Computational Design with Curved Creases

An Inquiry of David Huffman's Approach to Paperfolding
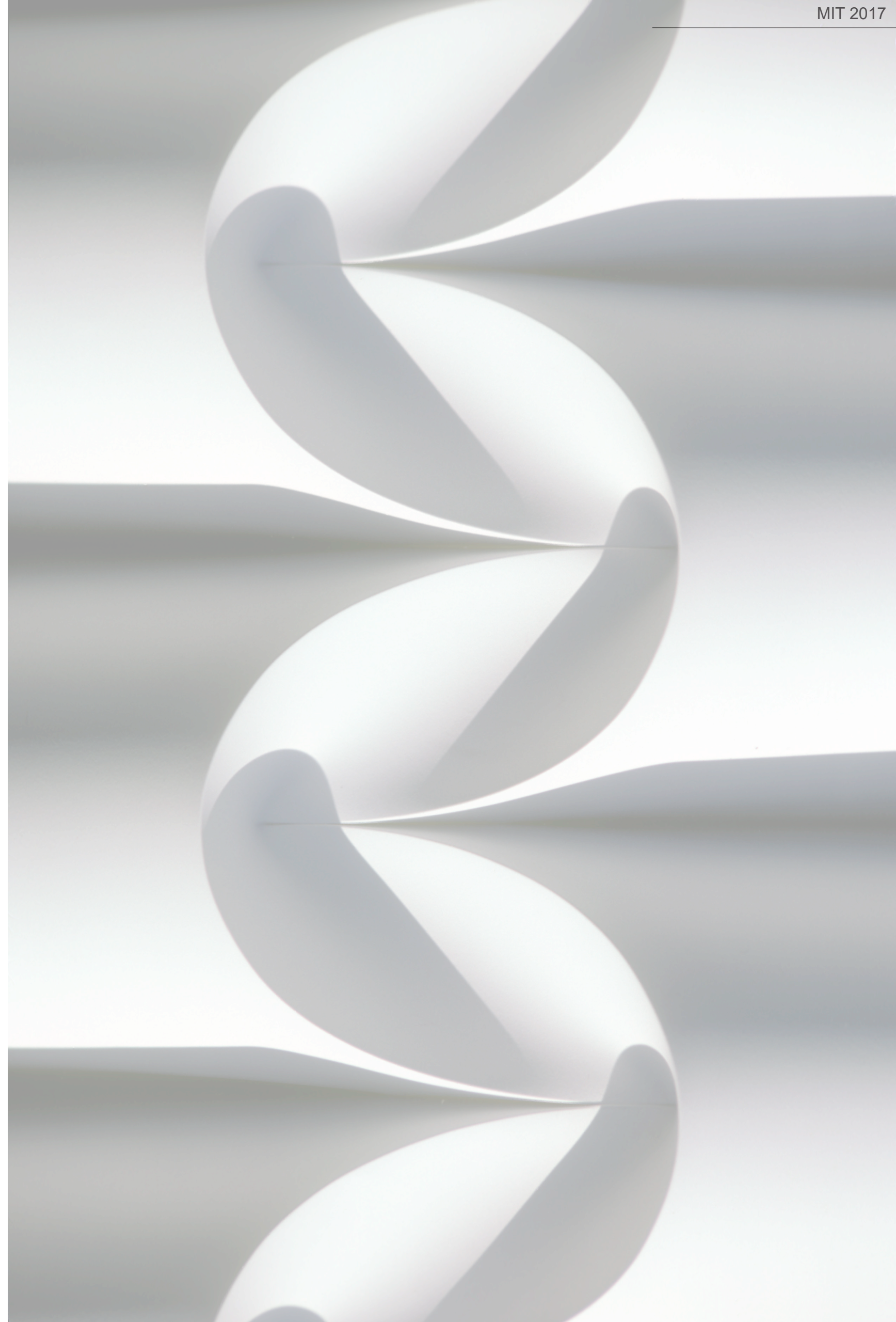
Duks Koschitz
Design and Computation Group, Dept. of Architecture, MIT

