6.851 Advanced Data Structures (Spring'10)

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$\boxed{\text{Problem 3}}$ *Due: Thursday, Feb. 25*

Be sure to read the instructions on the assignments section of the class web page.

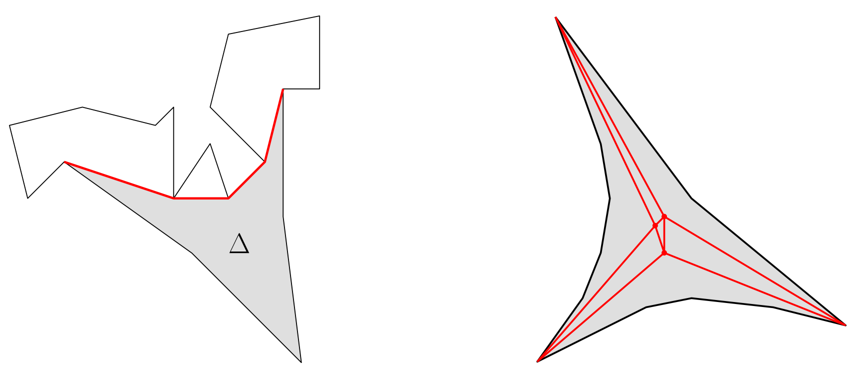
Ray Shooting in Simple Polygons.

1. Let $w_1, w_2, \ldots, w_k$ be positive weights we want to store in a weight balanced binary search tree (WBBST). We denote $\sum_{j \leq k} w_j$ by W. To built the WBBST we find the unique element w_r such that the sums $\sum_{j<r} w_j$ and $\sum_{j>r} w_j$ are both at most $W/2$. We make the element w_r the root of the WBBST and recurse on the two remaining subsets (left and right of w_r).

   (a) Show that we can search for an item with weight w_i in the WBBST in $O(1 + \log(W/w_i))$ time.

   (b) To search through all home-in situations that occur during the ray shooting algorithm we use a WBBST for every concave chain of the balanced pseudo-triangulation. For an edge we choose as weight its *bay-size* (= number of edges of the opposing polygon). Show that we can answer the searches for all home-in situations using $O(\log n)$ time in total.

2. By adding additional interior points we can transform a balanced pseudo-triangulation into a pseudo-triangulation where every pseudo-triangle has only one concave chain. Since we add at most 6 new pseudo-triangles any line intersects at most $O(\log n)$ of these special pseudo-triangles.
   Show how to triangulate the special pseudo-triangles using additional points, such that any line intersects at most $O(\log^2 n)$ triangles.



(i) (ii)

Figure 1: The bay-sizes of the red chain of Δ in (i) are from left to rigth: 7,3,1,6. Picture (ii) shows how to decompose a pseudo-triangle into pseudo-triangles with at most one concave chain using additional points.