

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF COMPUTER SCIENCE AND ELECTRICAL ENGINEERING

6.801/6.866 MACHINE VISION

Handed out: 2009 Nov 5th

Due on: 2009 Nov. 14th

Problem 1: Consider the binary image of a circular disk that is approximately d pixels across. We are going to estimate the expected error in finding the horizontal (x) component of the centroid. We break the image up into rows. Within each row that intersects the disk there is a contiguous set of pixels where $b_{ij} = 1$.



Now the value of first and last pixels in this sequence are somewhat uncertain in that additive image measurement noise might push the brightness measurements below threshold, or push adjacent pixels, now classified as “background,” above threshold. The centroid of the row of pixels then has an error with standard deviation some multiple (less than one) of the size of a pixel. Due to this error, the centroids of different rows intersecting the disc will not be exactly the same. In computing the centroid of the disc, we are in essence averaging these noisy estimates.

How do you expect the error in the x -component of the centroid to vary with the size d of the image of the disc? Can the location of a large disc be found more accurately than the location of a small disc?

Problem 2: Consider binary image data streaming in along a row. We feed the bits into an adder/accumulator that is reset to zero at the start of the row. Clearly at the end of the row the accumulator contains the number of bits that were ‘1’ in that row. Now consider a second adder/accumulator that takes its input not from the image data, but from the output of the first accumulator. Like the first accumulator, it adds its current input to the current total when a new pixel is scanned in.

- (a) Show that the contribution of a single bit in the image to the output of the second accumulator depends on its position in the image row—a bit at the end of the row being added in only once, while a bit at the start of the row is added in many times.

- (b) How would you use such a device (modified if needed) to compute the (horizontal) first moment without using multiplication (Hint: you may need a buffer, another accumulator, or some “post processing” of the result in the accumulator).
- (c) Suggest how you could obtain the second (and perhaps higher order) moments using additional accumulators, again without needing multiplication.

Problem 3: To compute the area, position and orientation of a region in the image plane we need the zeroth, first and second moments. These can be computed by accumulating appropriate sums of products as we step through the pixels of the region. Alternatively, we can first trace the outline of the region and then derive the moments from the outline. Assume that the region is represented by a polygon with vertices (x_i, y_i) for $i = 0, 1, \dots, N-1$ and that in traversing the polygon one goes around the region in an anti-clockwise fashion. Also assume that modulo N arithmetic is to be used on the indices so that e.g. $(x_N, y_N) = (x_0, y_0)$ and $(x_{-1}, y_{-1}) = (x_{N-1}, y_{N-1})$.

- (a) Show that the area or zeroth moment of the polygon is

$$m_{00} = \frac{1}{2} \sum_{i=0}^{N-1} (x_{i-1}y_i - x_i y_{i-1})$$

- (b) Show that the first moments (needed to compute the centroid) are given by

$$m_{10} = \frac{1}{6} \sum_{i=0}^{N-1} (x_{i-1} + x_i)(x_{i-1}y_i - x_i y_{i-1})$$

$$m_{01} = \frac{1}{6} \sum_{i=0}^{N-1} (y_{i-1} + y_i)(x_{i-1}y_i - x_i y_{i-1})$$

- (c) Show that the second moments (needed to compute the orientation) are given by

$$m_{20} = \frac{1}{12} \sum_{i=0}^{N-1} (x_{i-1}^2 + x_{i-1}x_i + x_i^2)(x_{i-1}y_i - x_i y_{i-1})$$

$$m_{11} = \frac{1}{24} \sum_{i=0}^{N-1} (2x_{i-1}y_{i-1} + x_{i-1}y_i + y_{i-1}x_i + 2x_i y_i)(x_{i-1}y_i - x_i y_{i-1})$$

$$m_{02} = \frac{1}{12} \sum_{i=0}^{N-1} (y_{i-1}^2 + y_{i-1}y_i + y_i^2)(x_{i-1}y_i - x_i y_{i-1})$$

Possibly useful facts:

- * Gauss's integral formula, a.k.a. Green's formula, a.k.a Stokes' formula relates certain area integrals to certain contour integrals.
- * A triangle with vertices $(0, 0)$, (x_{i-1}, y_{i-1}) , and (x_i, y_i) has area $(x_{i-1}y_i - x_i y_{i-1})/2$.
- * The centroid of a triangle is a third of the way along a line connecting the center of the base — formed by any two of the vertices — to the remaining vertex.
- * Once you have computed m_{10} you can easily derive m_{01} by means of a $\pi/2$ rotation of the coordinate system.
- * Once you have computed m_{20} you can easily derive m_{02} by means of a $\pi/2$ rotation of the coordinate system.
- * Once you have computed $(m_{20} - m_{02})/2$ you can easily derive m_{11} by means of a $\pi/4$ rotation of the coordinate system.
- * The second moment of a region about an axis that does not pass through the centroid equals the second moment about a parallel axis that does pass through the centroid plus the product of the area and the square of the distance from the axis from the centroid (parallel axis theorem).