2. Historical background of paperfolding

2.2 A history of curved crease paperfolding
Napkin folding
'The Bauhaus model'
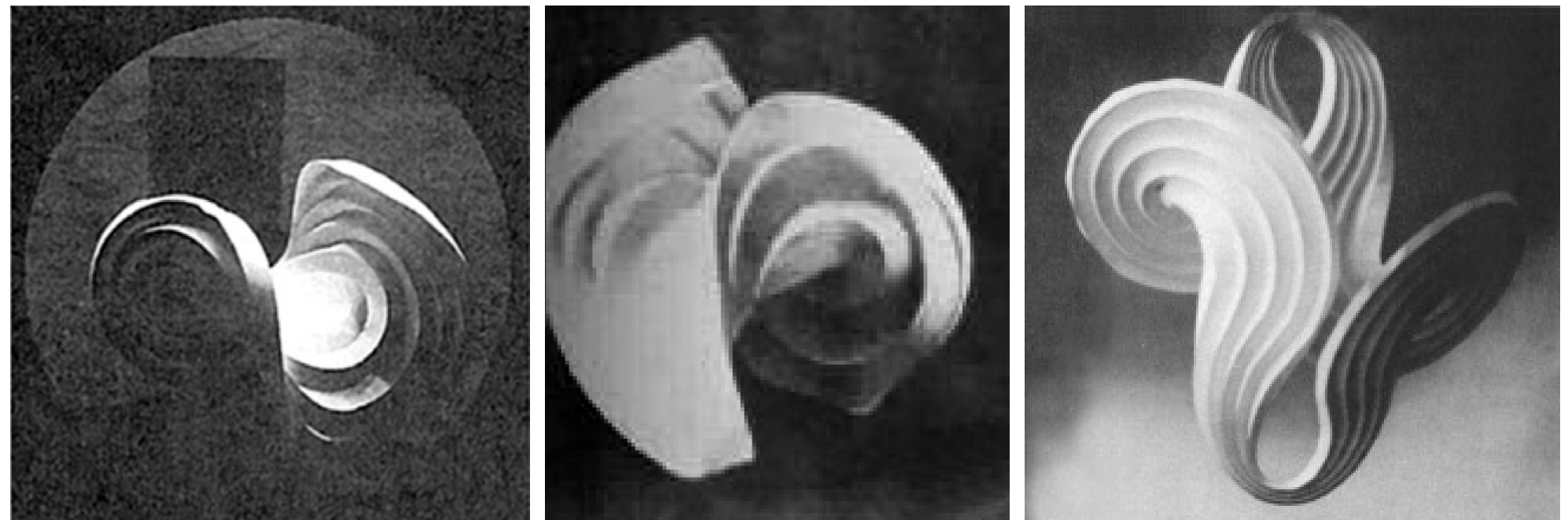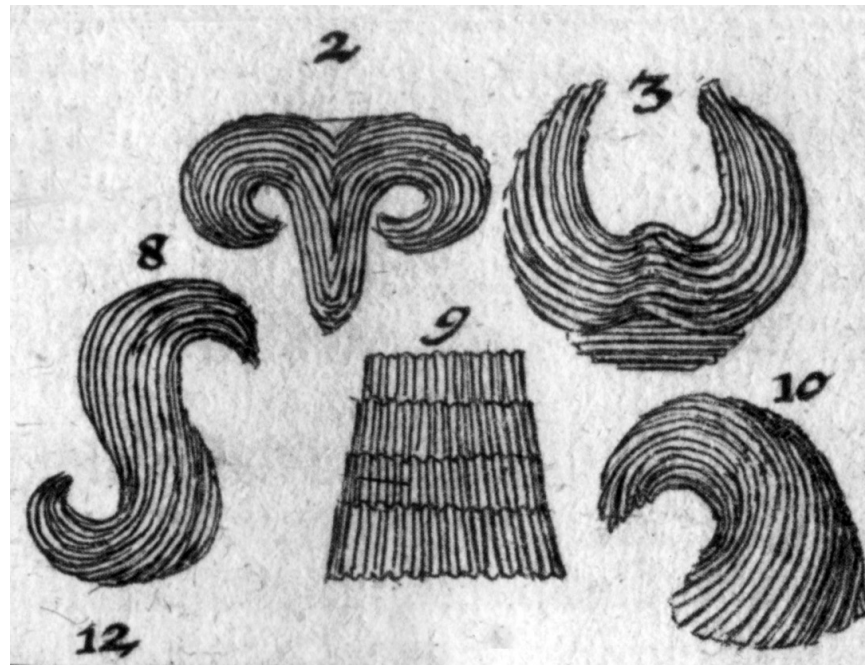David Huffman and Ronald Resch
Product design

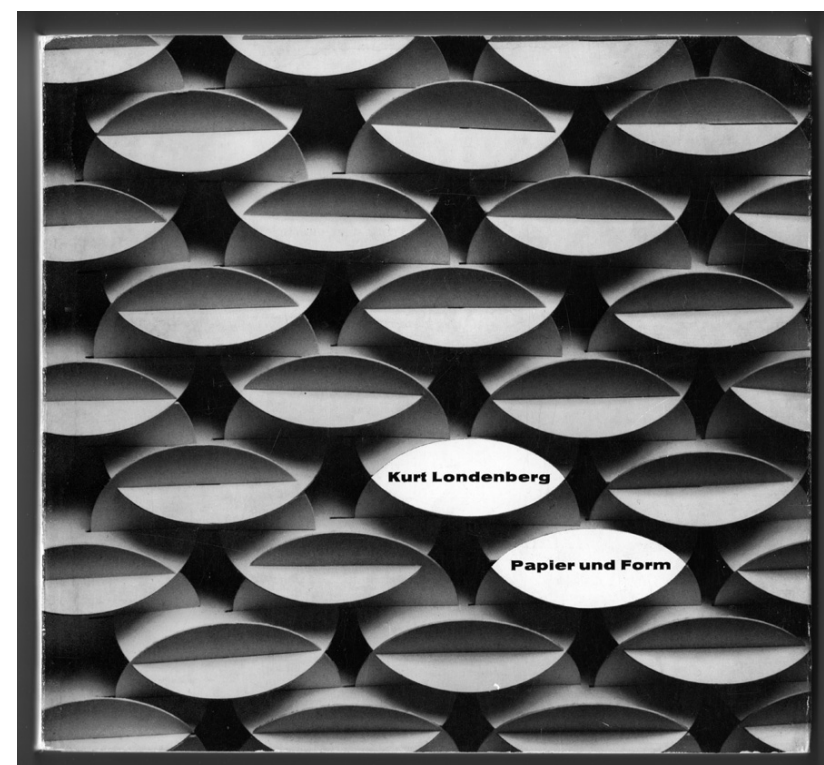**Fig 2.2.2** 'The Bauhaus model',1927 and 1937, a later version by Irene Schawinsky

