6.857 Lecture 4:
Hash Functions

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Most slides courtesy of Ron Rivest (Crypto 2008)

Outline

- Review hash function basics
- Revisit indistinguishability from RO
- MD5
- MD6

Review: Hash function basics (I)

- Hash function h: $\{0,1\}^* \longrightarrow \{0,1\}^d$

  maps arbitrary-length strings of data to fixed-length output (“digest”)

  in deterministic, public, “random” manner

Review: Hash function basics (II)

- Hash function typically consists of:
  - *Compression function*
    $f: \{0,1\}^c \times \{0,1\}^b \longrightarrow \{0,1\}^c$
    maps fixed-length input to fixed-length output
  - *Mode of operation* h^f
    how to apply f repeatedly to arbitrary-length input to get fixed-length output (of length d)

Review: Desirable properties (I)

- One-wayness (preimage resistance)
  - Infeasible, given $y \leftarrow_R \{0,1\}^d$, to find any x s.t. $h(x) = y$
- Collision resistance
  - Infeasible to find x, x' s.t. $x \neq x'$ and $h(x) = h(x')$
- Weak collision resistance (2nd preimage resistance)
  - Infeasible, given x, to find $x' \neq x$ s.t. $h(x) = h(x')$

Review: Desirable properties (II)

- Pseudorandomness
  - Infeasible to distinguish behavior from random oracle (RO)
- Non-malleability
  - Infeasible, given h(x), to produce h(x’), where x and x’ are “related”

Formal definitions

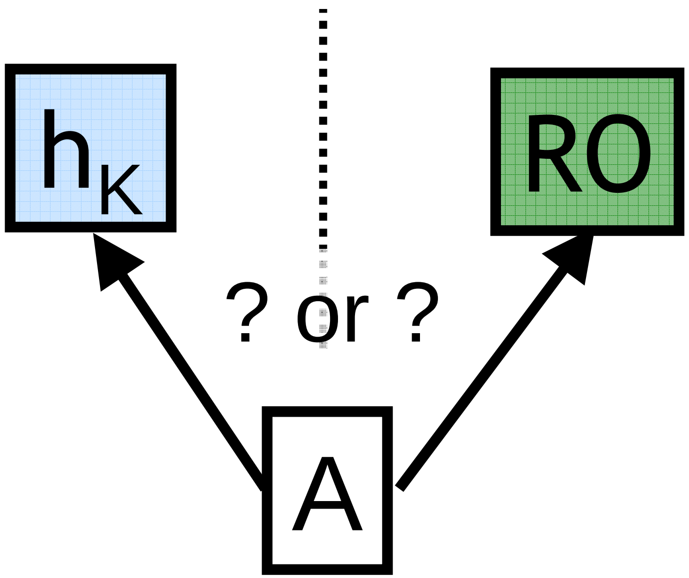
- Family of functions
  $H: \{0,1\}^k \times \{0,1\}^* \rightarrow \{0,1\}^d$
- For each $K \in \{0,1\}^k$, we have
  $h_K: \{0,1\}^* \rightarrow \{0,1\}^d$
- Security properties defined in terms of game played w/ adversary

Collision resistance

- Security game:
  - Adversary A gets $K \leftarrow_R \{0,1\}^k$
  - A outputs x, x'
  - A wins if $x \neq x'$ and $h(x) = h(x')$
- Advantage of A = probability that A wins
- H is collision resistant if no efficient adversary has more than negligible advantage

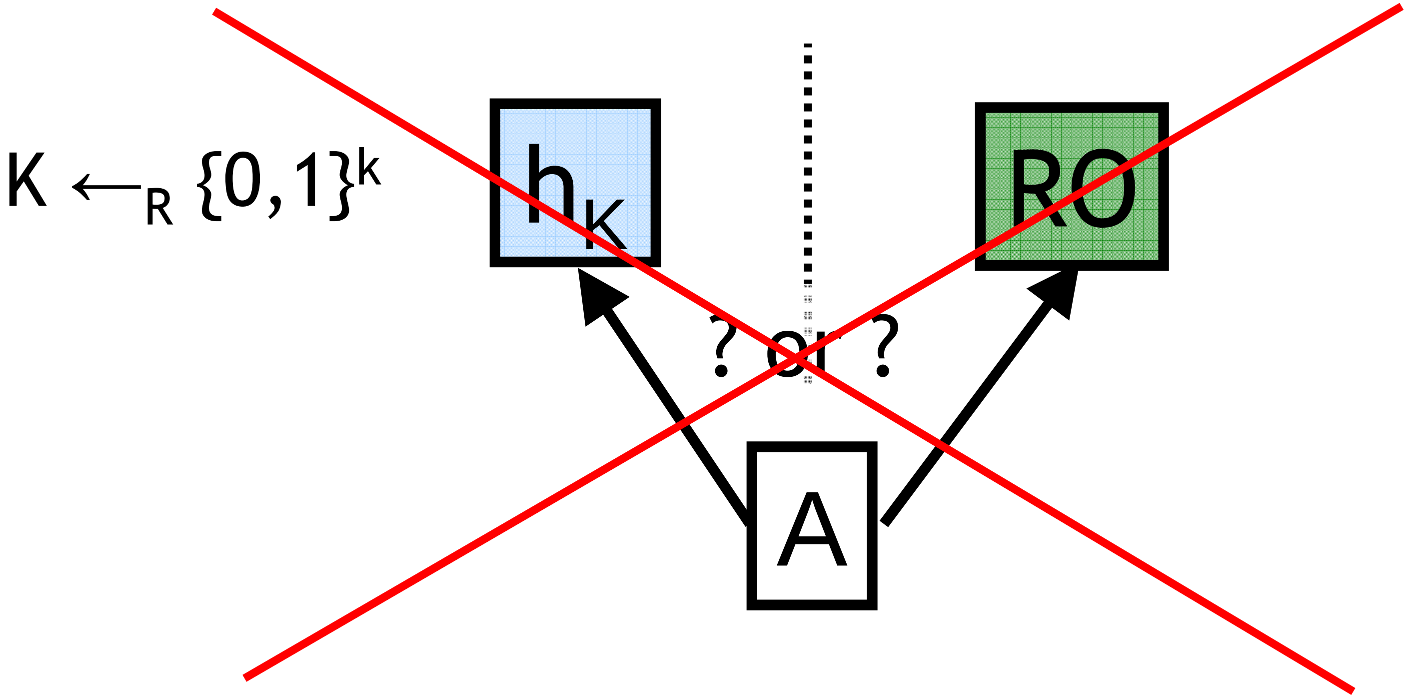
Indistinguishability from RO

$K \leftarrow_R \{0,1\}^k$



- A makes hash queries, i.e. outputs x, gets back $h_K(x)$ or RO(x) (depending on which world A is in)
- At end of game, A outputs 0 or 1
- Advantage of A = $|Pr[A^{h_K} = 1] – Pr[A^{RO} = 1]|$
- H is indistinguishable from RO if no efficient adversary has more than negligible advantage

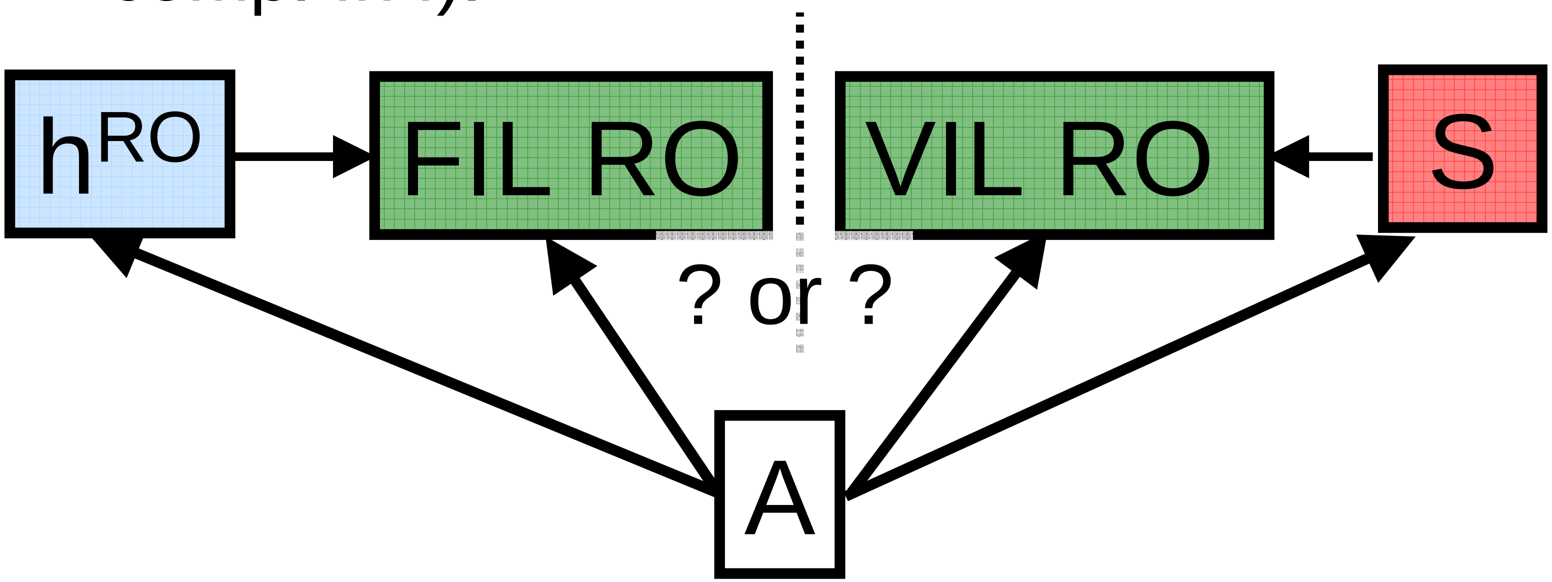
Indistinguishability from RO



- But h_K and f are *fixed, public* functions…
- No randomness in h_K, so it *will* be distinguishable from RO
- Adversary should have access to comp. fn f
- Need a new notion: "*indifferentiability*" from RO

