

6.857 Hash fns

3/10/21

$$h : \{0,1\}^* \rightarrow \{0,1\}^d$$

$h(x)$ = hash value, message digest

ROM = gives "random" value for each input x

Properties: DW, CR, TCR, PR
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 last time this time

- Applications:
- Password Storage
 - File modification
 - Digital Signature
 - Commitments
 - Voting Equipment Security
 - MD construction
 - Merkle Trees

Properties: $H = \{h_s\}$ $h_s(x) = y$ $y \in \{0,1\}^d$
 r_{seed}

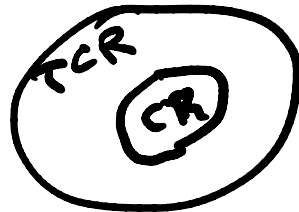
One-way: Given y in range of h
infeasible to find any x s.t. $h(x) = y$

Collision-Resistance: Infeasible to find x, x'
s.t. $h(x) = h(x')$

Target Collision Resistance:

Given x , infeasible to find $x' \neq x$
random \rightarrow s.t. $h(x') = h(x)$

Given x
compute $x' \in h^{-1}(x)$
 \Rightarrow collision



$\neg \text{TCR} \Rightarrow \neg \text{CR}$
 $\text{CR} \Rightarrow \text{TCR}$

Pseudo-random: infeasible to distinguish between

- box with PO
- " with h_s

Applications:

• Password storage

User has password pw

Naive

Table: (U, pw)
↑ name ↑ password

Modified: Store (U, h(pw)) in table

Requires: OW
→ pw (forbidden)

fails often in practice because pw's poorly chosen

⇒ exhaustive search works for Adv

$h_s(pw)$ often
↑ seed or salt
per system

$h_s(pw) = h(s || pw)$
↑ SHA-256

|| = concatenation operator

$h_s(u || pw)$

"ab" || "cd" = "abcd"

"ab" || "," || "cd"

- File modification detection

store $(s, h_s(F))$

$s = \text{seed}$

$F = \text{file}$

Securely
(on thumb)

↑ target

for each file F on disk

recompute $h_s(F)$

to see if it matches

Requires! TCR: Adv wants to find F'
st. $h_s(F') = h_s(F)$

- Digital Signatures:

PK public key for signature verification

SK

Signature on message: $SK(M)$

↳ might be long!
too long!

So: Signature as $SK(\underbrace{h_s(M)}_{\text{short!}})$

Requires: CR
not find m, m' $h_s(m) = h_s(m')$

• Commitments: (Auction)

bid m message

send commitment Com(m,r) to auctioneer

open commitment by revealing m, r

Binding: Com(m,r) binds me to revealing m later

Hiding: Com(m,r) doesn't reveal m
 $h_s(m||r) = \text{Com}(m,r)$

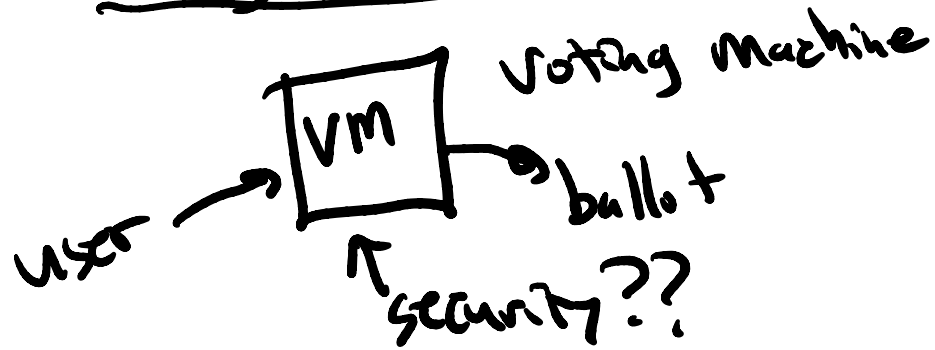
\neg Binding $\Rightarrow h_s(m||r) = h_s(m'||r')$ collision

\neg Hiding $\Rightarrow \underbrace{h_s(m||r)}_{\text{Com}(m||r)}$ reveals m, r

\hookrightarrow OW on m

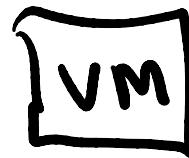
CR
OW
 \uparrow

Voting Equipment Security



VM Certified
for particular
software suite

electron:



is this
running
right software?

is this possible?

can you check without running software on VM

$hash(VM\ software) = h(certified\ software)$

is any part of VM trusted?

read VM memory from outside

- ① VM asked to check hash code!
- ② VM always says "yes, Oh!" (bug)
- ③ Doesn't check against certified software
checks
$$h(\text{VM software}) = h(\text{VM } \neq \text{ software})$$

Terrible!

- ④ Contract with vendor says
acceptance testing, must be done by vendor
not E.O. ! (Else voids warranty)

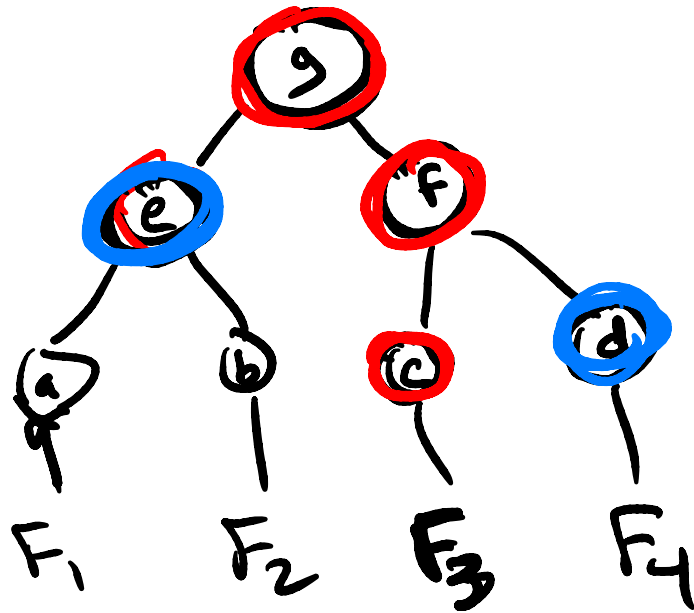
(∴ ⇒ Use paper ballots!)

Merkle Trees:

Goal: authenticate many files

F_1, F_2, \dots, F_{106}

Build a tree to authenticate all, given hsh(root)



~~h~~
 $e = h(a || b)$
 $\Rightarrow g = \text{hash value at root}$

F_3
c, d, e

Requires: CR