6.849: GEOMETRIC FOLDING ALGORITHMS, SPRING 2017

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# Problem Set 7 Solutions

*Due: Wednesday, April 5, 2017*

**Solve Problem 7.1 and *either* Problem 7.2 or 7.3.**

**Problem 7.1 [Mandatory, Collaboration exactly with your project group].**
Each week we will ask you to tell us about your progress from the last week on your final project. What have you been working on or thinking about? Did you run into any issues or questions? Did you reach any milestones? Did your project shift direction? (If you don't have progress from the last week, say so to get credit for this problem, but glance nervously at the impending deadline.)

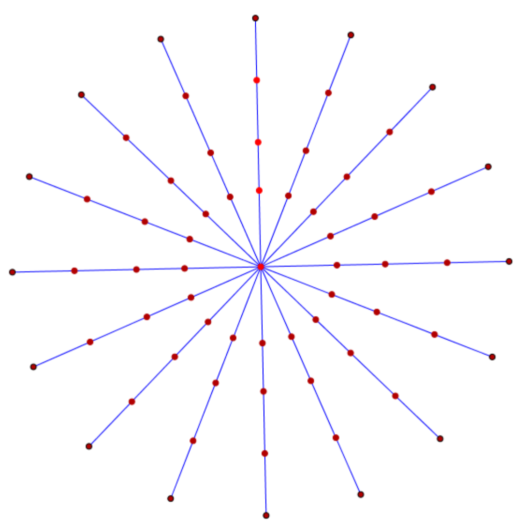
**Solve ONE of the two problems below.**

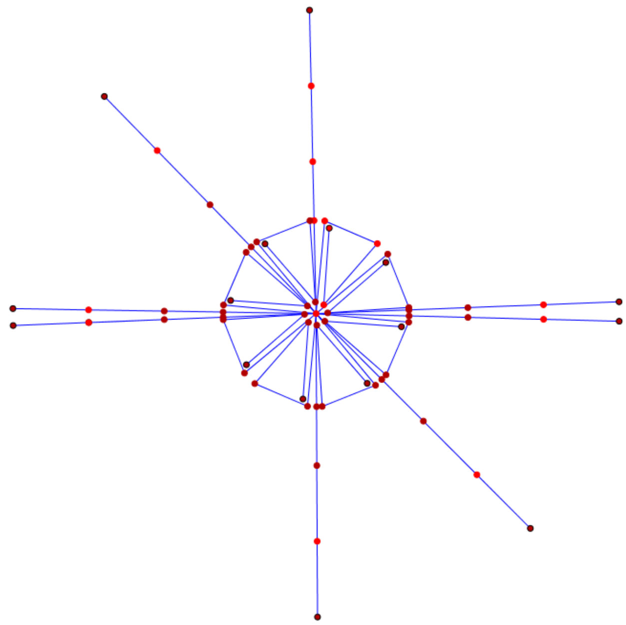
**Problem 7.2 [Collaboration OK].** For infinitely many $n$, find a tree linkage with $n$ bars whose configuration space has $2^{\Omega(n)}$ (exponentially many) connected components. In other words, there should be exponentially many configurations such that you can't reach any of them from any of the others without collisions.

**Solution by Jonathan Tidor:**

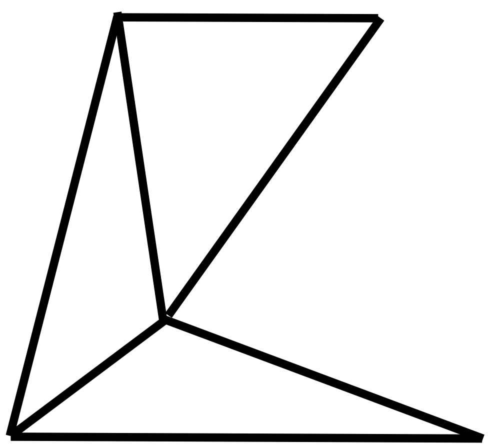
Consider a star with $2n$ arms coming out of a center vertex. Each arm consists of a bar of length $L$ connected to a bar of length 1 connected to a bar of length $L$ that is connected to another bar of length $L$. Pick $L \approx n/(2\pi)$. This is a tree linkage with $N = 8n$ bars. We claim that the configuration space has at least $\binom{2n}{n} \approx 2^{N/4}$ connected components.

