Problem Set 10

Due: Wednesday, November 12, 2003.

NONCOLLABORATIVE Problem 1. Consider the (nonparameterized) *vertex cover* problem: given a graph G, find a minimum-size set of vertices that covers all edges (i.e., every edge has at least one endpoint in the vertex cover). Show that vertex cover is fixed-parameter tractable with respect to the treewidth parameter. (**Hint:** The dynamic program is easier than dominating set.)

Problem 2. Suppose you're implementing a video game in which the player can walk around a planar environment made up of walls, and at any time the screen displays only the walls that are (partially) visible by the player. More precisely, the player is modeled as a single point; the walls are modeled as noncrossing line segments; two points are *visible* if the line segment connecting them does not intersect any walls except at its endpoints; and a wall is *visible* from a point if at least one point on the wall is visible from the point. Give an $O(n \lg n)$-time algorithm to compute the set of walls visible from the player. **Hint**: Use a line-sweep algorithm, but instead of sweeping a horizontal line downwards, sweep a half-line around a point.

Problem 3. The standard representation of a Voronoi diagram is a graph together with, for each vertex of the Voronoi diagram, a cyclic linked list of the incident edges in clockwise order around the vertex and, for each input point, a cyclic linked list of the vertices and edges around the Voronoi cell of that point.

(a) Show how to reduce the problem of sorting n numbers to the problem of computing the Voronoi diagram of $\Theta(n)$ points. Your reduction should take linear time, and can use standard arithmetic ($+$, $-$, $\cdot$, $/$, $\sqrt{}$) but cannot use trigonometric functions (sin, cos, etc.). (This is the *real RAM* model of computation.)

(b) Conclude that computing the Voronoi diagram of n points requires $\Omega(n \lg n)$ time in the worst case in the *algebraic decision tree* model of computation, in which the computation can branch based only on a binary decision of comparing two algebraic expressions (expressions involving inputs and $+$, $-$, $\cdot$, $/$, $\sqrt{}$), and the cost of a computation is the depth of that node in the tree.

Problem 4. The *width* of a set of points is the minimum distance between two parallel lines with the property that all of the points lie in the slab between the two parallel lines. Give an algorithm to compute the width of a point set in linear expected time. *This problem turns out to be unsolvable! Sorry!*

Problem 5. Submit a $\frac{1}{2}$- to 1-page writeup of your project plan, on a page separate from your problem set. The writeup can be the same as your other group members', but each person should submit one. It should include references.