6.851 Advanced Data Structures (Spring'07)

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Problem 7 – Solution

Hamming Weight via Word Packing.

Consider our input string x of length k and let $z = \lfloor 2 + \lg k \rfloor$. We assume that $k + 1$ is not a power of 2 (otherwise we shorten x by 1 by computing $x\%2$ and one shift right) and that z and k are relatively prime (otherwise set $z = z - 1$).

The key idea is to transform x into a bit string that still fits in a machine word and has the same number of ones as x but all the ones are in locations that are multiples of z. Then all we need to do is use the modulo operator $(\%(2^z - 1))$ on the new string (in $O(1)$ time).

1. Set A to be a string with z ones and k zeros between any two ones.
2. Compute $y = xA$ (y is a bitstring that consists of z copies of x).
3. Set B to be a string with k ones and z zeros between any two ones.
4. Return $(y \text{ AND } B)\%(2^z - 1)$.

AND-ing y with B is equivalent to partitioning y to consecutive blocks of length z and replacing every block with a 000..01 iff the rightmost bit of the block is 1. Because z and k are relatively prime, this operation makes sure that every 1 bit in x corresponds to some unique 1 bit in (y AND B) that appears in a location that are multiples of z. Thus, $(y \text{ AND } B)\%(2^z - 1)$ gives the answer.