Indifferentiability (Maurer et al. '04)

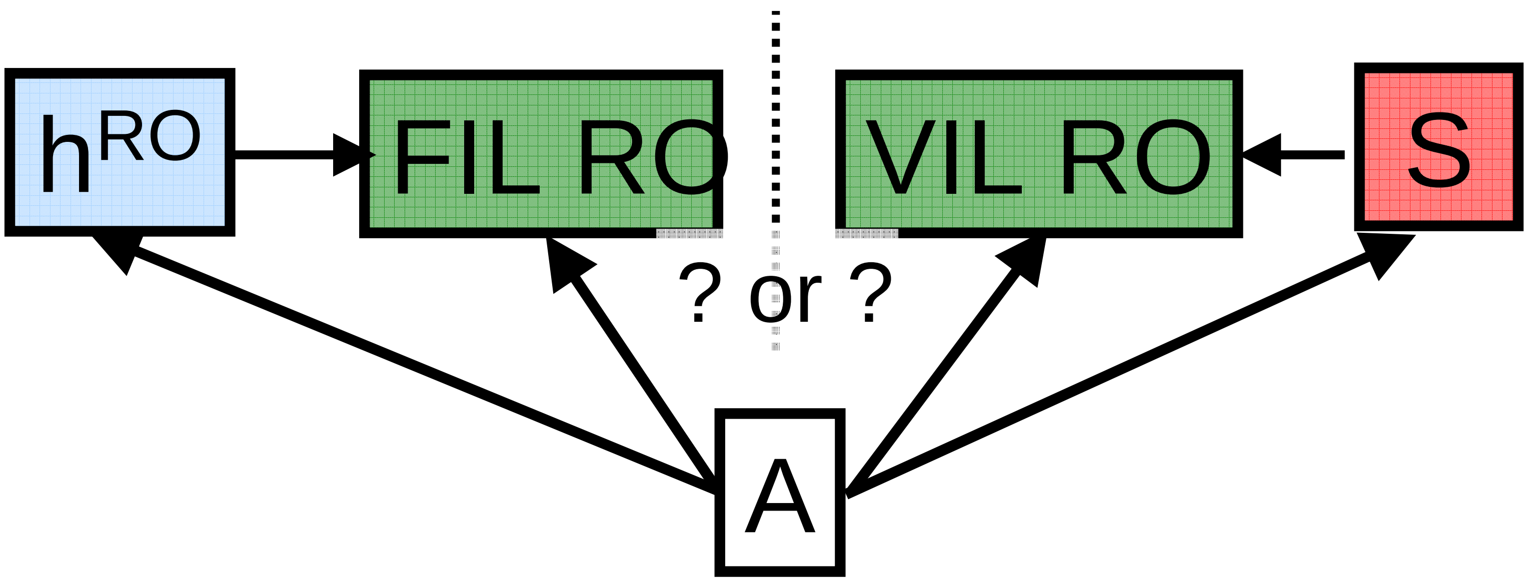
- Variant notion of indistinguishability appropriate when distinguisher has access to inner component (e.g. mode of operation h^f / comp. fn f).



- FIL = fixed input length, VIL = variable input length

Indifferentiability from RO

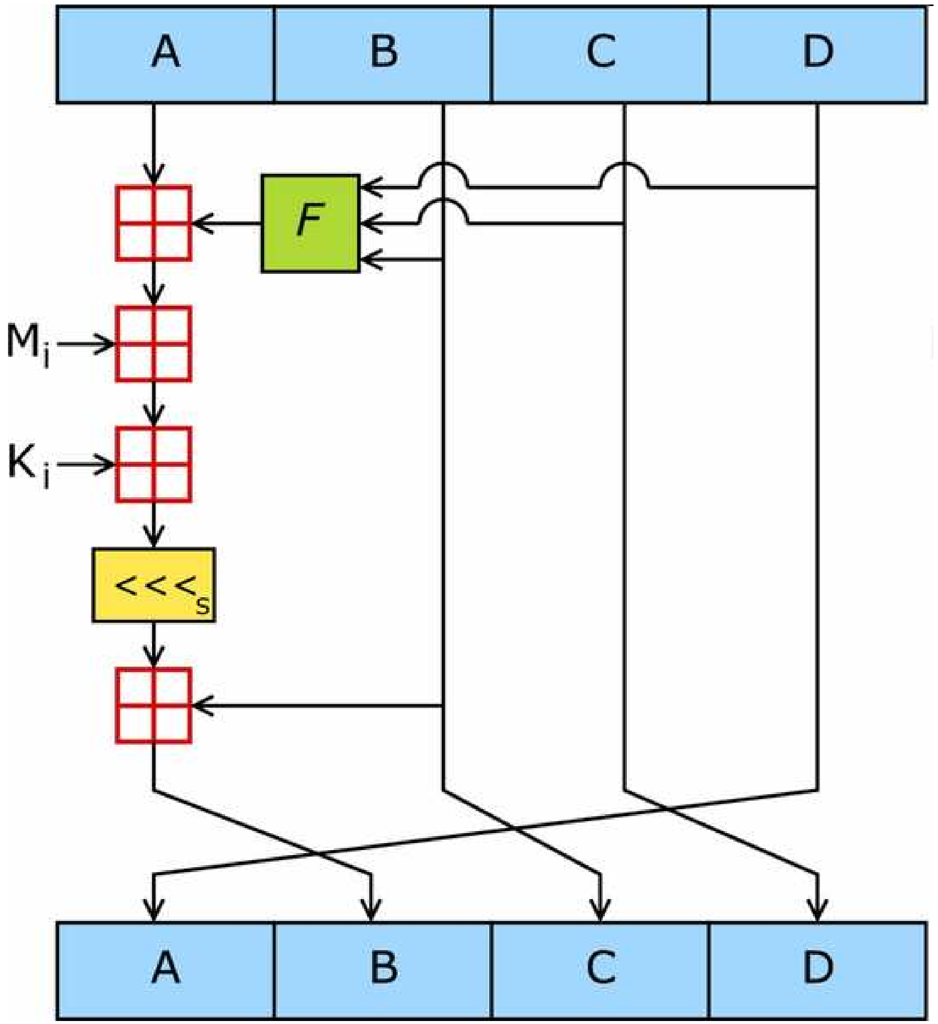
- Indifferentiability: ∃ simulator S s.t. no adversary can distinguish left from right with more than negligible advantage



MD5 compression function

- Chaining variable and output = 128 bits
- IV = fixed value
- 64 steps (4 rounds of 16 steps)
- 512-bit message block considered as 16 32-bit words

MD5 compression function



- M_i = 32-bit message word
- K_i = 32-bit constant, differs in each step
- $<<<_s$ = left bit rotation by s bits; s differs in each step
- ⊞: addition mod 2^{32}
- $F(x,y,z) = \begin{cases} (x \wedge y) \vee (\neg x \wedge z) \\ (x \wedge z) \vee (y \wedge \neg z) \\ x \oplus y \oplus z \\ y \oplus (x \wedge \neg z) \end{cases}$

  depending on round

Image source: http://en.wikipedia.org/wiki/File:MD5.png

Wang et al. break MD5 (2004)

- Differential cryptanalysis (re)discovered by Biham and Shamir (1990). Considers step-by-step ``difference'' (XOR) between two computations…
- Applied first to block ciphers (DES)…
- Used by Wang et al. to break collision-resistance of MD5
- Many other hash functions broken similarly; others may be vulnerable…

NIST SHA-3 competition!

