

6.851 ADVANCED DATA STRUCTURES (SPRING'12)

Prof. Erik Demaine TAs: Tom Morgan, Justin Zhang

Problem 6 *Due: Thursday, Apr. 5*

Be sure to read the instructions on the assignments section of the class web page. Remember to keep your solutions to one page!

Concise van Emde Boas. Develop and analyze a data structure that supports insert, delete, successor and predecessor in the word-RAM model in $O(\lg \lg u)$ worst-case time. Your data structure should use $O(u)$ bits of space. Note that the van Emde Boas data structure from lecture used $\Theta(u)$ words of space, and thus $\Theta(u \lg u)$ bits of space.

Union-Split-Find. Develop and analyze a word-RAM data structure to maintain a set of disjoint intervals of the form $[a, b)$ such that $a, b \in \mathcal{U}$. Your data structure should support the following operations in $O(\lg \lg u)$ time:

- **make** (a, b) : Create the interval $[a, b)$ (must not overlap existing intervals).
- **union** (a, b, c) : Merge the adjacent intervals $[a, b)$ and $[b, c)$ into $[a, c)$.
- **split** (a, b, k) : For $k \in [a, b)$, split the interval $[a, b)$ into $[a, k)$ and $[k, b)$.
- **find** (k) : Return the interval $[a, b)$ that contains k , or report that no interval contains k .