

## 6.851 ADVANCED DATA STRUCTURES (SPRING'07)

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Problem 3      *Due: Monday, Mar. 5*

Be sure to read the instructions on the assignments section of the class web page.

**Dynamic connectivity with key values.** Show how to modify the  $O(\lg^2 n)$  dynamic connectivity data structure by Holm, de Lichtenberg, and Thorup to maintain a key value in each node subject to two operations:

- $\text{find-min}(v)$ : find the minimum key of any node in the connected component containing node  $v$ .
- $\text{set-key}(v, x)$ : change the key stored in node  $v$  to value  $x$ .

The asymptotic cost of old operations should remain the same, and the new operations should be supported as fast as possible. (Assume that keys are totally ordered and can be compared in  $O(1)$  time.)

**Dynamic connectivity with path queries.** Show how to modify the  $O(\lg^2 n)$  dynamic connectivity data structure by Holm, de Lichtenberg, and Thorup to support a *path connectivity query*: given two vertices  $v$  and  $w$ , find a path between  $v$  and  $w$ , or report that no such path exists. This new query should be supported in time  $O(\lg n / \lg \lg n)$  time if there is no path and  $O(\lg n / \lg \lg n + k)$  time if the output path has length  $k$ .