TOPIC: 6.857 - Computer & Network Security

DATE: Rivest

FILE UNDER:

PAGE: L15.1

Admin:

Pset #4 posted
Quiz on 4/18 in-class

Today:

- Public-key encryption
- El Gamal PK encryption
- Semantic security
- DDH
- IND-CCA2 security
- Cramer-Shoup PK encryption

Readings:

Paar & Pelzl, chapters 6, 7, 8

Katz & Lindell, Chapter 10

Public-key encryption:

Let λ = "security parameter" (i.e. "key size")

Then $1^\lambda = \underbrace{11\cdots1}_{\lambda}$ λ 1's in a row. Length $= \lambda$

Need three algorithms:

① $\text{Keygen}(1^\lambda) \longrightarrow (PK, SK)$

② $E(PK, m) \longrightarrow c$

Encryption takes $m \in$ message space M

to $c \in$ ciphertext space C

(with given public key PK)

Encryption may be <u>randomized</u>.

③ $D(SK, c) \rightarrow m$

Decryption is <u>deterministic</u>

s.t. (Correctness condition)

$$(\forall (PK, SK))(\forall m)\ D(SK, E(PK, m)) = m$$

TOPIC:

DATE:

FILE UNDER:

randomized!

El-Gamal PK encryption (Taher El Gamal, 1984)

Let $G = \langle g \rangle$ be a cyclic group with generator g.

(Keygen may output description of g & G, given λ.)

Keygen:

Pick x at random from $[0 \ldots |G|-1]$

Let $SK = x$.

Let $PK = g^x$

Output (PK, SK) (& description of G, if needed)

Encryption:

Pick k at random from $[0 \ldots |G|-1]$

Assume message m represented as element of G.

Let $y = g^x$ be PK of recipient

Output $c = (g^k, m \cdot y^k)$ as ciphertext

Decryption:

Let $c = (a, b)$ be received ciphertext

Let $m = b / a^x$. Output m.

[Correctness follows since $a^x = g^{kx} = g^{xk} = y^k$.]

El Gamal encryption related to DH key exchange:

Alice $\xrightarrow{\quad y = g^x \quad (\text{via PKI?}) \quad}$ Bob

$\xleftarrow{\quad a = g^k \quad}$

DH key $= (g^k)^x$ $\qquad$ DH key

$= g^{kx}$ $\qquad$ $= (g^x)^k = g^{kx}$

$=$

$\xleftarrow{\quad b = m \cdot (\text{DHkey}) \quad}$

Encrypt by multiplying by DH key.

Decrypt by dividing by DH key.

How to define security for PK encryption?

We'll see two definitions:

① "semantic security" (Goldwasser & Micali)

② "adaptive chosen ciphertext attack" (ACCA) secure

(≈ to IND-CCA we saw for symmetric encryption)

"Game" definition of semantic security:

Phase I ("Find"):

- Examiner generates (PK, SK) using $\text{Keygen}(1^\lambda)$
- Examiner sends PK to Adversary
- Adversary computes for polynomial (in λ) time, then outputs two messages m_0, m_1 of same length, and "state information" s. [$m_0 \neq m_1$ required]

Phase II ("Guess"):

- Examiner picks $b \xleftarrow{R} \{0,1\}$, computes $c_* = E(PK, m_b)$
- Examiner sends c_*, s to Adversary
- Adversary computes for polynomial (in λ) time, then outputs $\hat{b}$ (his "guess" for b).

Adversary "wins" game if $\hat{b} = b$.

TOPIC:

DATE:

FILE UNDER:

Def: A PK encryption scheme is semantically secure if $\text{Prob}[\text{Adv wins}] \leq \frac{1}{2} + \text{negligible}$

Fact: In order for a PK encryption scheme to be semantically secure, it must necessarily be randomized. (Randomized encryption is necessary but not sufficient for semantic security.)

Is El Gamal PK encryption semantically secure?

DDH (Decision Diffie-Hellman Assumption):

Given a group G with generator g:

It is hard/infeasible to decide whether a given triple of elements was generated as

$$(g^a, g^b, g^c) \qquad [a, b, c \text{ random}]$$

or as

$$(g^a, g^b, g^{ab}) \qquad [a, b \text{ random}]$$

That is, if DDH holds in a group, you can't even <u>recognize</u> the DH key g^{ab} when it is given to you! (You can't distinguish it from a random element.)

<u>Theorem:</u> DDH $\Rightarrow$ CDH

<u>Proof:</u> If $\neg$CDH, then $\neg$DDH (contrapositive).

