

6.857 Rivest  
3/30/11 L15.1

## Admin: Quiz in-class Wed April 6<sup>th</sup>

Coverage: through Monday April 4<sup>th</sup>'s lecture.

Closed book but open notes. (Your own notes & posted notes only.)

## Outline:

- Secret-sharing with short shares
- Key establishment
  - Direct
  - Server-assisted: symmetric } Needham-Schroeder  
asymmetric } protocols

Next time: Large scale PKI:

X.509

SPKI/SDSI

cert revocation

Note: We have  $0 \leq m < p$  so each share is "as big as"  $m$ .  
Shares are  $(i, y_i)$ ; each  $y_i$ : "same size as"  $p$  or  $m$ .  
If file is very large: can break into pieces, share each piece. (byte)  
Put still, each person gets <sup>total</sup> share as big as original  $m$ .  $GF(2^8)$   
How to reduce share sizes?

Example: I have 100 MB file to share among  $n=20$  friends;  
I want  $t=10$  to be able to reconstruct it.  
With original scheme, each gets 100 MB share.  
Goal: each has  $100MB/t = 10$  MB share, ?

Idea 1: Give up information-theoretic (unconditional) security for computational security.

Idea 2: Encrypt message, then share ~~it~~ ciphertext, & share key.  
of ciphertext,

Idea 3: In secret-sharing, secret is set of all  $t$  coefficients, not just constant term. (No longer info-theoretic secure; each point on curve eliminates some polynomials.  $t$  shares reduces possibilities from  $p^t$  down to 1. ~~Each share reduces # polynomials by factor of  $p$ .~~)

Given  $(M, n, t)$

$M$  = message to be shared

(100 MB)

~~Key~~ AES key =  $s$

(128 bits)

Share  $s \Rightarrow n$  shares  $s_1, s_2, \dots, s_n$

s.t. any  $t$  can reconstruct  $s$

(info-theoretically secure)

Give  $i$ th party  $s_i, 1 \leq i \leq n$ . (and  $i$ )

Encrypt:  $C = \text{AES}_s(M)$

(100 MB)

Break  $C$  into chunks  $C_1, C_2, \dots, C_\ell$

(each  $t$  byte  
 $\ell = 100 \text{ MB} / t$ )

Let  $C' = C_k =$ 

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 $t$  bytes

$a_0^{(k)} a_1^{(k)} \dots a_{t-1}^{(k)}$  over  $GF(2^8)$

$f_k(x) = a_0^{(k)} + a_1^{(k)}x + \dots + a_{t-1}^{(k)}x^{t-1}$  [not random  
[ $t$ ] bytes used as coeffs]

Give party  $i$  share  $f_k(i)$  of  $k$ th chunk

$\ll 1$  byte long, chunk has size  $t$  bytes

Total size of party  $i$ 's share = 128 bits (for  $s_i$ )

+ 10 bits (for  $i$ )

+  $100 \text{ MB} / t$  for  $f_1(i), \dots, f_\ell(i)$

$\approx 100 \text{ MB} / t$

Reconstruction:  $t$  parties

reconstruct  ~~$s$~~

reconstruct  $f_k(x)$  for  $k=1, 2, \dots, \ell$

$\Rightarrow C$

decrypt  $\Rightarrow M$

Thm: If encryption function (e.g. AES) is secure, then

so is this secret sharing scheme. Even if adversary gets  $C$ , he gets no information on  $M$ . (aside from length)



## Threshold crypto (fully distributed)

- Can we avoid having secret  $s$  (in secret sharing of  $ky$ ) ever exist?
  - Dealing - make up shares  $s_i$ ,  $s$  is implicitly generated; never explicitly exists
  - Reconstruction - don't reconstruct  $s$   
instead, use shares ~~to~~ of key to produce shares of signature, then reconstruct signature

## How about RSA? [Fouque/Stem paper]

- generate  $PK = (n, e)$  in distributed manner
- no one party even knows factorization  $n = p \cdot q$
- yet each party has share  $d_i$  of decryption key  
yet  $p$  &  $q$  exist & have suitable properties (size, etc..)
- given message  $m$ , each party can compute share  $\sigma(d_i, m)$  of signature
- signature shares can be combined in public manner to produce  $\sigma(m)$

## Key management / key distribution

themes: crypto, keys, names, individuals, trust,  
identity, scaling, usability, certificates,  
PKI, trusted intermediaries

- keys need to be shared to be useful (at least PK part for PK crypto)
- how is such sharing to be arranged?

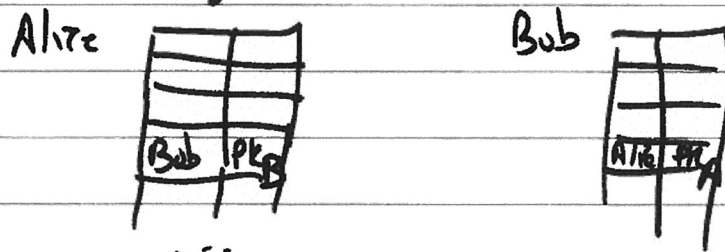
- Directly (by physical mtg)



meet in private (no eavesdroppers)  
recognize each other (authentication)

Share PK's, or symmetric keys

save in database: (when Alice has > 1 contact...)



note appearance of names tied to entries ...

privacy not needed if PK's are exchanged (as opposed to symm. keys)

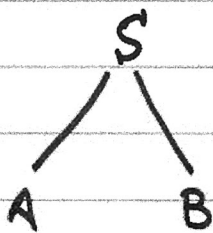
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Alice or Bob could be a computer  
(e.g. Alice installs key in computer Bob,  
or computer Alice gives user Bob her public key)

Such direct meetings are ~~the~~ necessary foundation of  
key mgt, as we'll see...