- Input: 0 to 2^{64}-1 bits, size not known in advance
- Output sizes 224, 256, 384, 512 bits
- Collision-resistance, preimage resistance, second preimage resistance, pseudorandomness, …
- Simplicity, flexibility, efficiency, …
- Due Halloween '08

MD5 was designed in 1991…

- Same year WWW announced…
- Clock rates were 33MHz…
- Requirements:
  - $\{0,1\}^* \longrightarrow \{0,1\}^d$ for digest size d
  - Collision-resistance
  - Preimage resistance
  - Pseudorandomness
- What's happened since then?
- Lots…  
- What should a hash function --- MD6 --- look like today?

Design Considerations / Responses

Memory is now ``plentiful''…

- Memory capacities have increased 60% *per year* since 1991
- Chips have 1000 times as much memory as they did in 1991
- Even ``embedded processors'' typically have at least 1KB of RAM

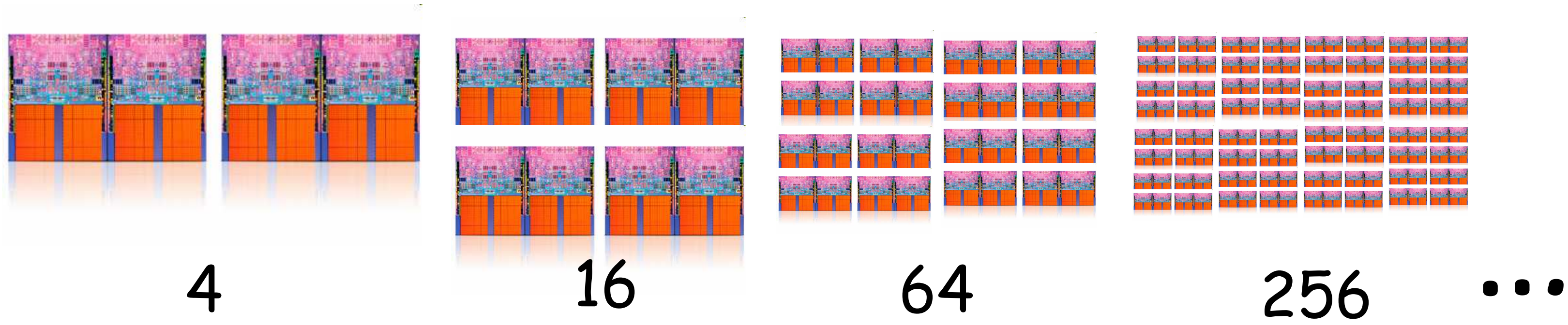
So… MD6 has…

- *Large* input message block size:
  512 *bytes* (not 512 bits)
- This has many advantages…

Parallelism has arrived

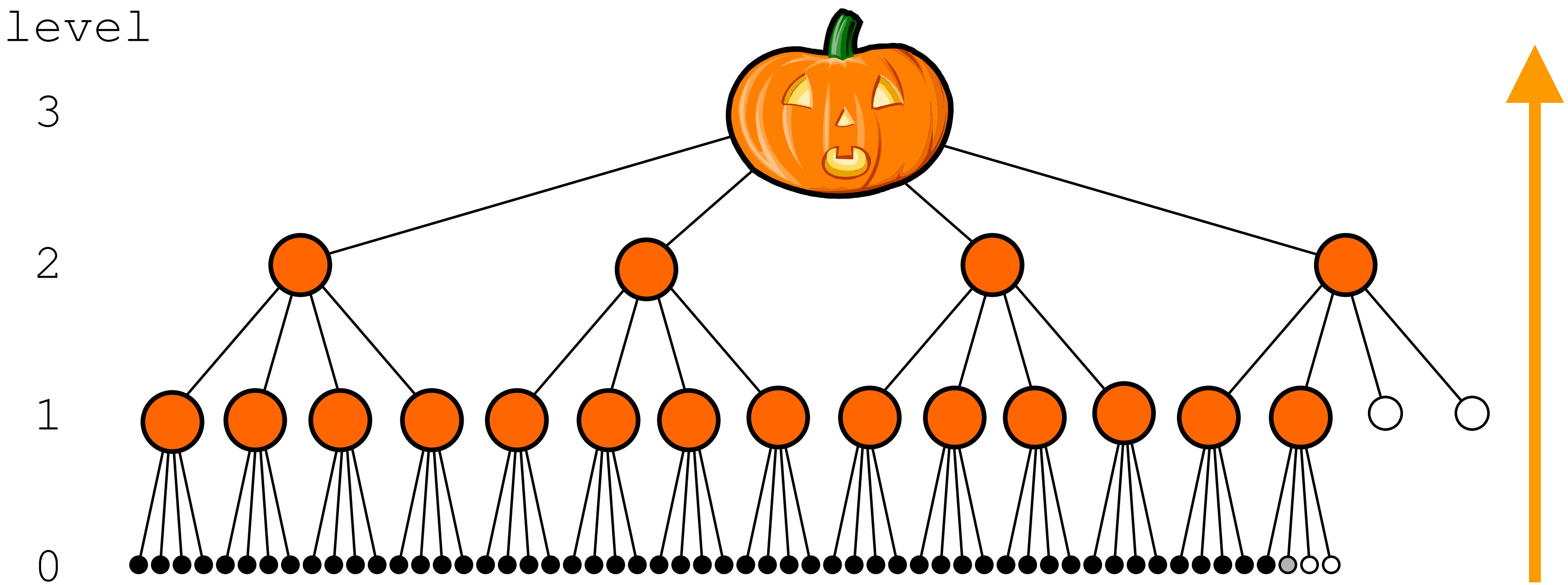
- Uniprocessors have "hit the wall"
  - Clock rates have *plateaued*, since power usage is quadratic or cubic with clock rate:

    $P = VI = V^2/R = O(\text{ freq}^2)$ (roughly)
- Instead, *number of cores* will double with each generation: tens, hundreds (thousands!) of cores coming soon



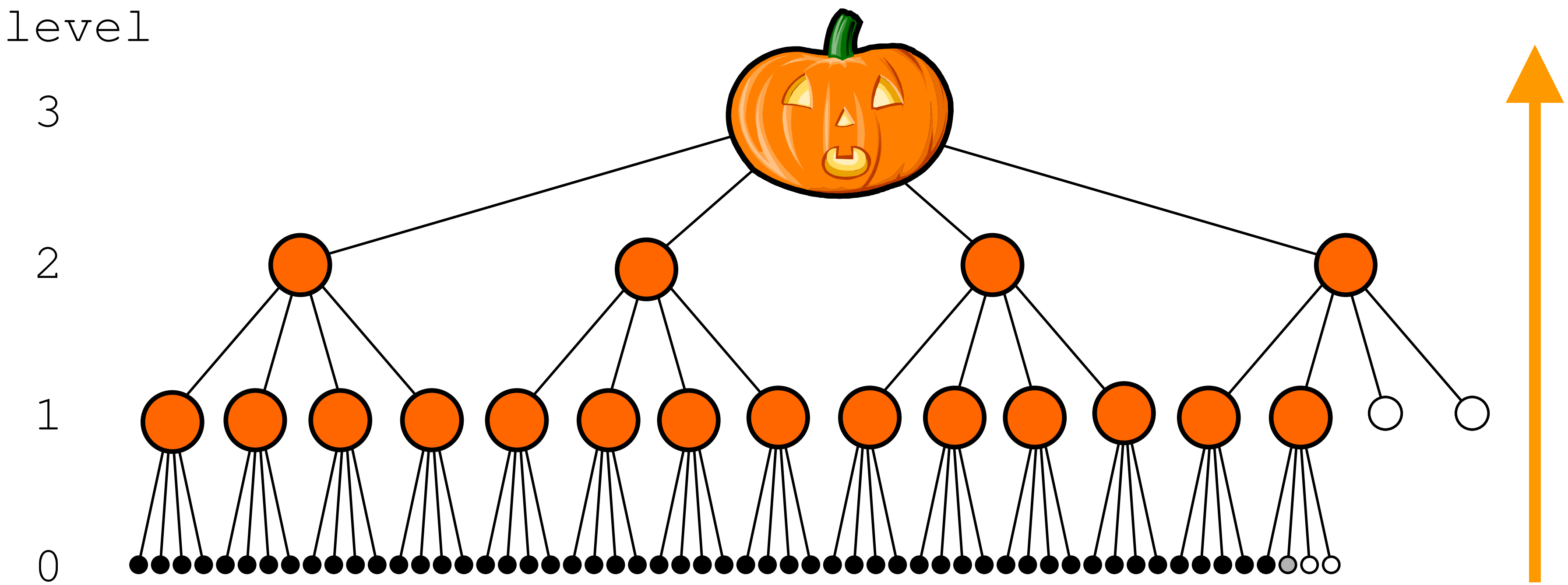
So… MD6 has…

- Bottom-up tree-based mode of operation (like Merkle-tree)
- 4-to-1 compression ratio at each node



Which works very well in parallel

- Height is $\log_4$(number of nodes)