Problem 4: This problem is about the brightness gradient — and higher partial derivatives — in a rotated coordinate system. Consider the transformation

$$\begin{aligned} s &= x \cos \theta + y \sin \theta \\ t &= -x \sin \theta + y \cos \theta \end{aligned}$$

- (a) Solve for x and y in terms of s and t .
- (b) Express the brightness gradient components b_s and b_t in the rotated coordinate system in terms of the brightness gradient components b_x and b_y in the original coordinate system.
- (c) If the s -axis lies in the direction of the brightness gradient (b_x, b_y) , show that, in this special case

$$\cos \theta_0 = \frac{b_x}{\sqrt{b_x^2 + b_y^2}} \quad \text{and} \quad \sin \theta_0 = \frac{b_y}{\sqrt{b_x^2 + b_y^2}}$$

- (d) Using the angle θ_0 from part (c), show that the slope in the direction of the brightness gradient (b_x, b_y) is $b_s = \sqrt{b_x^2 + b_y^2}$ and that the slope in the direction $(-b_y, b_x)$ perpendicular to the brightness gradient is $b_t = 0$.
- (e) Show that the second partial derivative in the direction of the s -axis is

$$b_{ss} = b_{xx} \cos^2 \theta + 2b_{xy} \sin \theta \cos \theta + b_{yy} \sin^2 \theta$$

- (f) Using the angle θ_0 from part (c), show that the second partial derivative in the direction of the brightness gradient is

$$b_{ss} = \frac{b_{xx}b_x^2 + 2b_{xy}b_xb_y + b_{yy}b_y^2}{b_x^2 + b_y^2}$$

while the second partial derivative in the direction perpendicular to the brightness gradient is

$$b_{tt} = \frac{b_{xx}b_y^2 - 2b_{xy}b_xb_y + b_{yy}b_x^2}{b_x^2 + b_y^2}$$

Problem 5: Here we consider a method for computing the “Euler number” of a continuous “almost binary” image $b(x, y)$, where

$$0 \leq b(x, y) \leq 1$$

Consider a rotationally symmetric image $b(x, y) = f(r)$. If needed, assume $b(x, y) = 1$ for $r < R - \epsilon$ and $b(x, y) = 0$ for $r > R + \epsilon$ — with a smooth monotonic transition between $f(r) = 1$ at $r = R - \epsilon$ to $f(r) = 0$ at $r = R + \epsilon$.

- (a) Show that

$$b_x = f'(r)\frac{x}{r} \quad \text{and} \quad b_y = f'(r)\frac{y}{r}$$

and hence that

$$b_x^2 + b_y^2 = f'(r)^2$$

- (b) Show that

$$b_{xx} = f''(r)\frac{x^2}{r^2} + f'(r)\frac{y^2}{r^3}$$

$$b_{xy} = f''(r)\frac{xy}{r^2} - f'(r)\frac{xy}{r^3}$$

$$b_{yy} = f''(r)\frac{y^2}{r^2} + f'(r)\frac{x^2}{r^3}$$

- (c) Show that

$$\frac{b_{xx}b_y^2 - 2b_{xy}b_xb_y + b_{yy}b_x^2}{b_x^2 + b_y^2} = \frac{f'(r)}{r}$$

- (d) By changing to polar coordinates, show that

$$\iint \frac{f'(r)}{r} dx dy = -2\pi$$

independent of the radius R — or the exact nature of the “edge transition” $f(r)$ — as long as $f(r) = 1$ for $r < R - \epsilon$ and $f(r) = 0$ for $r > R + \epsilon$.

Now consider

$$E = -\frac{1}{2\pi} \iint \frac{b_{xx}b_y^2 - 2b_{xy}b_xb_y + b_{yy}b_x^2}{b_x^2 + b_y^2} dx dy$$

- (e) What is E for $b(x, y) = f(r)$ as defined above? What is E for $b(x, y) = f(2r)$? What is E if $b(x, y) = 1 - f(r)$? What is E for $b(x, y) = f(r) - f(2r)$? What is the geometric/topological interpretation of these result?
- (f) What is E for $b(x, y) = f(r_1) + f(r_2)$ where $r_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}$ and $r_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2}$ and the separation between (x_1, y_1) and (x_2, y_2) is larger than say $2R$. What is the geometric/topological interpretation of these result?