6.851 Advanced Data Structures (Spring'10)

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Problem 7 Sample Solutions

Finding the most significant 1 bit.

(a) We define $C_0 = 0^b$, and $(C_1) = 10^{b-1}$ $A_x = \big[x \texttt{ | } (x + \tilde{}C_1)\big] \texttt{ \& } C_1$

Step-a(x)
$A_x \leftarrow x + 01^{b-1}$
$A_x \leftarrow A_x \texttt{ | } x$
return $A_x \texttt{ \& } 10^{b-1}$

(b) $B_x = \big((A_x \texttt{ >> } (b-1)) \texttt{ * } (0^b 1)^{b-1} \texttt{ >> } (w-b)\big) \texttt{ \& } 1^b$

Steps-ab(x)
$A_x \leftarrow$ Step-a(x)
$B_x \leftarrow A_x \texttt{ >> } (b-1)$
$B_x \leftarrow (B_x \texttt{ * } [0^b 1]^{b-1}) \texttt{ >> } (w-b)$
return $B_x \texttt{ \& } 1^b$

(c) Note that the order of the chunks has changed in part (b). We need to find the *least* significant 1 bit. Also, not that $-x = \tilde{}x + 1$.
$C_x = b - (B_x \texttt{ \& } -B_x)$

Steps-abc(x)
$B_x \leftarrow$ Step-b(x)
$C_x \leftarrow B_x \texttt{ \& } -B_x$
return $b - C_x$

(d) Let $\alpha = \sum_{i=0}^{b-1} (1 \texttt{ << } i) \texttt{ << } i$, then by setting $y = (x \texttt{ >> } C_x) \texttt{ \& } 1^b$, and $z = (y \texttt{ * } [0^{b-1}1]^b) \texttt{ \& } \alpha$, we have reduced the problem to parts (a)-(c).

Full Algorithm

Most-Significant(x)
$C_x \leftarrow$ Steps-abc(x)
$y \leftarrow (x \texttt{ >> } C_x) \texttt{ \& } 1^b$
$z \leftarrow (y \texttt{ * } [0^{b-1}1]^b) \texttt{ \& } \alpha$
$C_z \leftarrow$ Steps-abc(z)
return $(C_x \texttt{ << } b + C_z)$