6.851 Advanced Data Structures (Spring'07)

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Problem 6 *Due: Monday, Apr. 2*

Be sure to read the instructions on the assignments section of the class web page.

On Weak d-universal Hash Families. Recall that a set $\mathcal{H}$ of hash functions is a *weak d-universal family* if, for all $x, y \in U$ with $x \neq y$,

$$\Pr_{h \leftarrow \mathcal{H}} \Big\{ h(x) = h(y) \Big\} = \frac{d}{m}.$$

Let $U = \mathbb{Z}_2^\ell$ (the set of bit vectors of length ℓ). For a given $k \times \ell$ binary matrix M, we define a hash function $h_M : U \to \mathbb{Z}_2^k$ as $h_M(x) = M \cdot x$, where additions and multiplications are done modulo 2. Show that the family $\mathcal{H} = \{h_M \mid M \text{ is a binary } k \times \ell \text{ matrix}\}$ is weakly 1-universal.

Deterministic y-fast tries. Suppose you have a dynamic perfect hash function h such that:

- h is constructible in deterministic linear time;
- $h(x)$ can be evaluated in $O(1)$ worst case, deterministic time;
- insertions and deletions take $O(\lg^5 u)$ worst case time;

Use h to modify the y-fast trie data structure to support insertion, deletion, predecessor, and successor in $O(\lg \lg u)$ amortized deterministic (rather than randomized) time.

Range Existence Queries. Given a set S of integers, the *range existence query* $req(a, b)$ asks whether there is any element in $S \cap \{a, a+1, \ldots, b\}$. Suggest an $O(n \lg u)$-space data structure that stores a static set S of n integers from $\mathcal{U} = \{0, 1, \ldots, u-1\}$ and answers range existence queries in expected $O(1)$ time.

Hint: Think why LCA queries in a perfect binary tree are easy.