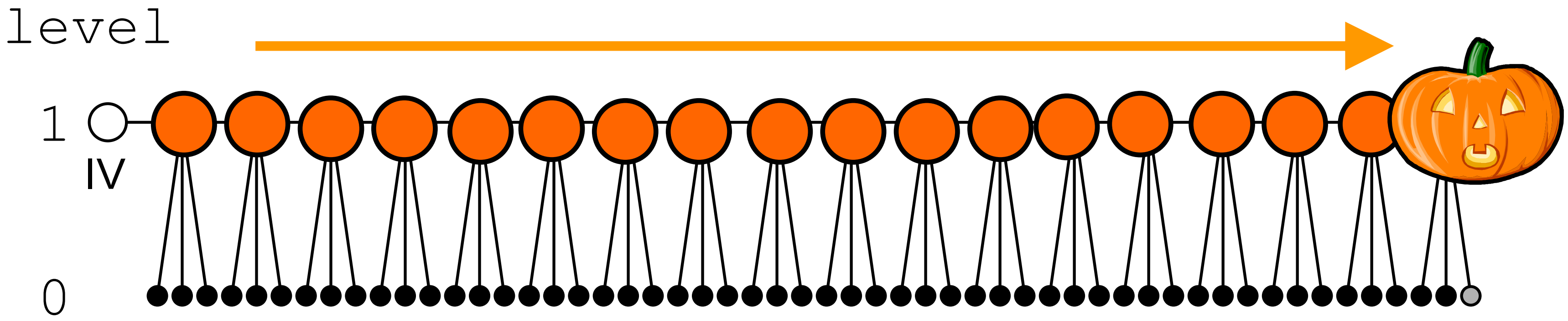


But… most CPU's are small… 

- Storage proportional to tree height may be too much for some CPU's…

So… MD6 has…

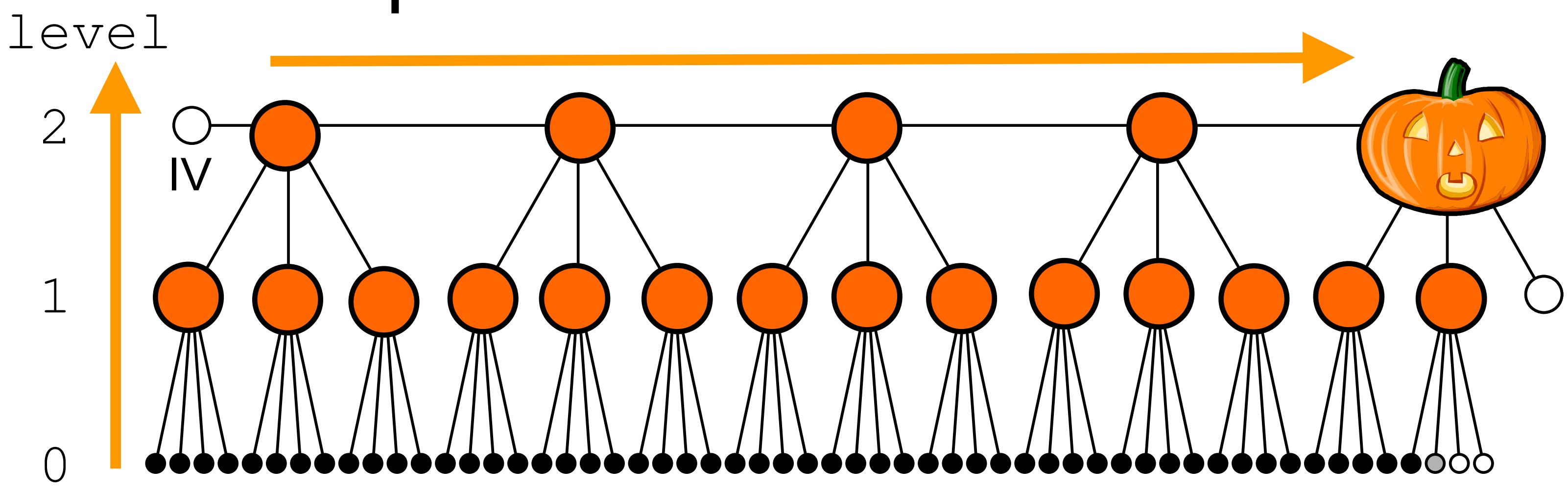
- Alternative sequential mode

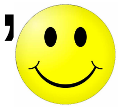


- (Fits in 1KB RAM)

Actually, MD6 has…

- a smooth sequence of alternative modes: from purely sequential to purely hierarchical… L parallel layers followed by a sequential layer, $0 \leq L \leq 64$
- Example: L=1:

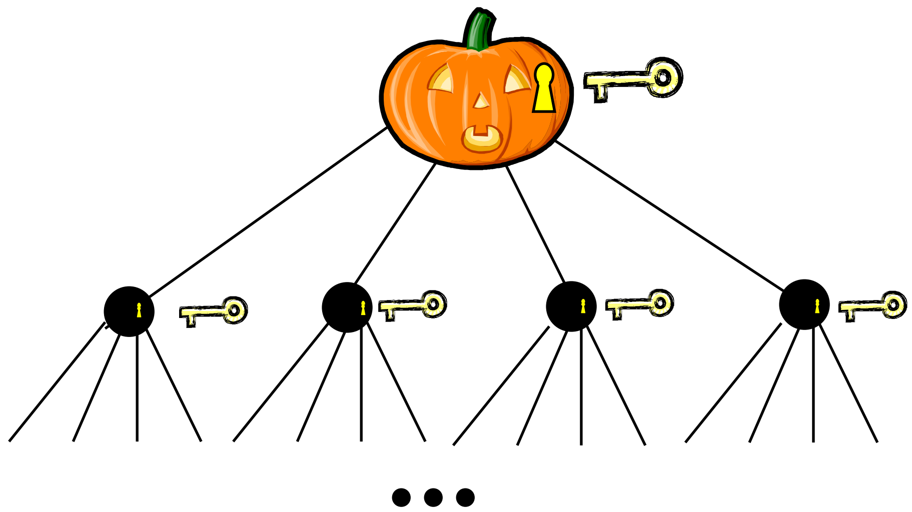


Hash functions often ``keyed”  

- Salt for password, key for MAC, variability for key derivation, theoretical soundness, etc…
- Current modes are “post-hoc”