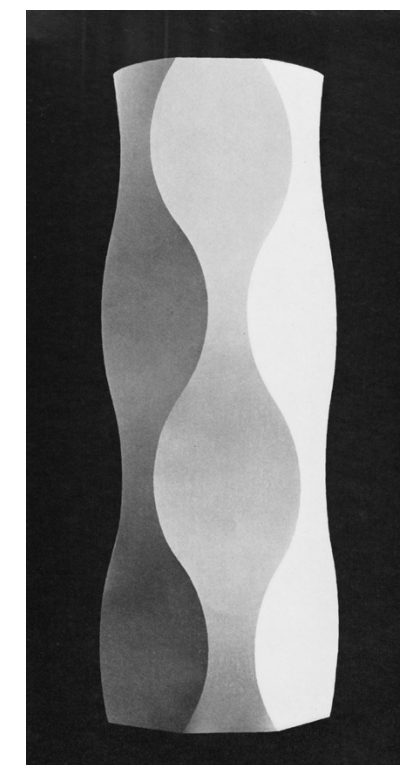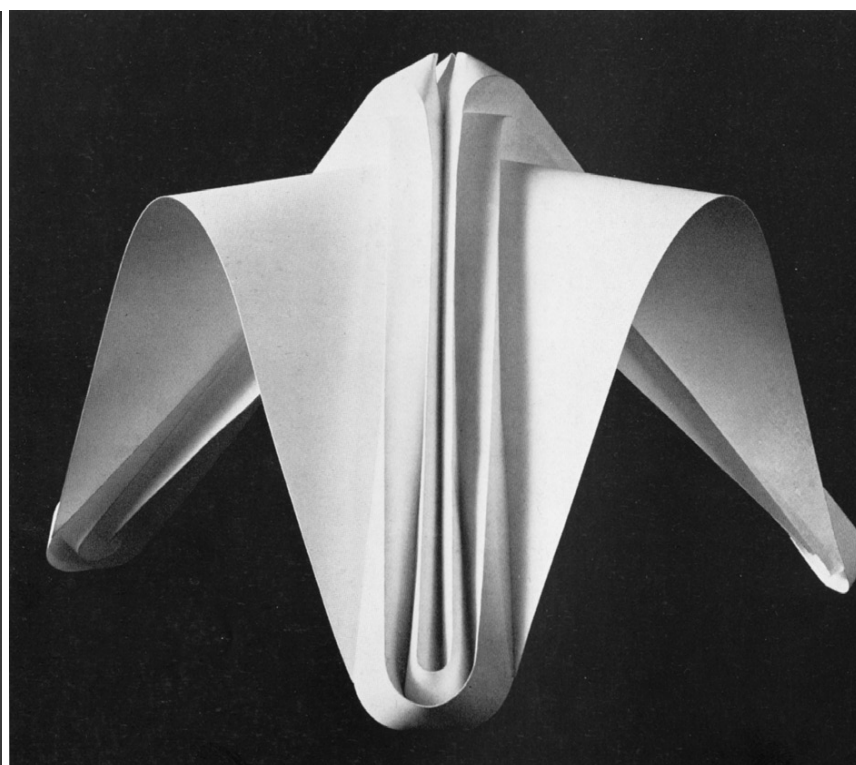**Fig 2.2.5** Cover and a design in 'Papier und Form' (1972, Kurt Londenberg)

Fig 2.2.6 Designs in 'Forms of paper' (1971, Hiroshi Ogawa)

