Administrivia

PS1 out today, homework groups

Term project ideas
- slashdot ("security")
- www.iacr.org/eprint

Outline

Hash functions
- Intro
- Random Oracle Model
- Desirable Properties
- Applications

Introduction

What is a hash fn?

Maps arbitrary strings of data to fixed-length output in deterministic, public, "random" manner.

$$h: \{0,1\}^* \longrightarrow \{0,1\}^d$$

strings of arbitrary length ≥ 0

strings of length d

Hash Functions

No secret key. All operations public.
Anyone can compute h, poly time computation

Examples: MD4, MD5, SHA-1, SHA-256, SHA-512

	MD4, MD5	SHA-1	SHA-256	SHA-512
d:	128	160	256	512
broken (CR):	2^6 ✓ 2^{37}	✓? 2^{69}		

Ideal: Random Oracle
(not achievable in practice)

Oracle: on input $x \in \{0,1\}^*$
if x not in book
flip coin d times to determine $h(x)$
record $(x, h(x))$ in book
else: return y where $(x,y) \in$ book

Gives random answer every time, except as required for consistency with previous answers. (h must be deterministic)

In practice, ∄ RO so need something "pseudo random"

Desirable Properties

OW (1) "one-way" (pre-image resistance)

Infeasible, given $y \in_R \{0,1\}^d$ to find any x s.t. $h(x) = y$

↑ "pre-image" of y

CR (2) Collision-resistance (strong collision resistance)

Infeasible to find x, x', s.t. $x \neq x'$ and $h(x) = h(x')$ (a "collision")

TCR (3) Weak collision resistance (target CR, 2nd pre-image resistance)

Infeasible given x, to find $x' = x$ s.t. $h(x) = h(x')$

PRF (4) Pseudo-randomness

Behavior indistinguishable from RO

NM (5) Non-malleability

Infeasible, given $h(x)$, to produce $h(x')$ where x and x' are "related" (e.g. $x' = x+1$)

Informal definitions. Formal requires family of hash functions

Facts

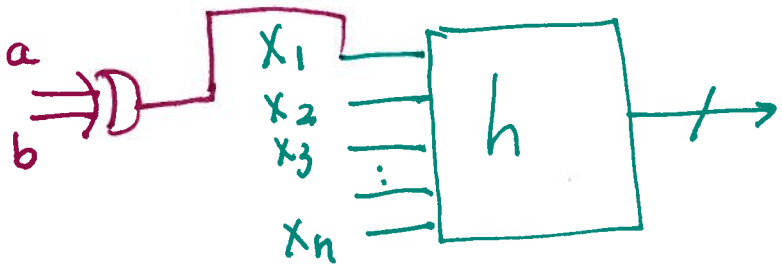
h is CR $\Rightarrow$ h is TCR (but not reverse)

h is OW $\not\Leftrightarrow$ h is CR, TCR (neither impl. holds)

Collisions can be found in $O(2^{d/2})$ – birthday attack

Inversion can be found in $O(2^d)$

Examples



OW $\not\Rightarrow$ TCR

h(x) is OW, CR

$h'(a, b, x_2, .. x_n)$ is still OW, but not TCR

$$h'(x) = \begin{cases} 0 \,||\, x & \text{if } |x| \le n \\ 1 \,||\, h(x) & \text{otherwise} \end{cases}$$

h is OW, CR, but h' is TCR, not OW

TCR $\not\Rightarrow$ OW

Applications

(1) Password storage

- Store $h(PW)$, not PW, on computer
- Use $h(PW)$ to compare against $h(PW')$ where PW' is the typed password

Disclosure of $h(PW)$ should not reveal PW

Need OW.

(2) File modification detector

- For each file F, store $h(F)$ securely (on DVD)
- check if F modified by recomputing $h(F)$
- need TCR (adversary wants to change F but not $h(F)$)

(3) Digital signatures

PK_A : Alice's Public key

SK_A = Alice's Private key

Signing : $\sigma = sign(SK_A, M)$

Verify : $verify(M, \sigma, PK_A) = true/false$

Adversary wants to forge a signature that verifies

For large M, easier to sign $h(M)$ $\sigma = sign(SK_A, h(M))$

Need CR, don't need OW. Alice gets Bob to sign x, then claims he signed x', if $(h(x) = h(x'))$