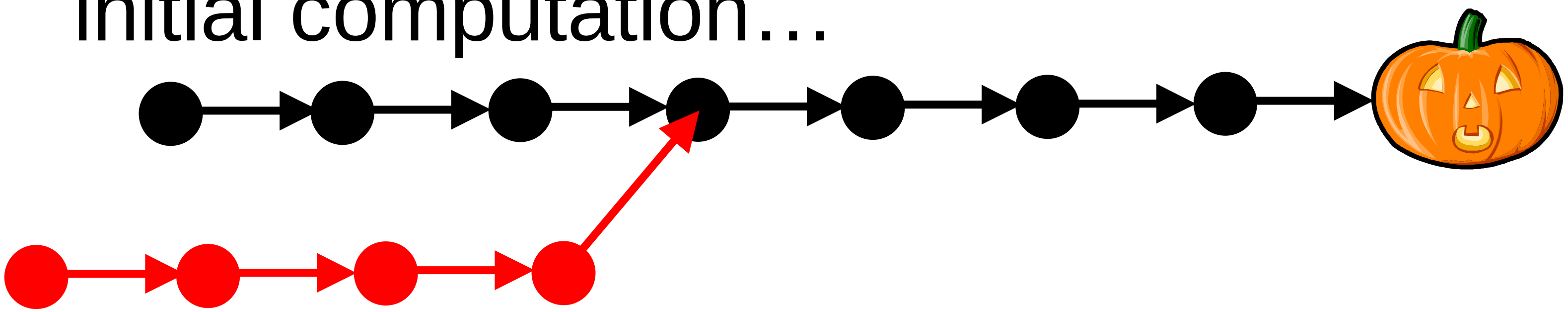
So… MD6 has…

- *Key input K* of up to 512 bits
- K is input to *every* compression function



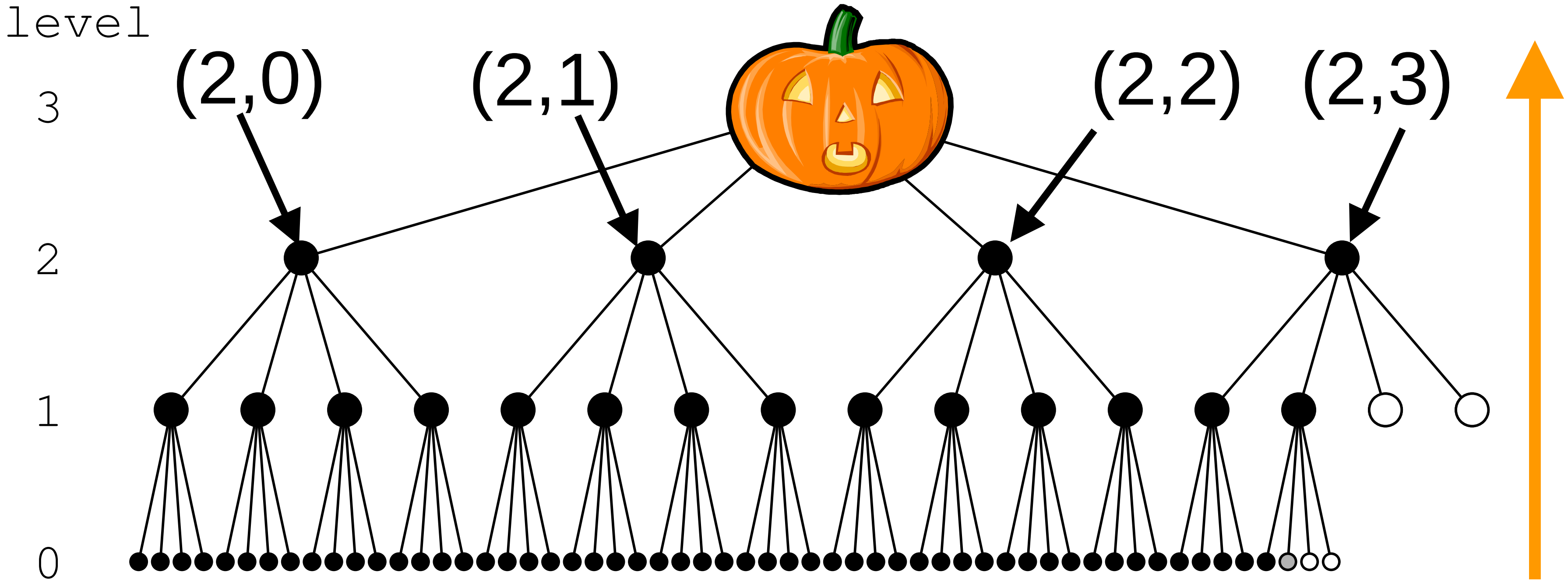
Generate-and-paste attacks 

- Kelsey and Schneier (2004), Joux (2004), …
- Generate sub-hash and fit it in *somewhere*
- Has advantage proportional to size of initial computation…



So… MD6 has…

- 1024-bit intermediate (chaining) values
- root truncated to desired final length
- Location (level,index) input to each node



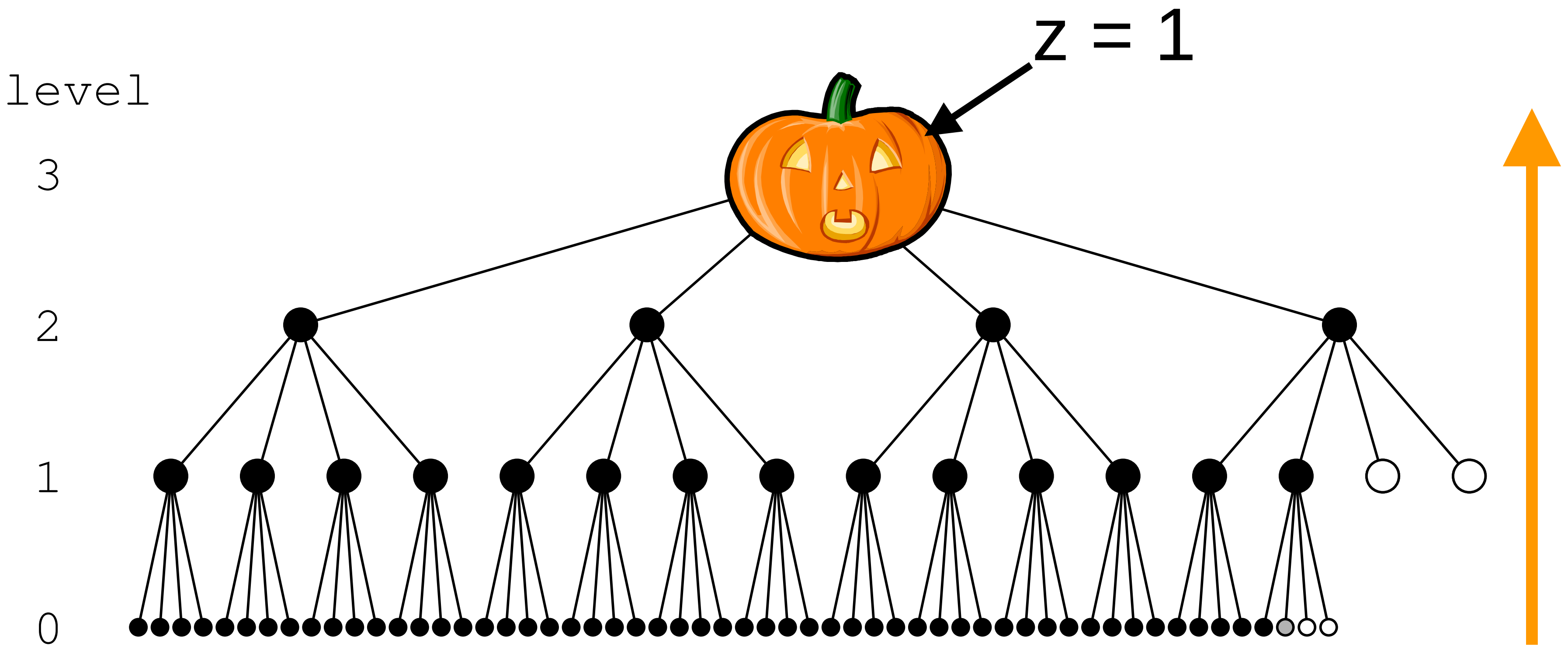
Extension attacks… 

- Hash of one message useful to compute hash of another message (especially if keyed):

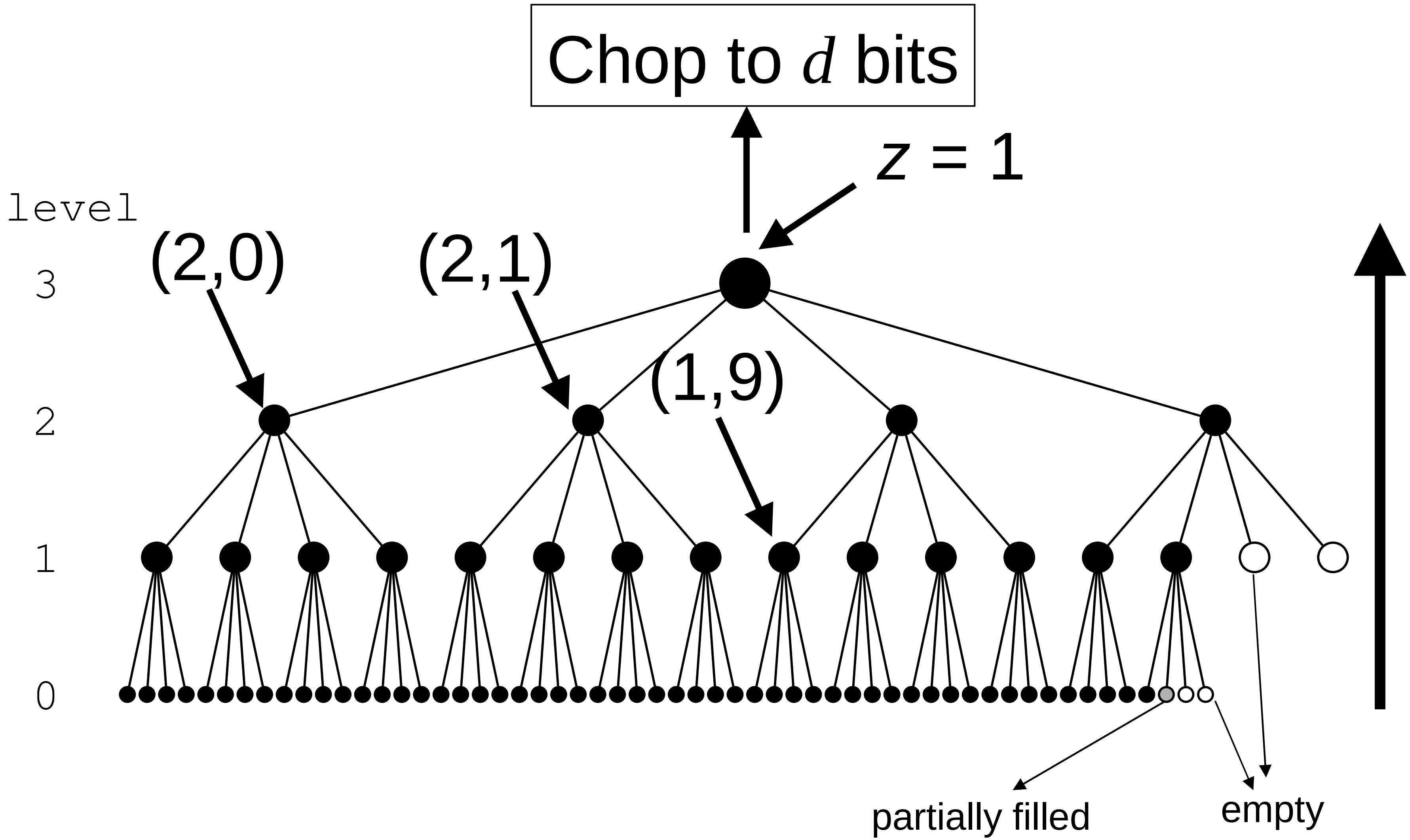
$$H(\ K \,||\, A \,||\, B\ ) = H(\ H(\ K \,||\, A) \,||\, B\ )$$