2. Historical background of paperfolding

2.2 A history of curved crease paperfolding
Napkin folding
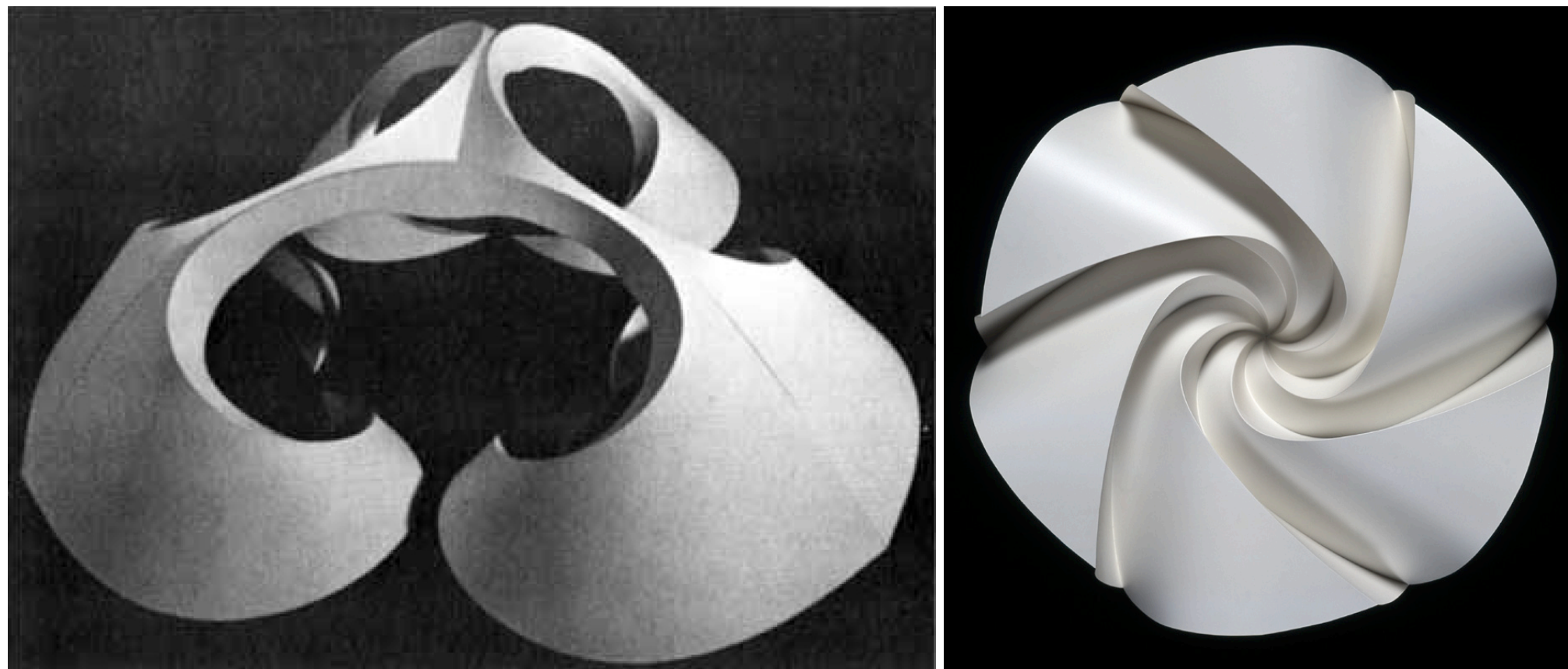'The Bauhaus model'
David Huffman and Ronald Resch
Product design

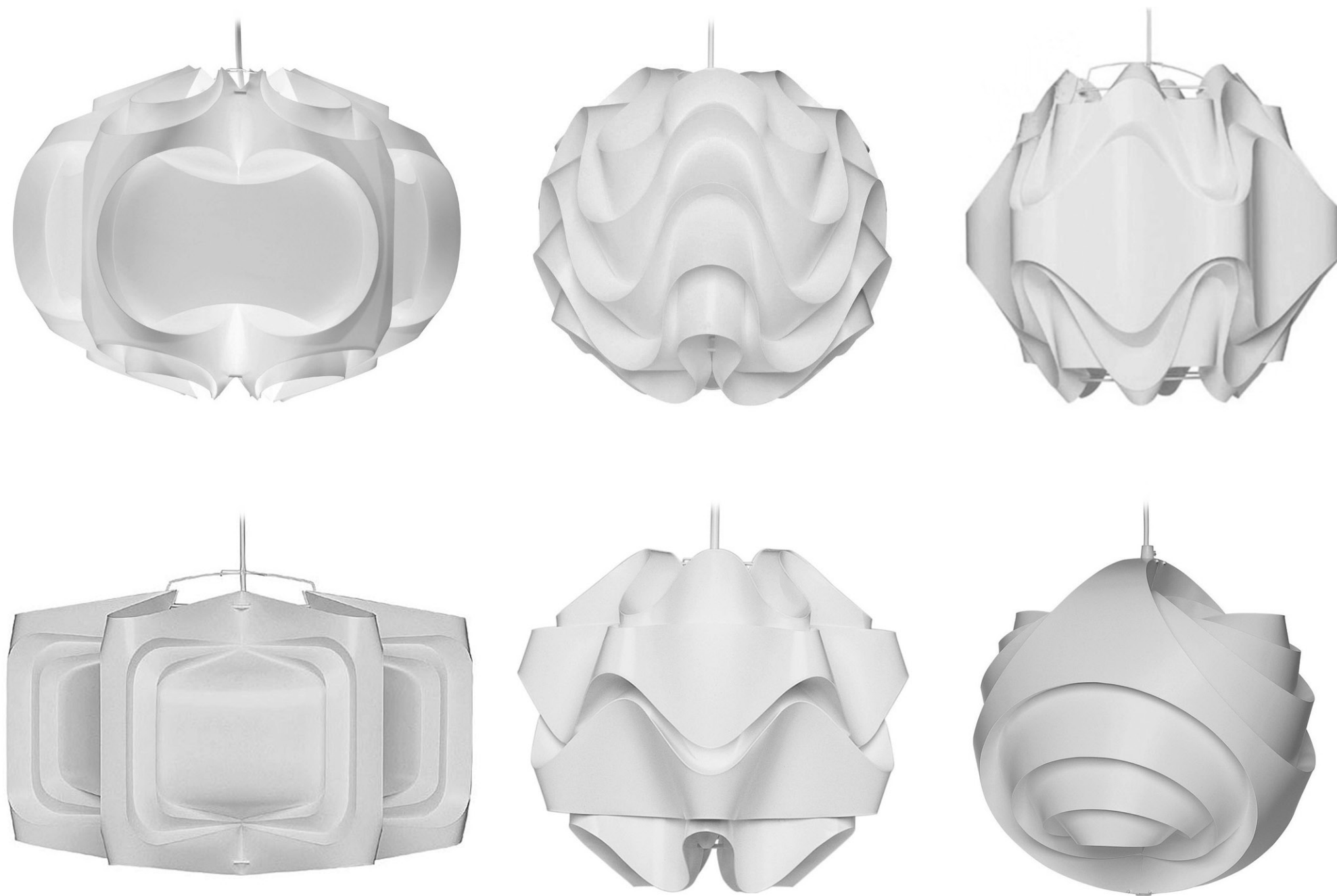**Fig 2.2.7** 'Space Curve' (Ron Resch), Vinyl model (David Huffman)

