6.851: Advanced Data Structures, Spring 2021

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Problem Set 4 Solutions

Due: Thursday, March 18, 2021

Problem 4.1 [Generically Speaking]. Show that any point set S can be transformed into a generic point set S' preserving the strict ordering of coordinates and approximately preserving OPT.

Solution: It suffices to solve the problem one dimension at a time:

Lemma 1. *For any point set S, there exists a point set S' such that*

(a) No two points in S' share an x coordinate.

(b) Strict relative ordering of points is preserved.

(c) $|\mathrm{OPT}(S')| = O(|\mathrm{OPT}(S)|)$.

Proof. Let $T = \mathrm{OPT}(S)$ and, for any x, let $S_x = [(x, y_1), (x, y_2), \ldots, (x, y_m)] \subseteq S$ denote the list of all points in S with that x coordinate, sorted in increasing order of y.

Let $\Delta x > 0$ be smaller than any positive difference between x coordinates of points in S. Then it is also smaller than any positive difference between x coordinates of points in T. Let $\varepsilon = \Delta x/n$.

Now we construct two sets $S' \subseteq T'$:

(a) For each $(x, y_i) \in S_x$, add $(x + i\varepsilon, y_i)$ to S' and T'.

(b) For each $(x, y_{i+1}) \in S_x$, add $(x + i\varepsilon, y_{i+1})$ to T'.

(c) For each $(x, y) \in T$, add (x, y) and $(x + n\varepsilon, y)$ to T'.

Informally, for each vertical segment S_x we add a diagonal line going up and to the right to S' and a backwards "N"[1] to T'.

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*     X . . . X *
*     X . . X * X
X     X . . . . X
*     X . X * . X
X ->  X . . . . X
*     X X * . . X
*     X * . . . X
*     * . . . . X
S/T       S'/T'
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Figure 1: "*" denotes a point in both S' and T', while "X" denotes a point in S' only.

It is easy to see that T' is satisfied, and that $|\mathrm{OPT}(S')| \le |T'| \le 4|T| = 4|\mathrm{OPT}(S)|$. ∎

[1] Not to be confused with a forwards "И", which has a thinner diagonal stroke and more symmetrical serifs.