6.851 Advanced Data Structures (Spring'07)

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Problem 1 *Due: Monday, Feb. 19*

Be sure to read the instructions on the assignments section of the class web page.

Move-To-Front Variations. The Move-To-Front heuristic moves the accessed item x all the way to the front of the list. In this question we examine the behavior of Move To Front when the accessed item is moved forward but not all the way to the front.

Prove that if we move the accessed item x forward by a fixed number of locations k, then the competitive ratio is not $O(1)$. What is the worst-case competitive ratio?

In contrast, prove that if we move x some fixed fraction of the distance to the front (i.e., if x is at index i in the list and our fixed fraction is $0 < r < 1$, we move x exactly $\lceil r \cdot i \rceil$ locations forward), then the performance of that heuristic is within a constant factor of the optimal. What factor do you obtain?

Splay Trees Bounds. Recall the *working-set bound*: if $t_i(y)$ is the number of distinct elements (including y) accessed since the last time y was accessed before time i, then the amortized cost to access x_i is $O(1 + \lg t_i(x_i))$. This bound says that, if accesses concentrate on a smaller set of elements (the working set), then the cost is logarithmic in this set, not in n. Equivalently, the bound says that if you access something you accessed recently, the access is cheap.

Also recall the *entropy bound*: if element x occurs f_x times in a sequence of m accesses, then the amortized cost to access an element is $O(\frac{1}{m} \sum_x f_x \lg(m/f_x))$. For binary search trees, the entropy bound is equivalent to static optimality, up to constant factors.

Prove that the working-set bound implies the entropy bound.