So… MD6 has…

- ``Root bit'' (aka "z-bit") input to each compression function:



Putting it all together…



Side-channel attacks 

- Timing attacks, cache attacks…
- Operations with data-dependent timing or data-dependent resource usage can produce vulnerabilities.
- This includes data-dependent rotations, *table lookups* (S-boxes), some complex operations (e.g. multiplications), …

So… MD6 uses…

- Operations on 64-bit words
- The following operations *only:*
  - XOR $\oplus$
  - AND $\bigcirc\!\!\!\!\wedge$
  - SHIFT by fixed amounts:

    $x \;\; >> \;\; r$ $\qquad >>$

    $x \;\; << \;\; \ell$ $\qquad <<$

Security needs vary… 

- Already recognized by having different digest lengths d (for MD6: $1 \leq d \leq 512$)
- But it is useful to have reduced-strength versions for analysis, simple applications, or different points on speed/security curve.

So… MD6 has … 

- A variable number r of rounds.
  (Each round is 16 steps.)
- Default r depends on digest size d :

$$r = 40 + (d/4)$$

d	160	224	256	384	512
r	80	96	104	136	168

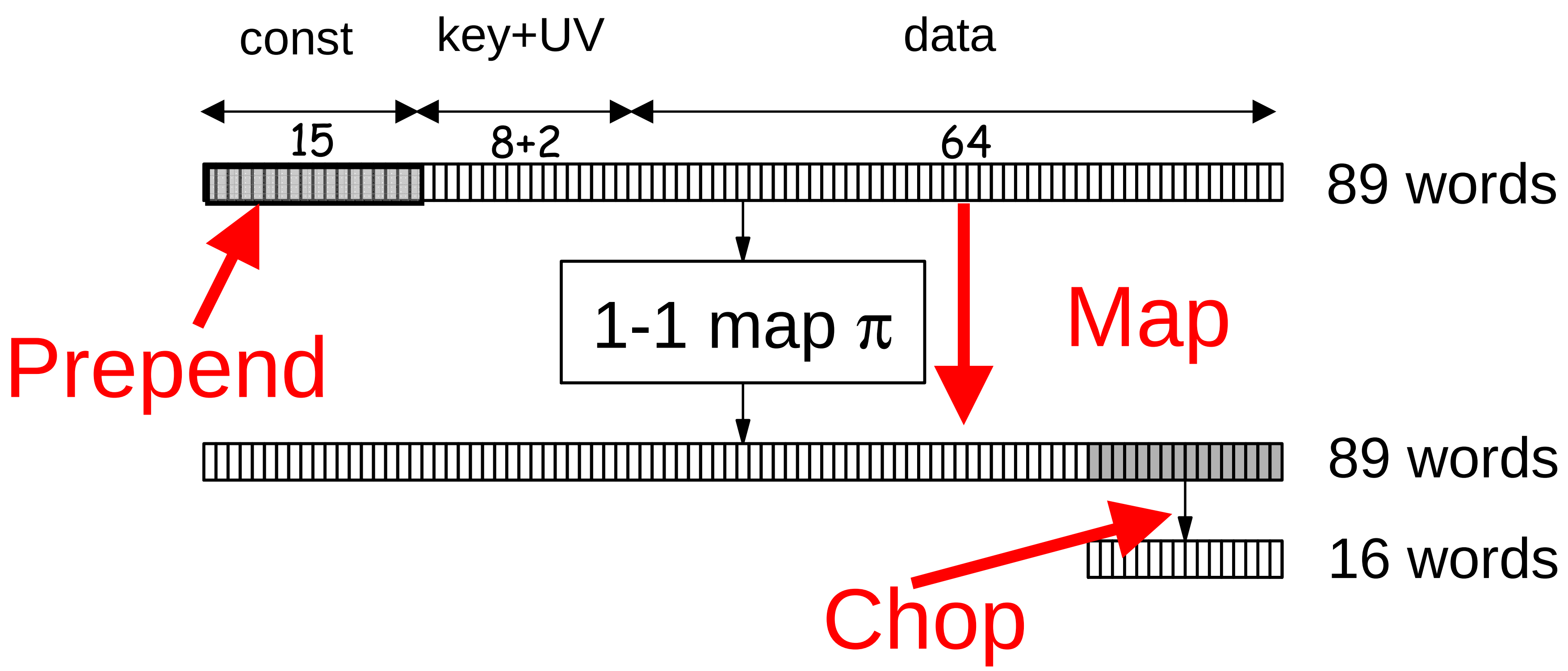
- But r is also an (optional) input.

MD6 Compression function

Compression function inputs

- 64 word (512 byte) data block
  - message, or chaining values
- 8 word (512 bit) key K
- 1 word U = (level, index)
- 1 word V = parameters:
  - Data padding amount
  - Key length (0 ≤ keylen ≤ 64 bytes)
  - z-bit (aka ``root bit'')
  - L (mode of operation height-limit)
  - digest size d (in bits)
  - Number r of rounds
- 74 words total

Prepend Constant + Map + Chop



Simple compression function:

Input: A[0 .. 88] of A[0 .. 16r + 88]

for i = 89 **to** 16 r + 88 :

$x = S_i \oplus A[i-17] \oplus A[i-89]$

$\oplus (A[i-18] \wedge A[i-21])$

$\oplus (A[i-31] \wedge A[i-67])$

$x = x \oplus (x >> r_i)$

$A[i] = x \oplus (x << \ell_i)$

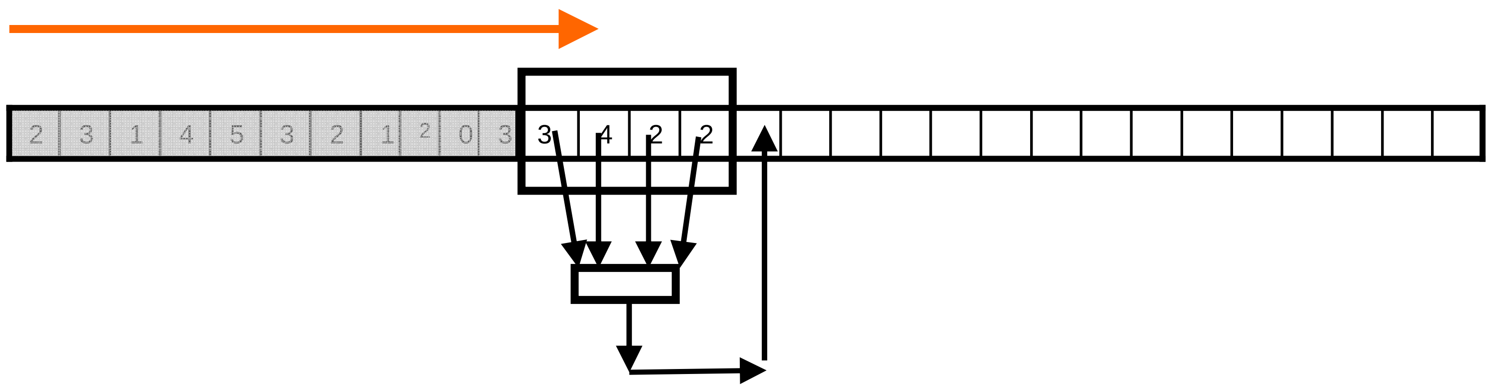
return A[16r + 73 .. 16r + 88]

Constants

- Taps 17, 18, 21, 31, 67 optimize diffusion
- Constants S_i defined by simple recurrence; change at end of each 16-step round
- Shift amounts repeat each round (best diffusion of 1,000,000 such tables):

