6.851: Advanced Data Structures, Spring 2021

Prof. Erik Demaine, Josh Brunner, Dylan Hendrickson, Yevhenii Diomidov

Problem Set 7

Due: Thursday, April 8, 2021

Problem 7.1 [Van Emde Boas in Space]. Recall the "saving space" version of the van Emde Boas predecessor data structure described in Lecture 11:[1]

A van Emde Boas structure V of universe size u consists of:

- $V.\min$ = the minimum element in V, not stored recursively
- $V.\max$ = the maximum element in V, stored recursively
- $V.\mathrm{summary}$ = a van Emde Boas structure of universe size $\sqrt{u}$, representing which clusters are nonempty (excluding $V.\min$)
- $V.\mathrm{cluster}$ = a **hash table** mapping a cluster number in $\{0, 1, \ldots, \sqrt{u}-1\}$ to a van Emde Boas structure of universe size $\sqrt{u}$ representing the elements in that cluster, but **only for clusters that are nonempty** (excluding $V.\min$)

(a) Prove matching (up to constant factors) **upper and lower bounds** on the worst-case space occupied by a "saving space" van Emde Boas structure of universe size u storing n elements. (Count the number of **words** of space, as an asymptotic function of n and u, ignoring constant factors.) As long as your upper and lower bounds match up to constant factors, we will accept weak lower bounds of the form "for any sufficiently large n, there is a u such that the space is at least $f(n, u)$". Equivalently, it suffices to produce an infinite family of instances with arbitrarily large n; you can assume a relation between n and u, as long as n can become arbitrarily large. On the other hand, your upper bound must work for all n and all u.

(b) How can you modify the data structure to achieve $O(n)$ words of space?

[1] http://courses.csail.mit.edu/6.851/spring21/lectures/L11.html?notes=7