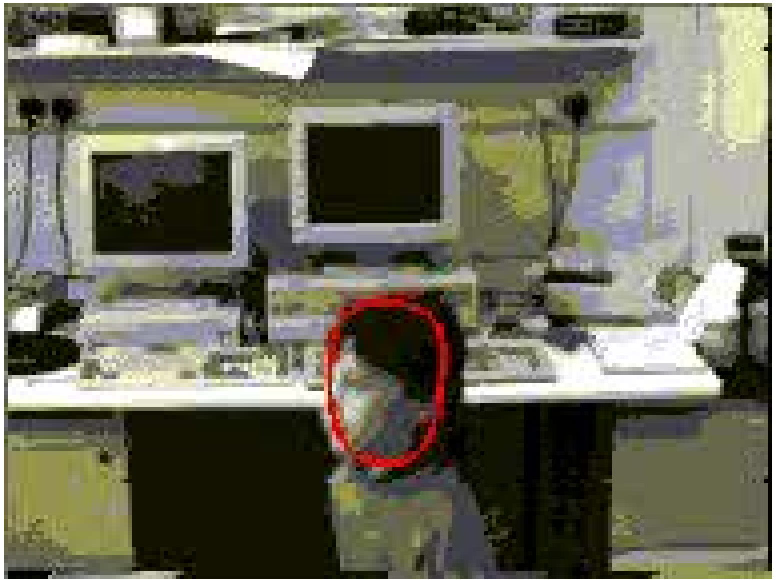
- hands
- bodies
- leaves

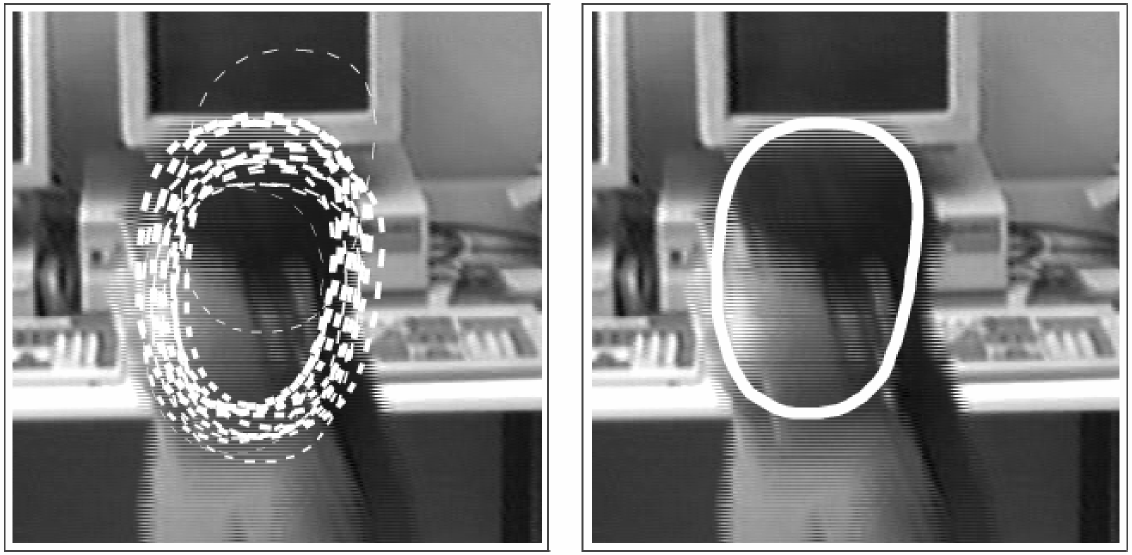
Contour tracking



[Isard 1998]

Head tracking





(a) (b)

[Isard 1998]

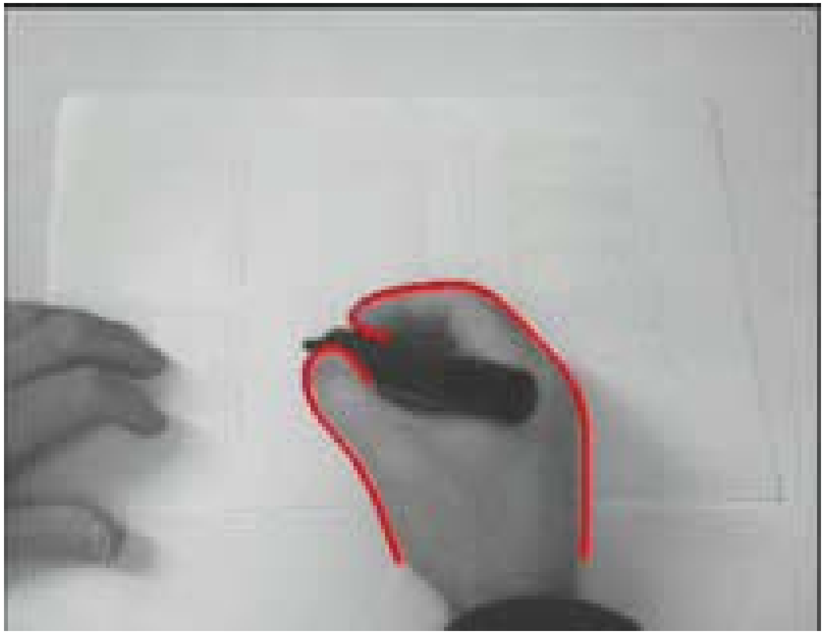
Leaf tracking

[Isard 1998]

Hand tracking

[Isard 1998]

Mixed state tracking



[Isard 1998]

Outline

- Sampling densities
- Particle filtering

[Figures from F&P except as noted]