6.849: Geometric Folding Algorithms, Spring 2017

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Problem Set 6

Due: Wednesday, March 22, 2017

Solve Problem 6.1 and *either* Problem 6.2 or 6.3.

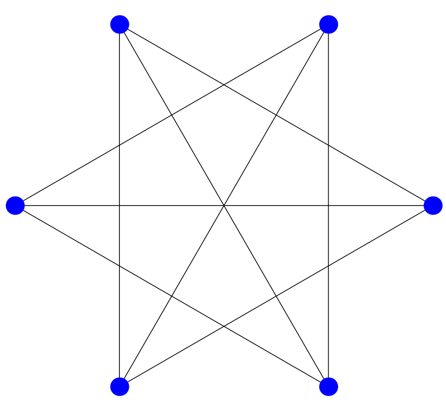
Problem 6.1 [Mandatory, Collaboration OK]. On Problem Sets 1–6, we will ask you to write a problem (solved or unsolved) related to the material covered in class. The problem should be original to the best of your knowledge, so be creative and diverse! Folding can be applied to mathematics, computation, engineering, architecture, biology, and beyond, so write a problem that is related to a field that interests you. If you write a problem whose solution can be solved from the material covered in class, then we may adapt your problem for future problem sets. If you pose a problem whose solution is not yet known, we may try to solve it in class during our open problem sessions, or it may become inspiration for a class project. Feel free to include solutions or commentary for your problem. While writing a problem is required, your submission will be graded generously, so have fun and share with us your exploration of the course material.

(On future problem sets, we will instead be asking about progress on your project.)

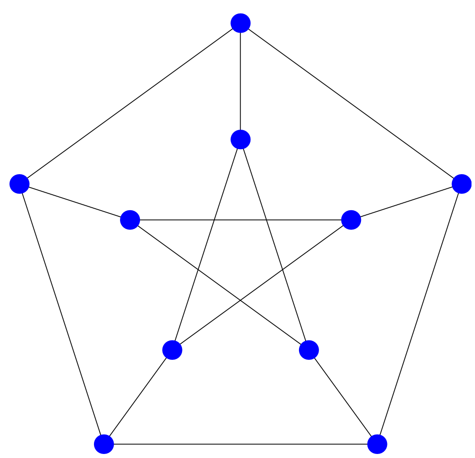
Solve ONE of the two problems below.

Problem 6.2 [Collaboration OK]. Analyze how many bars are used in (the corrected) Kempe's linkage construction from Lecture 9. Specifically, given a target polynomial $f \in \mathbb{R}[x, y]$ of total degree n,[1] and a closed disk B in the plane, show that the constructed linkage that traces $S = B \cap \{(x, y) \in \mathbb{R}^2 : f(x, y) = 0\}$ uses $O(n^2)$ bars.

Problem 6.3 [Collaboration OK]. For each of the following graphs, characterize as generically flexible, minimally generically rigid, or redundantly generically rigid.

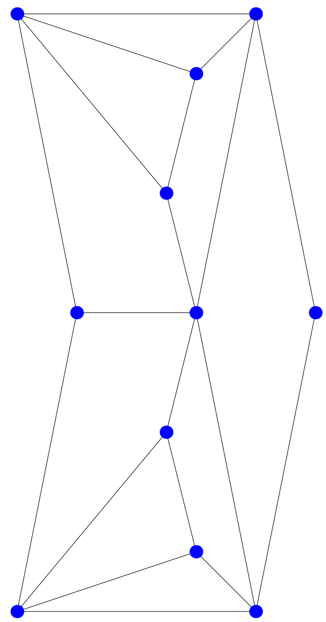


(a)

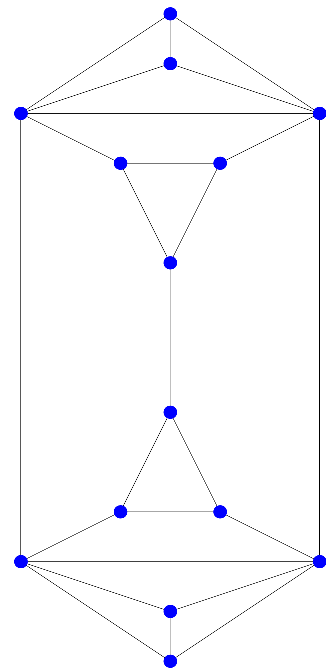


(b)

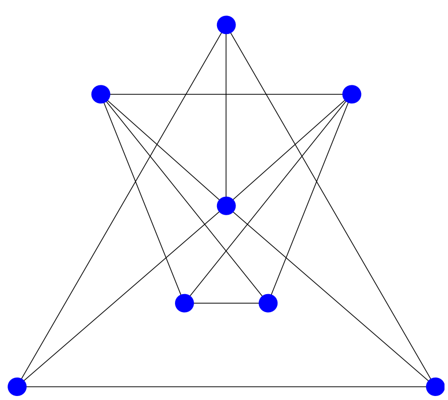
[1] The *total degree* of a polynomial $\sum_i c_i x^{a_i} y^{b_i}$ is $\max_i(a_i + b_i)$.



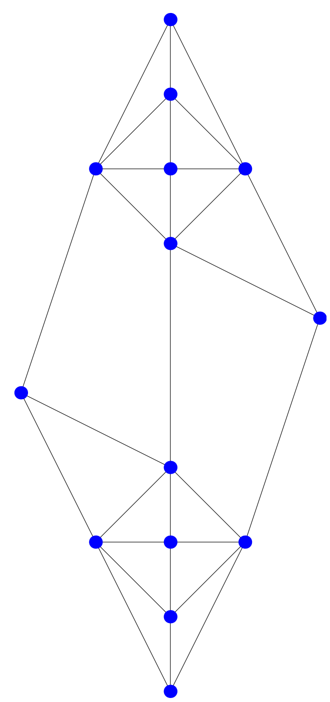
(c)



(d)



(e)



(f)