6.851 Advanced Data Structures (Spring’12)
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Problem 3  Due: Thursday, Mar. 8

Be sure to read the instructions on the assignments section of the class web page.

Geometric basics. Recall that in class we introduced the geometric view for binary search tree execution. Using only this view, prove that for any set of \(m\) queries on \(n\) items, there is a BST that will answer the queries in total time \(O(m \lg n)\). You should reason only about point sets, not about BSTs.

Working-set is harder. In class we introduced the entropy bound and the working-set property for BSTs. The entropy bound holds if all searches in the BST have amortized time \(O(\sum_{k=1}^{n} p_k \lg \frac{1}{p_k})\), where \(p_k\) is the fraction of the time that key \(k\) is queried. The working-set property holds if the time to search for an element \(x_i\) is \(O(\lg t_i)\), where \(t_i\) is the number of elements queried since the last access to \(x_i\). Prove that any BST with the working-set property also has the entropy bound.