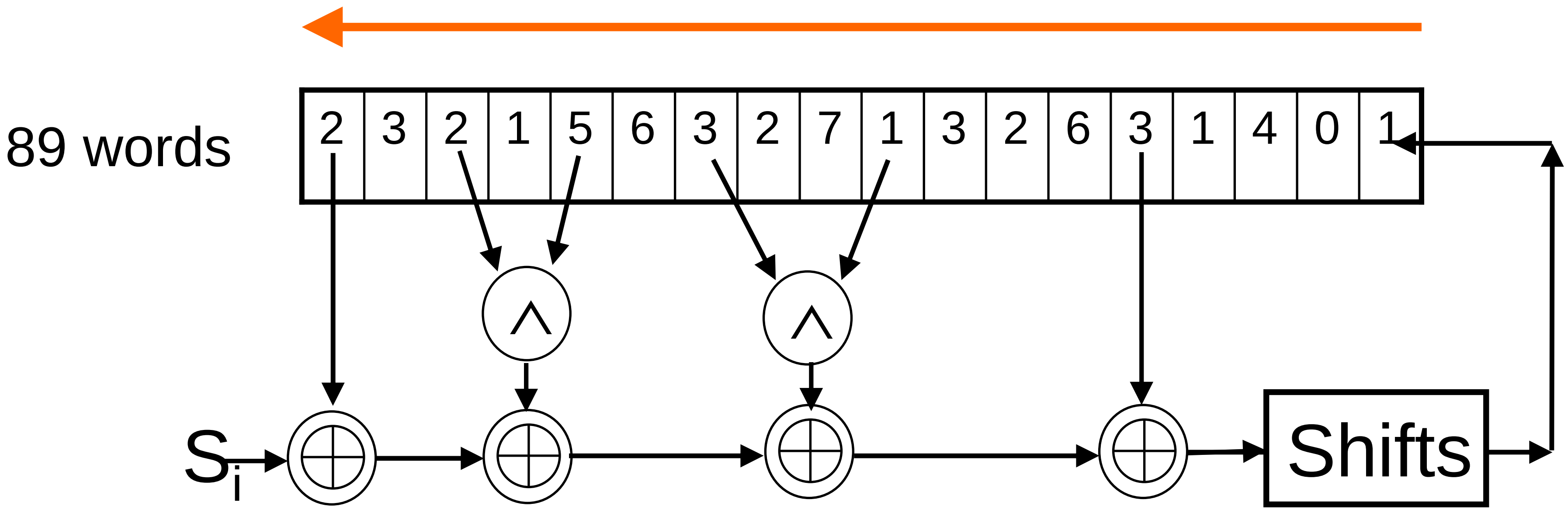
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
r_i	10	5	13	10	11	12	2	7	14	15	7	13	11	7	6	12
ℓ_i	11	24	9	16	15	9	27	15	6	2	29	8	15	5	31	9

Large Memory (sliding window)



- Array of 16r + 89 64-bit words.
- Each word computed as function of preceding 89 words.
- Last 16 words computed are output.

Small memory (shift register)



- Shift-register of 89 words (712 bytes)
- Data moves right to left

Security Analysis

Generate-and-paste attacks (again)

- Because compression functions are “location-aware”, attacks that do speculative computation hoping to “cut and paste it in somewhere” don’t work.

Analyzing mode of operation

General approach:

If compression function f is “secure”,

then mode of operation $MD6^f$ is “secure”

e.g.,

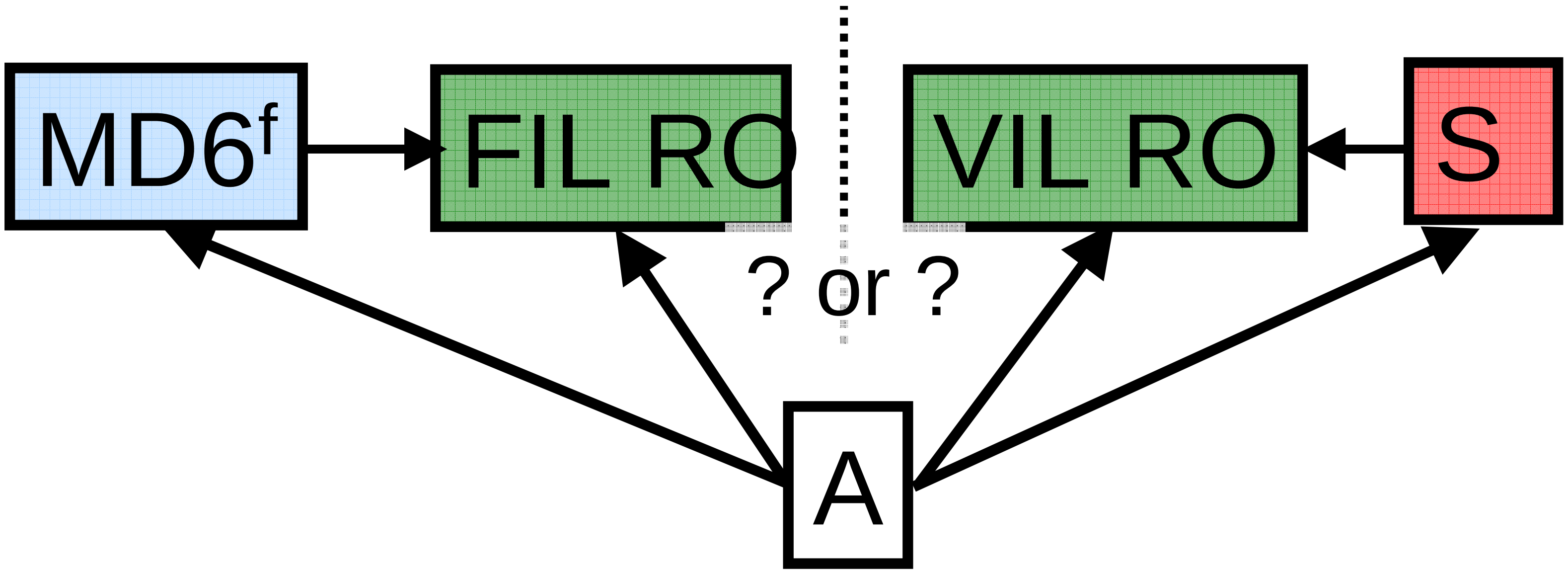
- f collision-resistant $\Rightarrow$ $MD6^f$ collision-resistant
- f preimage-resistant $\Rightarrow$ $MD6^f$ preimage-resistant
- f PRF $\Rightarrow$ $MD6^f$ PRF

Property preservations

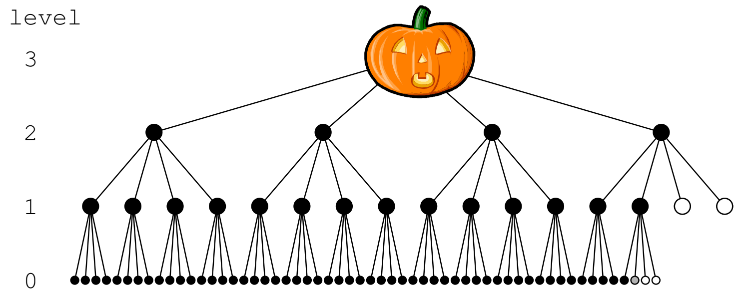
- **Theorem.** If f is collision-resistant, then $MD6^f$ is collision-resistant.
- **Theorem.** If f is preimage-resistant, then $MD6^f$ is preimage-resistant.
- **Theorem.** If f is a FIL-PRF, then $MD6^f$ is a VIL-PRF.
- **Theorem.** If f is a FIL-MAC and root node effectively uses distinct random key (due to z-bit), then $MD6^f$ is a VIL-MAC.
- (See thesis by Chris Crutchfield.)

Indifferentiability (Maurer et al. '04)

- Variant notion of indistinguishability appropriate when distinguisher has access to inner component (e.g. mode of operation MD6^f / comp. fn f).

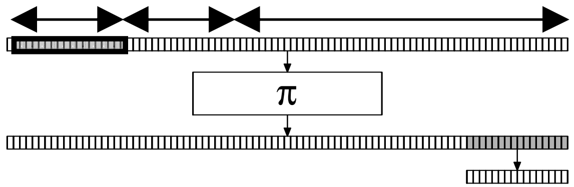


Indifferentiability (I)



- **Theorem.** The MD6 mode of operation is indifferentiable from a random oracle (viewing compression function as RO)
- **Proof:** Construct simulator for compression function that makes it consistent with any VIL RO and MD6 mode of operation…
- **Advantage:** $\epsilon \leq 2\, q^2 / 2^{1024}$
  where q = number of calls (measured in terms of compression function calls).

Indifferentiability (II)



- **Theorem.** MD6 compression function f^{π} is indifferentiable from a FIL random oracle (with respect to random permutation π).
- **Proof:** Construct simulator S for π and π^{-1} that makes it consistent with FIL RO and comp. fn. construction.
- **Advantage:** $\epsilon \leq q / 2^{1024} + 2q^2 / 2^{4672}$

Differential attacks don't work

- **Theorem.** *Any* standard differential attack has less chance of finding collision than standard birthday attack.
- *Proven only for MD6 with large number of rounds.