To see this, pick $n$ of the $2n$ arms to stick out and fold the other $n$ arms into isosceles triangles. This configuration is locked, in the sense that the $n$ arms folded in make a locked configuration (though the other $n$ arms can wave around). Thus we have described $\binom{2n}{n}$ configurations, none of which can reach each other.

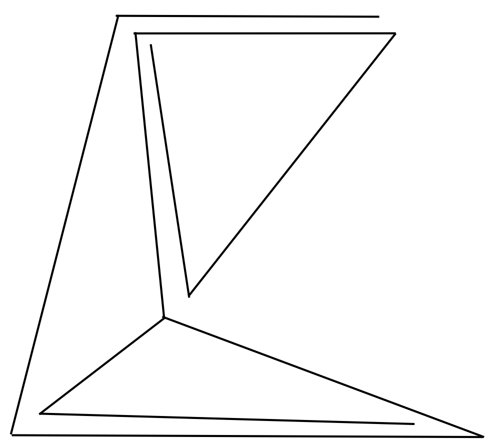




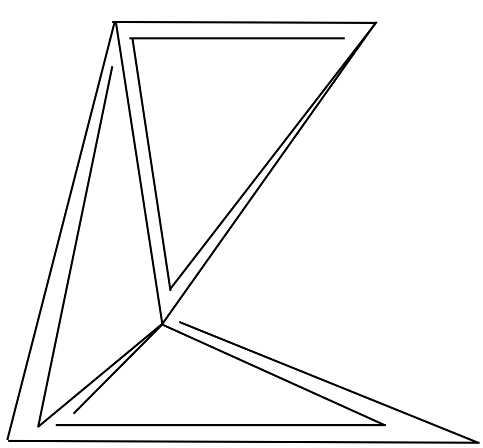
**Problem 7.3 [Collaboration OK].** For each of the three linkages (a), (b), (c) presented below, either find an infinitesimal motion of it or prove it rigid (in particular, using Rule 1 and Rule 2). The top left image shows the underlying graph for the other three linkages.



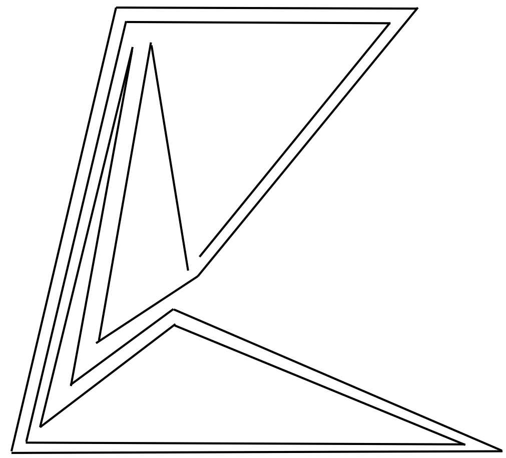
underlying graph



(a)



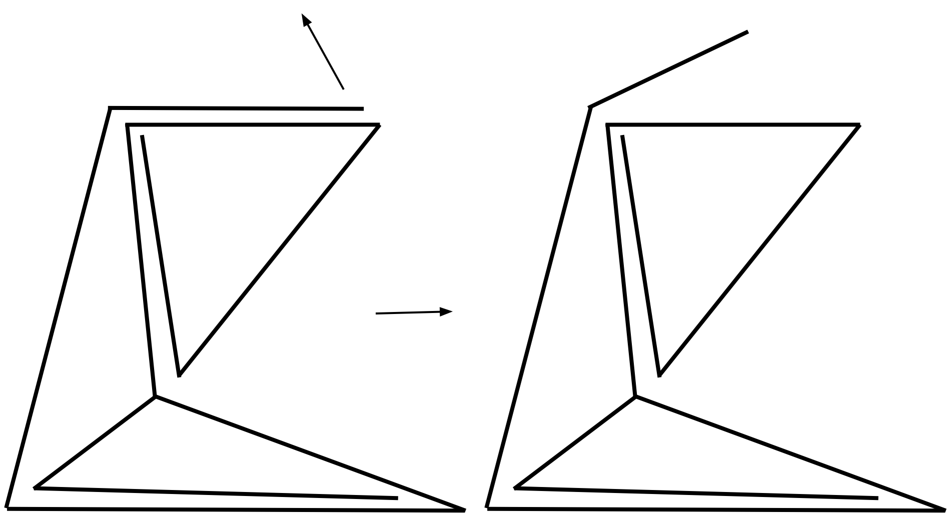
(b)



