TODAY: Dynamic graphs
- new results

New results: (partial list)

Worst-case dynamic connectivity:
- recall best previous was $O(\sqrt{n})$ update
- $O(\log^4 n)$ insert, $O(\log^5 n)$ delete,
  $O(\frac{\log n}{\log \log n})$ query correct with high probability
  [Kapron, King, Mountjoy - SODA 2013]
- idea: to find replacement edge, look for edge with odd # endpts. in $A$
  $\Rightarrow$ in XOR of edge names $\forall$ endpt. $\in A$,
evenly occurring edges cancel (& finds edge if only one)
- $\sum$ random $\pm 1$ per edge $\forall$ endpt. $\in A$
estimates $\sqrt{\text{size of cut}}$
- use sketching from streaming algorithms
Transitive closure: directed, \( s \rightarrow t \) path?
- \( m^{1/3-o(1)} \) update or query (\& product \( \geq m^{2/3-o(1)} \))
or \( m^{4/3-o(1)} \) preprocessing
ASSUMING no \( O(n^{2-\varepsilon}) \) algorithm for 3SUM
given \( n \) integers, do any 3 sum to 0? \( \leq n \) [Abboud & Williams - arXiv 2014]

**Single-source reachability:** directed, \( s \) fixed
- \( O(m+n) \) update or query trivial
  \& best known for sparse graphs
- \( O(n^{1.495}) \) query [Sankowski - SODA 2007]
  via fast Boolean matrix multiplication
- any \( O(n^{3-\varepsilon}) \) update \& query, \& \( O(n^{3-\varepsilon}) \) preproc.
  \( \Rightarrow \) \( O(n^{3-\varepsilon}) \) Boolean matrix multiplication [AW]
- incr/decremental: ditto for worst-case updates
- decremental: \( O(n^{0.984+\varepsilon}) \) expected am. update,
  \( O(1) \) query [Henzinger, Krinninger, Nanongkai - STOC 2014]

**Single-source reachability counting:**
- \# nodes reachable from fixed node \( s \)
  \( \geq m^{1-o(1)} \) update or query, or \( n^{w(1)} \) preproc. [AW]
ASSUMING strong exponential time hypothesis
- \( \forall \varepsilon > 0 \exists k \text{ s.t. } k\text{-CNF can't be solved in } 2^{(1+\varepsilon)n} \cdot n^{O(1)} \# \text{ variables} \)
Single-source shortest paths: directed & weighted
- \(O(m + nlgn) = O(n^2 lgn)\) update/query via Dijkstra
- any incr/decremental \(O(n^2 \varepsilon)\) update & query & \(O(n^3 \varepsilon)\) preprocessing \(\Rightarrow O(n^3 \varepsilon)\) algorithm for all-pairs shortest paths with polynomial integer edge weights [AW]
- decremental \((1+\varepsilon)\)-approximate: \(O(n^{0.984+\varepsilon})\) [HKN]
- decremental \((1+\varepsilon)\)-approx. unweighted undirected: \(O(n^{0.8} n/m + m^{o(1)})\) update, \(O(1)\) query
  [Henzinger, Krinninger, Nanongkai - SODA 2014]

Distance oracles: static weighted undirected graph,
- \(O(1)\)-time \((1+\varepsilon)\)-approx. s-t shortest-path weight
- \(O(\frac{1}{\varepsilon} n^{1+\varepsilon})\) space & \(O(\frac{1}{\varepsilon} mn^{\varepsilon})\) preproc.
  [Thorup & Zwick - J.ACM 2005]
- \(O(n lg n polylg lg n)\) space & \(O(1/\varepsilon)\) query for planar
  [Kawarabayashi, Sommer, Thorup - SODA 2013]
Problem 1: Add to link-cut trees the op.:
\[ \text{exert}(x) \text{: reroot } x \text{'s tree to have root } x \text{ while preserving } O(lg n) \text{ amortized/op.} \]
\[ \Rightarrow O(lg n) \text{ dynamic connectivity in trees} \]

[Sleator & Tarjan 1985]

Problem 2: Decremental single-source reachability in a DAG in \( O(m+q) \) total time \( \Rightarrow O(1) \) amortized if all edges deleted

[Italiano - IPL 1988]