Summary

- MD6 is:
  - Arguably secure against known attacks
  - Relatively simple
  - Highly parallelizable
  - Reasonably efficient

MD6 Team

- Dan Bailey
- Sarah Cheng
- Christopher Crutchfield
- Yevgeniy Dodis
- Elliott Fleming
- Asif Khan
- Jayant Krishnamurthy
- Yuncheng Lin
- Leo Reyzin
- Emily Shen
- Jim Sukha
- Eran Tromer
- Yiqun Lisa Yin

- Juniper Networks
- Cilk Arts
- NSF

THE END

↓

MD6

↓

03744327e1e959fbdcdf7331e959cb2c28101166

Round constants S_i

- Since they only change every 16 steps, let S'_j be the round constant for round j.
- S'_0 = `0x0123456789abcdef`
- $S'_{j+1} = (S'_j \lll 1) \oplus (S'_j \wedge \text{mask})$
- mask = `0x7311c2812425cfa0`

Software Implementations

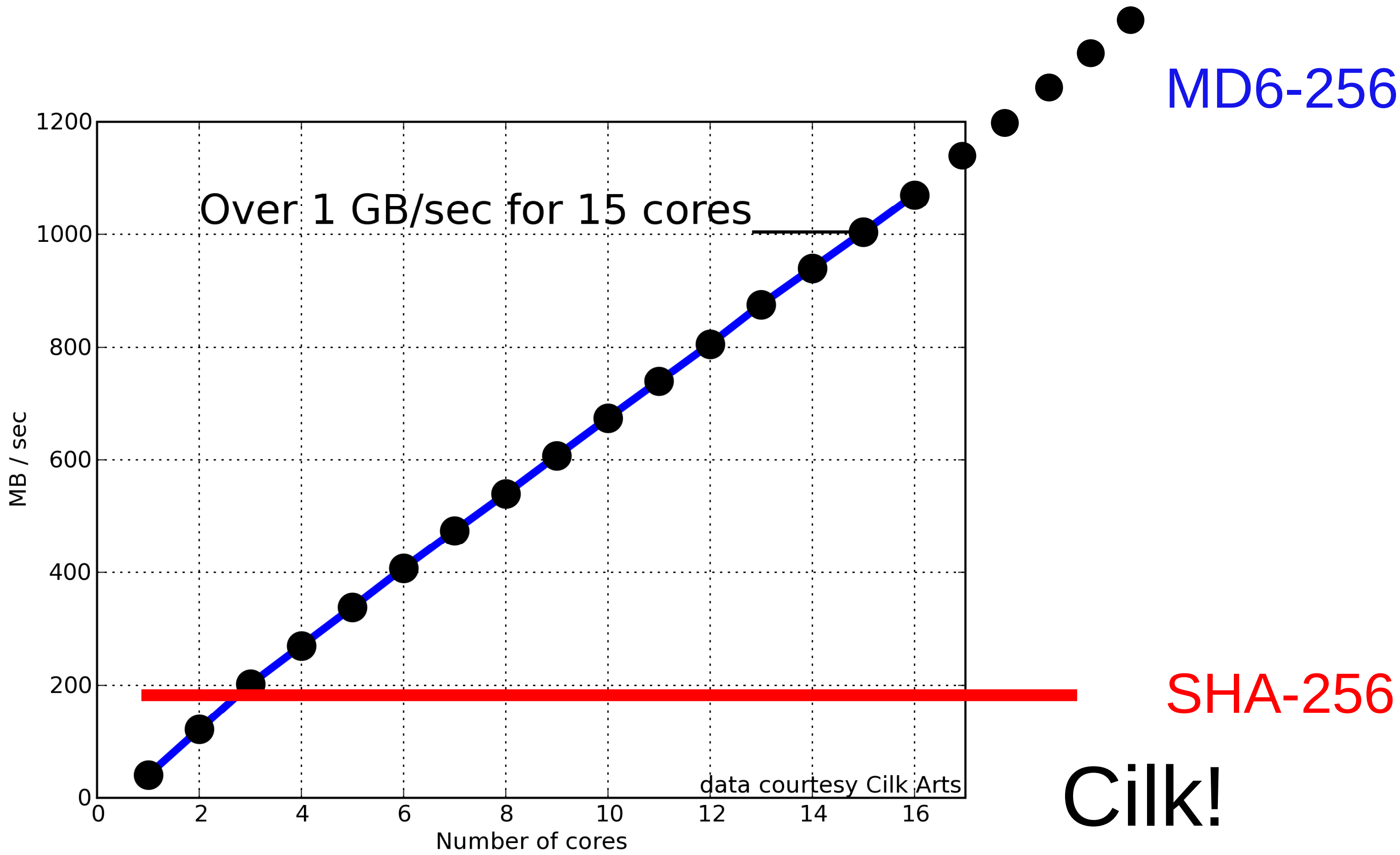
Software implementations

- Simplicity of MD6:
  - *Same* implementation for all digest sizes.
  - *Same* implementation for SHA-3 Reference or SHA-3 Optimized Versions.
  - Only optimization is *loop-unrolling* (16 steps within one round).

NIST SHA-3 Reference Platforms

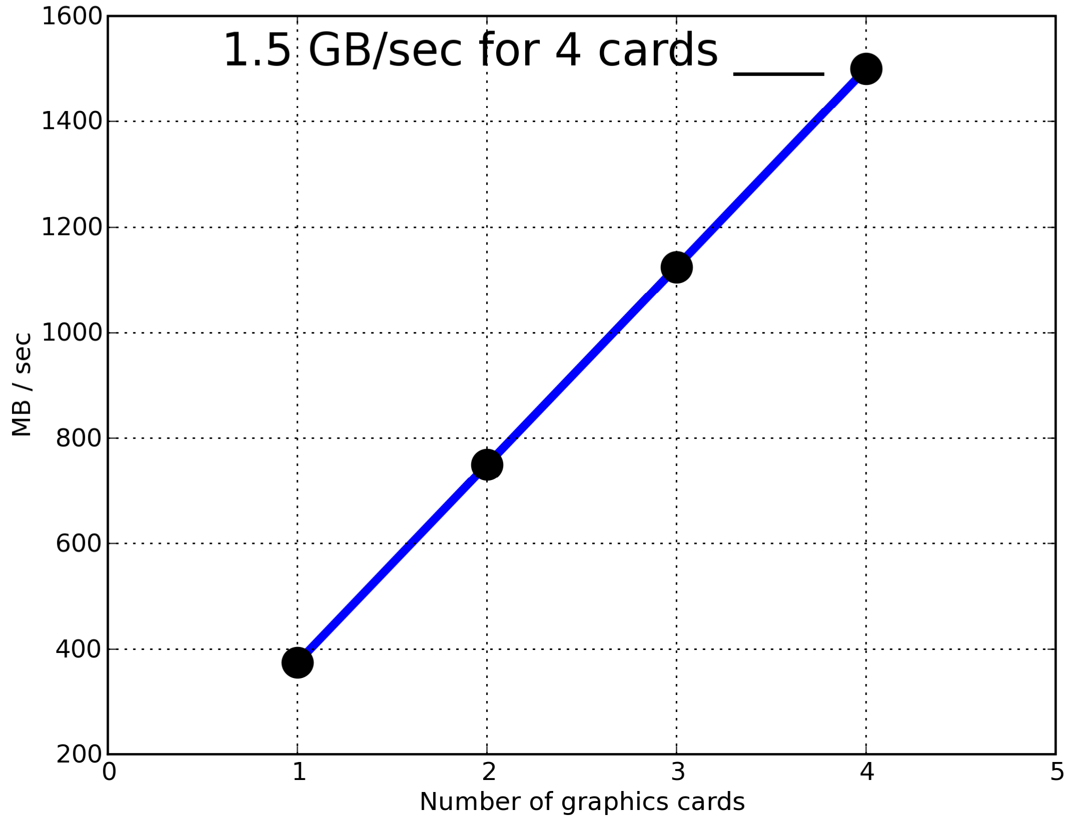
	32-bit	**64-bit**
MD6-160	44 MB/sec	97 MB/sec
MD6-224	38 MB/sec	82 MB/sec
MD6-256	35 MB/sec	77 MB/sec
MD6-384	27 MB/sec	59 MB/sec
MD6-512	22 MB/sec	49 MB/sec
SHA-512	38 MB/sec	202 MB/sec

Multicore efficiency

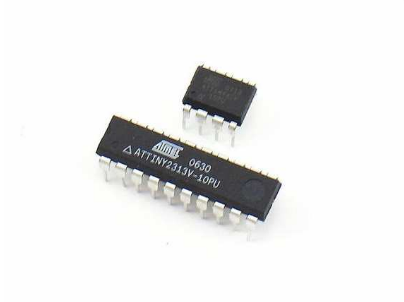


Efficiency on a GPU

- Standard $100 NVidia GPU
- 375 MB/sec on one card



8-bit processor (Atmel)



- With L=0 (sequential mode), uses less than 1KB RAM.
- 20 MHz clock
- 110 msec/comp. fn for MD6-224 (gcc actual)
- 44 msec/comp. fn for MD6-224 (assembler est.)

Hardware Implementations

FPGA Implementation (MD6-512)

- Xilinx XUP FPGA (14K logic slices)
- 5.3K slices for round-at-a-time
- 7.9K slices for two-rounds-at-a-time
- 100MHz clock
- 240 MB/sec (two-rounds-at-a-time) (Independent of digest size due to memory bottleneck)