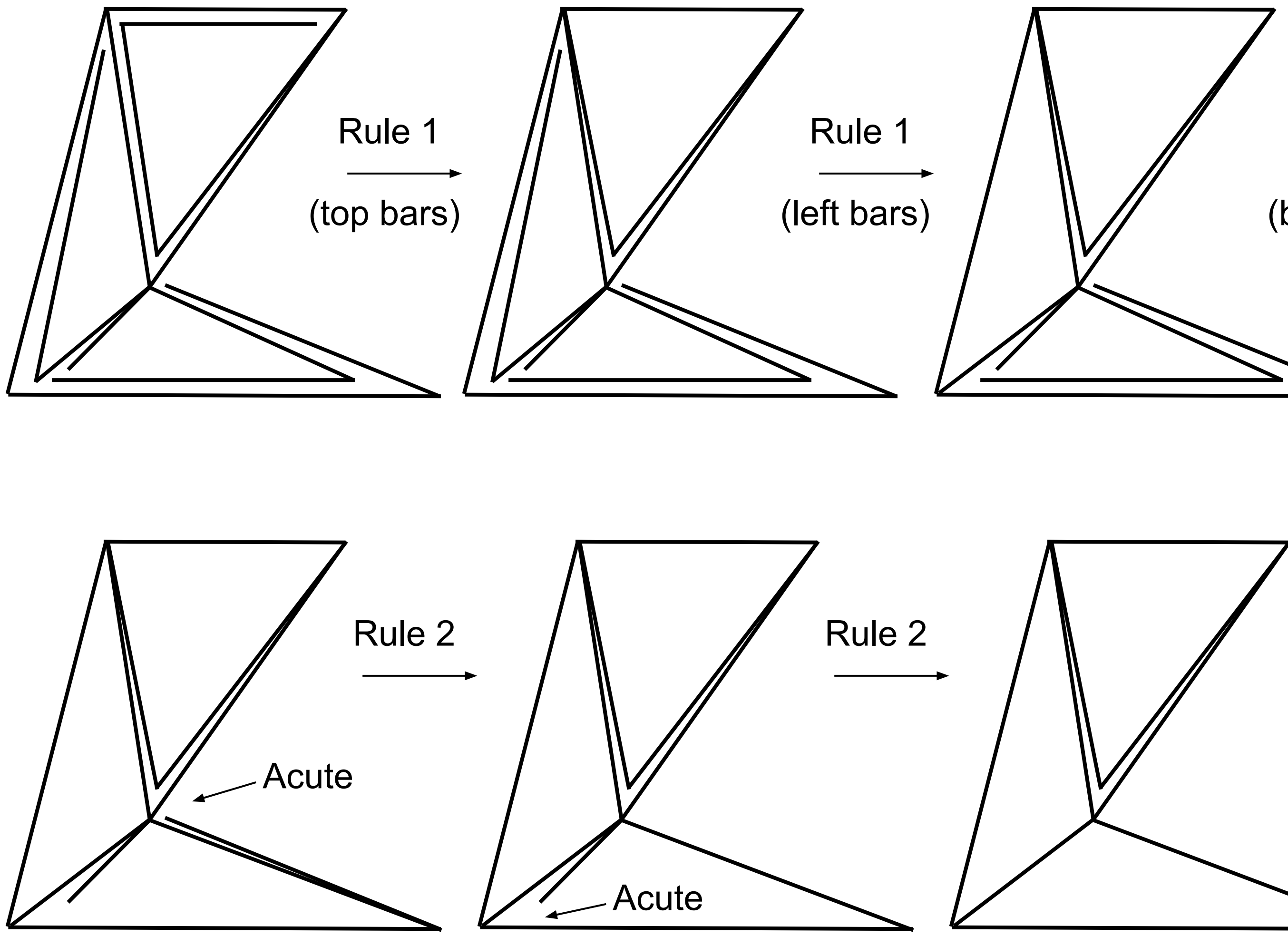
(c)

**Solution by Nur Muhammad Shafiullah:**

(a) Not rigid. Motion:



(b) Rigid.




and the final structure is rigid.

(c) Non-rigid, as it's a chain, and thus the Carpenter's Rule Theorem applies to it. Alternately, as the two bottom triangles are close together and are hinged only at one infinitesimally small region, there's an infinitesimal motion that rotates the bottom triangle.