Fig 2.2.9 Lamp designs #171 to 178 by Poul Christiansen, 1969 to early 2000's

2. Historical background
of paperfolding

2.3 Mathematical and computational approaches for curved creases

Fig 2.3.4 'Space curve' renderings (1971-72, Ephraim Cohen, Paper model (1971-72, Ron Resch)

2. Historical background of paperfolding

2.3 Mathematical and computational approaches for curved creases

Developable surfaces
Constructive geometric approach
Differential geometric analysis
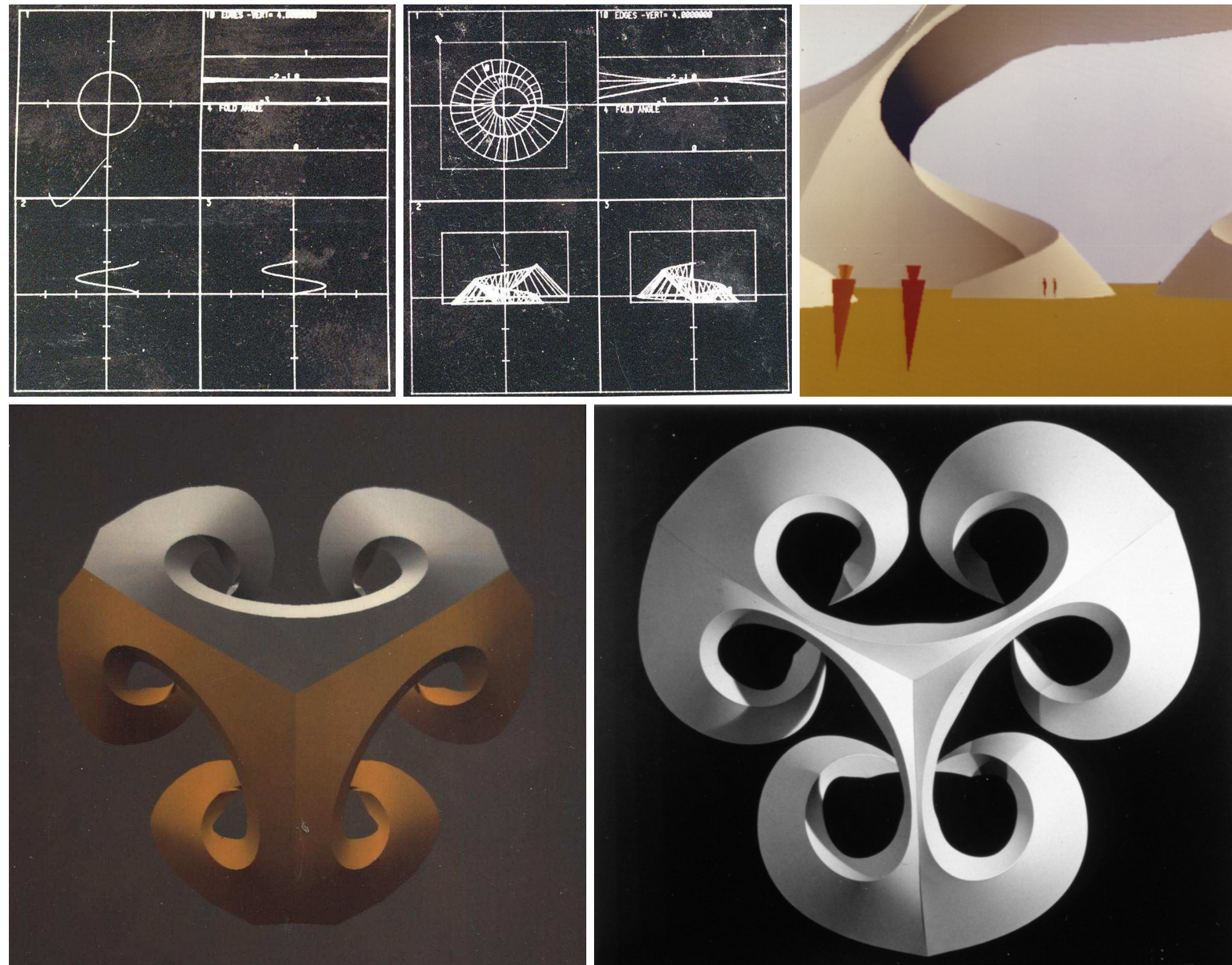Inverse calculation of a crease
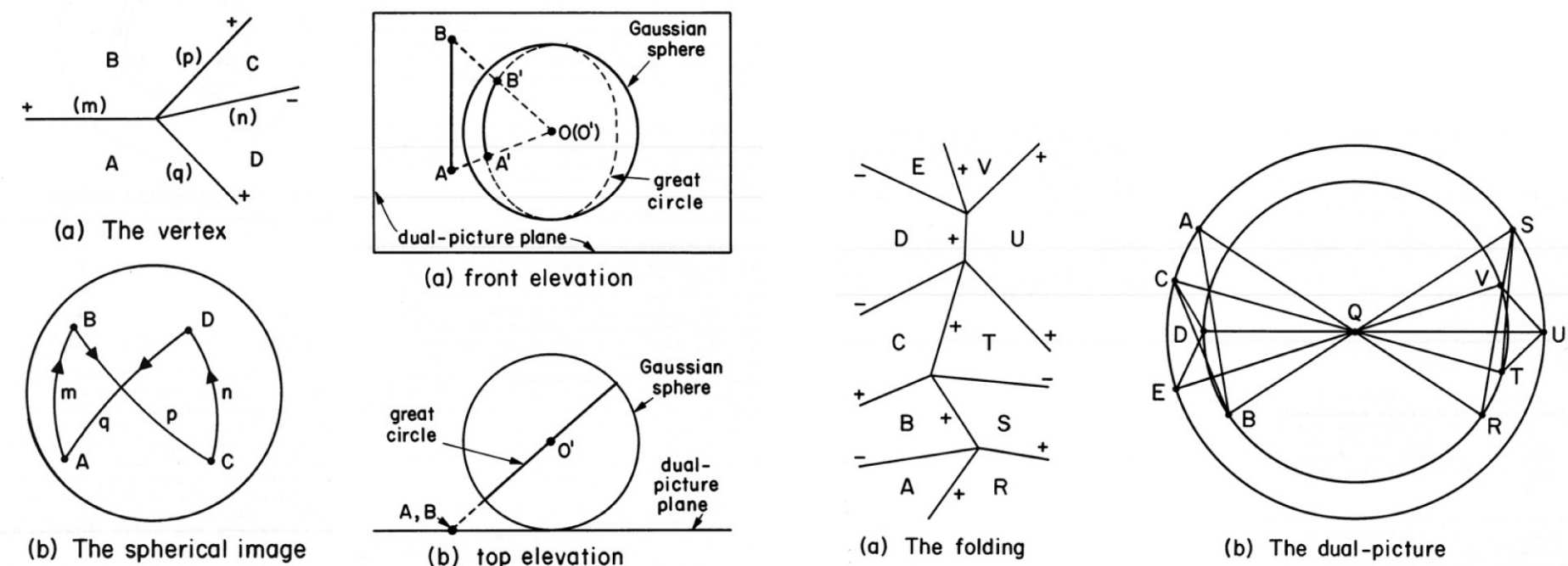Discrete geometric approach