If you can compute g^{ab} from g^a and g^b (i.e. $\neg$CDH) then you can decide if given third element <u>is</u> g^{ab} (i.e. $\neg$DDH). $\blacksquare$

<u>Theorem</u> (Tsiounis & Yung):

El Gamal is semantically secure in G

$\Longleftrightarrow$

DDH holds in G

- Semantic security may not be enough for some applications.
- El Gamal is <u>malleable</u>:

  Given $E(m) = (g^k, m \cdot y^k)$

  it is easy to produce $E(2m) = (g^k, (2 \cdot m) \cdot y^k)$

  without knowing m!
- More generally, El Gamal is <u>homomorphic</u>:

$$\begin{array}{lll} \text{Given} & c_1 \in E(m_1) & = (g^r, m_1 \cdot y^r) \\ \text{\& given} & c_2 \in E(m_2) & = (g^s, m_2 \cdot y^s) \\ \hline \text{can produce} & c_1 \cdot c_2 & = (g^{r+s}, (m_1 \cdot m_2) \cdot y^{r+s}) \\ & & \in E(m_1 \cdot m_2) \end{array}$$

- Product of ciphertexts yields an encryption of product of plaintexts.
- Special case: multiplying by $E(1) = (g^s, y^s)$ <u>re-randomizes</u> encryption.

TOPIC:

DATE:

FILE UNDER:

- What is stronger notion of security for PK encryption? (e.g. one that excludes malleability...)

- "IND-CCA2 secure" (ACCA secure = secure under adaptive chosen ciphertext attack)
  ≈ IND-CCA secure defn we saw for symmetric enc.

- Similar to semantic security defn, except that Adv allowed access to decryption oracle, too.
  (He has PK so access to encryption oracle already there.)
  (As before, may <u>not</u> use oracle to decrypt challenge ciphertext during "guess" phase.)

IND-CCA2 (ACCA) security game:

Phase I ("Find"):

- Examiner generates (PK, SK) using Keygen(1^λ)
- Examiner sends PK to Adversary
- Adversary computes for polynomial (in λ) time,

new ⇒ having access to a decryption oracle $D(SK, \bullet)$

then outputs two messages m_0, m_1 of same length, and "state information" s. [$m_0 \neq m_1$ required]

Phase II ("Guess"):

- Examiner picks $b \xleftarrow{R} \{0,1\}$, computes $c_* = E(PK, m_b)$
- Examiner sends c_*, s to Adversary
- Adversary computes for polynomial (in λ) time,

new ⇒ having access to a decryption oracle $D(SK, \bullet)$ except on input c_*

then outputs $\hat{b}$ (his "guess" for b).

Adversary wins if $\hat{b} = b$.

Def: PK encryption method is IND-CCA2 secure (ACCA-secure) if

$$\text{Prob}[\text{Adv wins}] \leq \frac{1}{2} + \text{negligible}$$

TOPIC:

DATE:

FILE UNDER:

How to make El Gamal IND-CCA2 secure?

- Cramer–Shoup method is such an extension of El Gamal.
- Let G_q be a group of prime order q

  (e.g. $G_q = Q_p$, where $p = 2q+1$, p & q prime).
- **Keygen:**

$$g_1, g_2 \xleftarrow{R} G_q$$

$$x_1, x_2, y_1, y_2, z \xleftarrow{R} Z_q$$

$$c = g_1^{x_1} g_2^{x_2}$$

$$d = g_1^{y_1} g_2^{y_2}$$

$$\boxed{h = g_1^{z}}$$

EG

$$PK = (g_1, g_2, c, d, h)$$

H = hash fn mapping G_q^3 to Z_q

$$SK = (x_1, x_2, y_1, y_2, z)$$

- **Enc(m) [where $m \in G_q$]:**

$\boxed{r \xleftarrow{R} Z_q}$ EG

$\boxed{u_1 = g_1^r}$ EG

$u_2 = g_2^r$

$\boxed{e = h^r \cdot m}$ EG

$\alpha = H(u_1, u_2, e)$

$v = c^r d^{r\alpha}$

ciphertext $= (\underline{\underline{u_1}}, u_2, \underline{\underline{e}}, v)$ EG

- **Decrypt (u_1, u_2, e, v):**

$\alpha = H(u_1, u_2, e)$

Check: $u_1^{x_1 + y_1\alpha} u_2^{x_2 + y_2\alpha} \stackrel{?}{=} v$

If not equal, <u>reject</u>

else output $\boxed{m = e / u_1^z}$ EG

Note: $u_1^{x_1} u_2^{x_2} = g_1^{rx_1} g_2^{rx_2} = c^r$

$u_1^{y_1} u_2^{y_2} = d^r$

$\boxed{u_1^z = g_1^{rz} = h^r}$ EG

TOPIC:

DATE:

FILE UNDER:

Theorem: Cramer-Shoup is IND-CCA2 secure (i.e. secure against adaptive chosen ciphertexts) if

1. DDH holds in G_q
2. H satisfies a certain condition ($\approx$ "target collision resistance)

Thus, our strongest notion of security for PK encryption is in fact <u>achievable</u>, albeit at some cost in terms of speed & complexity.