Indirect / Two-link / TTP (trusted third party) or server



Alice & Bob can't meet in  
person, but they have each  
met with trusted third party S  
& exchanged keys.

They can then request S to broker a "key-setup" operation  
so that A & B end up sharing a key, more-or-less as  
if they had actually met.

However, as we'll see, they need to trust S to behave  
properly & setup protocol (aka "key exchange" needs to  
be well designed).

Needham & Schroeder proposed 2 such protocols: one for  
symmetric keys, and one for public keys.

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## NS Symmetric Protocol

$K_{AS}$  = key shared between A & S } already set up  
 $K_{BS}$  similarly for B & S

"nonce" means a "use once" value

$N_A$  nonce generated by Alice

$N_B$  " " " Bob

could be from a counter, or a long random value...

Used to protect against certain forms of "reply attack"...

~~$K_{AB}$~~   $K_{AB}$ : key that gets setup here between A, B (and S)

$\{M\}_k$ : message M encrypted & authenticated with key k  
(e.g. encrypt M using AES in suitable mode, ~~then~~ & by k,  
append  $MAC_{k_2}(M)$ , where  $k_1$  &  $k_2$  derived from k)  
(literature often is vague about properties of  $\{\}_k$ ...)

Protocol:

- ①  $A \rightarrow S: A, B, N_A$       hi, I'm Alice, & I want to talk with Bob  
 $N_A$  is my "request nonce" ...
- ②  $S \rightarrow A: \{N_A, K_{AB}, B, \underbrace{\{K_{AB}, A\}}_{\text{blob}}\}_{K_{AS}}$       Oh here's key  $K_{AB}$  & blob to give B
- ③  $A \rightarrow B: \{K_{AB}, A\}_{K_{BS}}$       A knocks on B's door  
B decrypts & checks blob
- ④  $B \rightarrow A: \{N_B\}_{K_{AB}}$       B challenges A with  $K_{AB}$
- ⑤  $A \rightarrow B: \{N_B - 1\}_{K_{AB}}$       A responds

- who knows  $K_{AB}$ ?    A, B, S
- S must be trusted: can pretend to be A to B or vice versa, ...
- note role of names: handles by which to identify parties  
addresses to which msgs can be sent  
text strings that can be included in messages
- if no nonces in ①, ② & cut out ④, ⑤: could replay earlier session to A or B  
(can do same if  $K_{AB}$  later compromised...)  $\rightarrow$  fix with timestamps  
(Kerberos)



# PK protocol

$\begin{cases} K_{PX}, K_{SX} \\ \text{public \& secret keys of X} \end{cases}$   
S has  $K_{PA}, K_{PB}$   
A & B have  $K_{PS}$

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①  $A \rightarrow S: A, B$

PK req

②  $S \rightarrow A: \{K_{PB}, B\}_{K_{SS}}$

signed "cert" for B's PK

③  $A \rightarrow B: \{N_A, A\}_{K_{PB}}$

knock

encrypted, bound together

"psst! I'm A, and here's  $N_A$ ... talk to me"

PK req

new  
secret } ③'  $B \rightarrow S: B, A$

③''  $S \rightarrow B: \{K_{PA}, A\}_{K_{SS}}$

signed "cert" for A's PK

④  $B \rightarrow A: \{N_A, N_B\}_{K_{PA}}$

encrypted, bound together  
(non malleable...)

⑤  $A \rightarrow B: \{N_B\}_{K_{PB}}$

yep, I'm really here...

• ③' & ③'' could be replaced by including blob/cert  $\{K_{PA}, A\}_{K_{SS}}$  in ②

• at end only A & B know  $N_A$  &  $N_B$ ; eavesdroppers don't...

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Attack! (Gavin Lowe) 17 yrs later!

automated analysis

intruder  $I$  gets  $A$  to initiate communication with  $B$   
• then passes knock  $\{N_A, A\}$  on to  $B$  (after re-encrypting with  $K_{PB}$ )

•  $B$  responds with  $\{N_A, N_B\}_{K_{PA}}$ , which  $I$  sends to  $A$

•  $A$  sends  $\{N_B\}_{K_{PI}}$  to  $I$ .  $I$  decrypts it & gets  $N_B$

•  $I$  sends  $\{N_B\}_{K_{PB}}$  to  $B$

Now  $B$  thinks he is sharing  $N_A$  &  $N_B$  only with  $A$ , but  $I$  knows  $N_B$ , WRONG

Fix: ④  $B \rightarrow A: \{N_A, N_B, B\}$

Moral: Be explicit in protocols!

(e.g. give session id both ways, & identities;

give hash of shared transcript in each message  
(i.e. of all previous messages...)

Huge literature on such key-establishment protocols...