Fig 2.3.5 Mapping of trace on Gauss sphere, crease pattern and corresponding dual (Huffman)
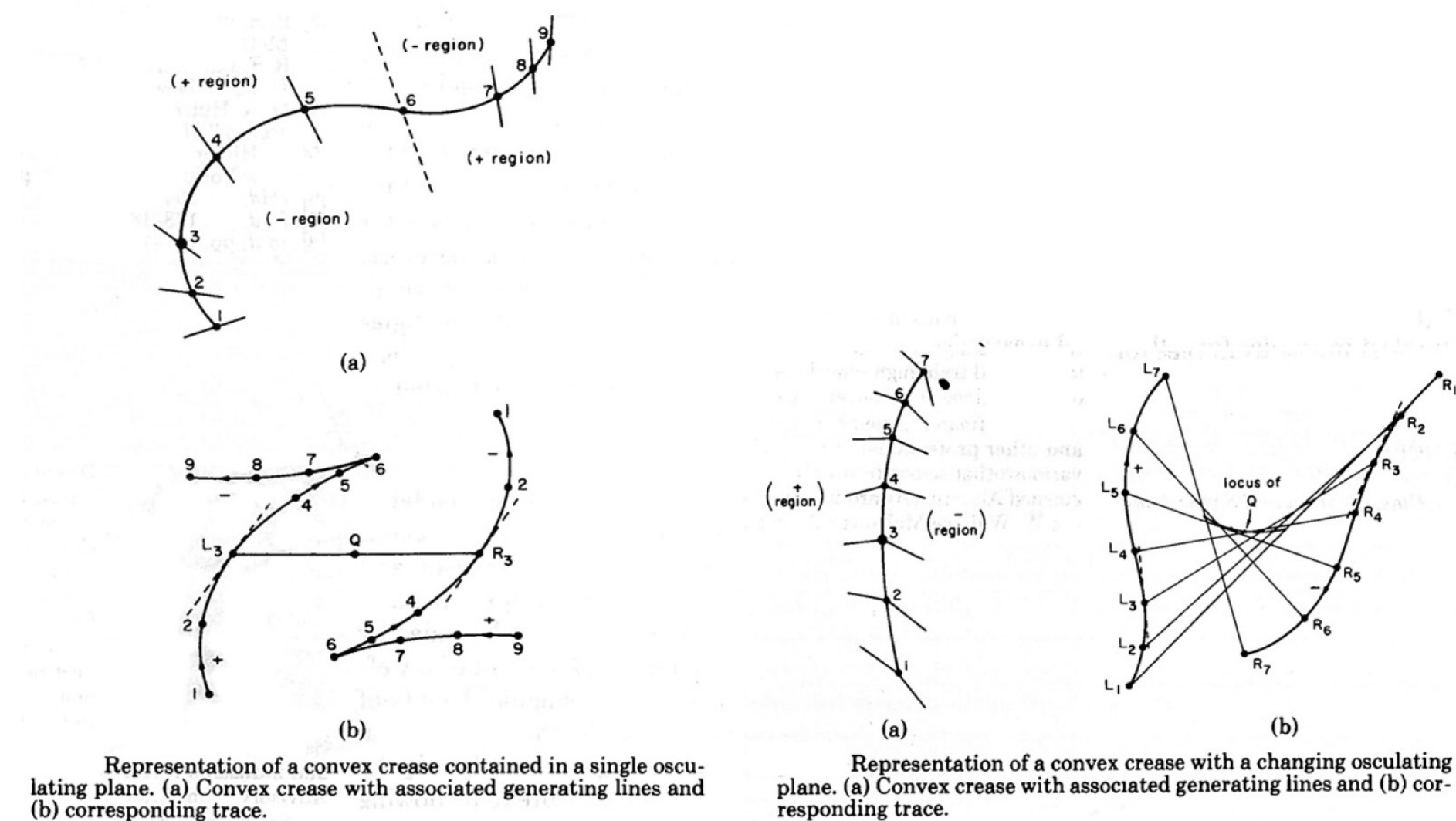

Representation of a convex crease contained in a single osculating plane. (a) Convex crease with associated generating lines and (b) corresponding trace.

Representation of a convex crease with a changing osculating plane. (a) Convex crease with associated generating lines and (b) corresponding trace.

