Problem Set 1, Part a

Due: Thursday, February 21, 2008
Problem sets will be collected in class. Please hand in each problem on a separate page.
Students who agree to let us hand out their writeups can help us by writing elegant and concise solutions and formatting them using LaTeX.

Reading:

Chapters 1 and 2 of *Distributed Algorithms*.
Chapter 3, Sections 3.1-3.6
Chapter 4, Sections 4.1-4.3

Reading for next week: Chapter 4, Section 4.4
Chapter 5, Section 5.1
Chapter 6, Sections 6.1-6.3

Problems:

Note: In all homework problems, you are free to invoke theorems proved in the book without re-proving them.

1. (a) Exercise 3.2, part (a).
   (b) Exercise 3.2, part (b).
   (c) Exercise 3.2, part (c).
   (d) Give a specific UID assignment for which $\Theta(n \log n)$ messages are sent.
2. Exercise 3.8.
3. Exercise 3.9. Prove correctness and prove the complexity bound. (You may find this hard.)
4. Consider the problem of determining the size of a synchronous bidirectional ring with UIDs using a comparison-based algorithm. Each process should output the correct number n of processes.
   (a) Design an algorithm that solves this problem. Prove the worst-case number of messages required to solve the problem with this algorithm. Try to get the smallest value you can for this measure.
   (b) Prove the best *lower bound* you can on the number of messages required to solve this problem in the worst case.
   (c) Provide an upper and lower bound for the following two problems. For both, assume they are comparison-only and in the same bidirectional ring environment. (Hint: You might find it easier to generalize your previous results to something that covers both as opposed to starting from scratch for each).
      - Each process should output the parity of n (i.e., whether n is odd or even).
      - Each process outputs whether the ring is "ordered" (i.e., UIDs increase clockwise around the ring except from the process with the maximum UID to the one with minimum UID).
5. Exercise 4.6