Ray Shooting in Simple Polygons.

1. Let $w_1, w_2, \ldots, w_k$ be positive weights we want to store in a weight balanced binary search tree (WBBST). We denote $\sum_{j \leq k} w_j$ by $W$. To built the WBBST we find the unique element $w_r$ such that the sums $\sum_{j < r} w_j$ and $\sum_{j > r} w_j$ are both at most $W/2$. We make the element $w_r$ the root of the WBBST and recurse on the two remaining subsets (left and right of $w_r$).

(a) Show that we can search for an item with weight $w_i$ in the WBBST in $O(1 + \log(W/w_i))$ time.

(b) To search through all home-in situations that occur during the ray shooting algorithm we use a WBBST for every concave chain of the balanced pseudo-triangulation. For an edge we choose as weight its bay-size ($=\text{number of edges of the opposing polygon}$). Show that we can answer the searches for all home-in situations using $O(\log n)$ time in total.

2. By adding additional interior points we can transform a balanced pseudo-triangulation into a pseudo-triangulation where every pseudo-triangle has only one concave chain. Since we add at most 6 new pseudo-triangles any line intersects at most $O(\log n)$ of these special pseudo-triangles. Show how to triangulate the special pseudo-triangles using additional points, such that any line intersects at most $O(\log^2 n)$ triangles.

Figure 1: The bay-sizes of the red chain of $\Delta$ in (i) are from left to right: 7,3,1,6. Picture (ii) shows how to decompose a pseudo-triangle into pseudo-triangles with at most one concave chain using additional points.