Fig 2.3.6 Crease in plane and dual, General case and dual (Huffman)

3. David Albert Huffman (DAH)

3.1 DAH, a visual biography

Figure 1. Catalog of labelled-line configurations possible at nodes in pictures of trihedral objects.
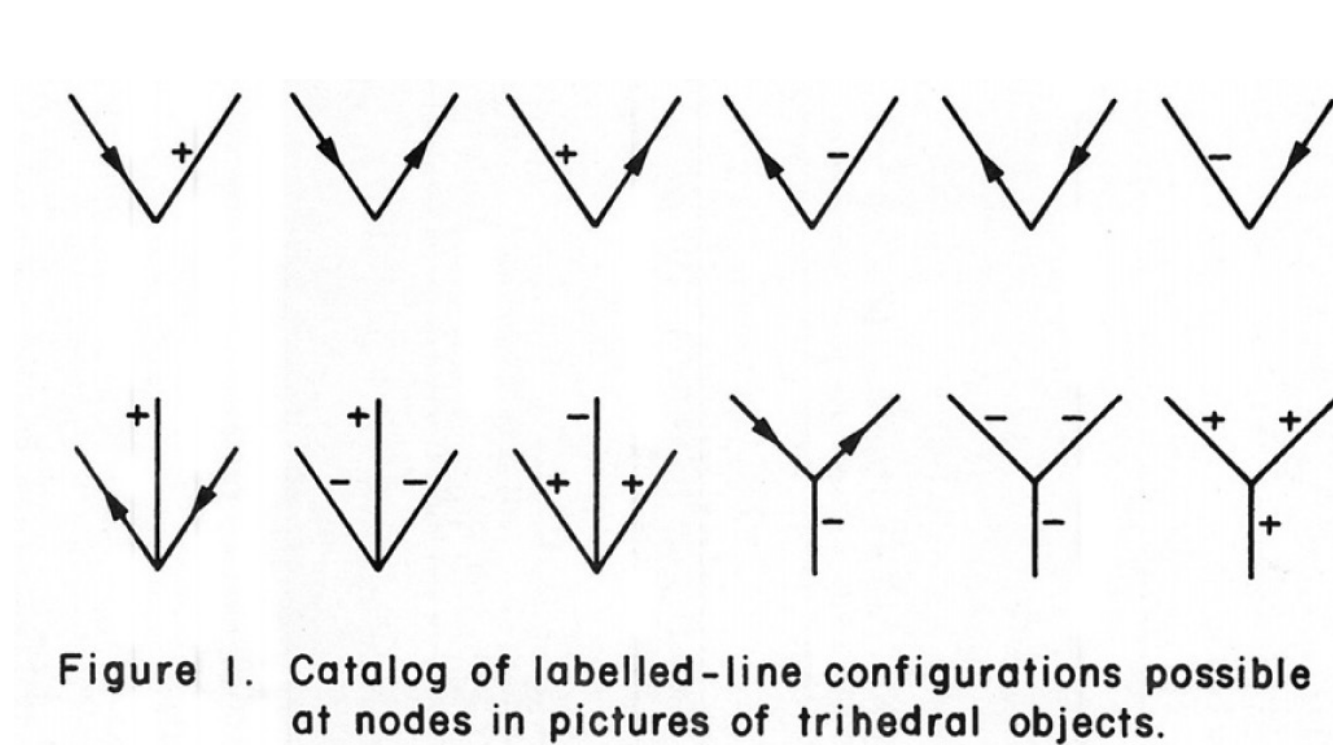
Fig 3.2.1 Huffman's labeling convention for trihedral objects

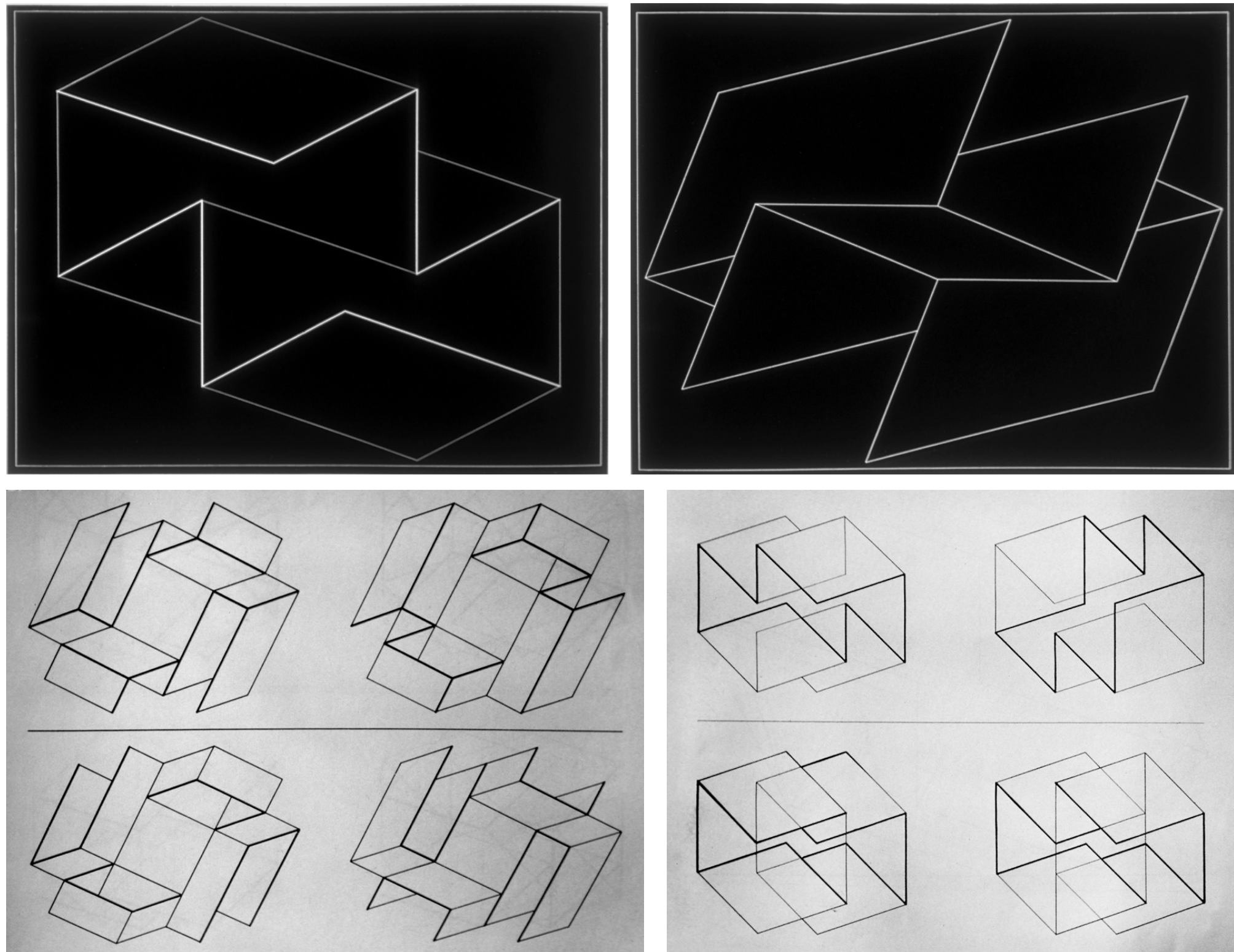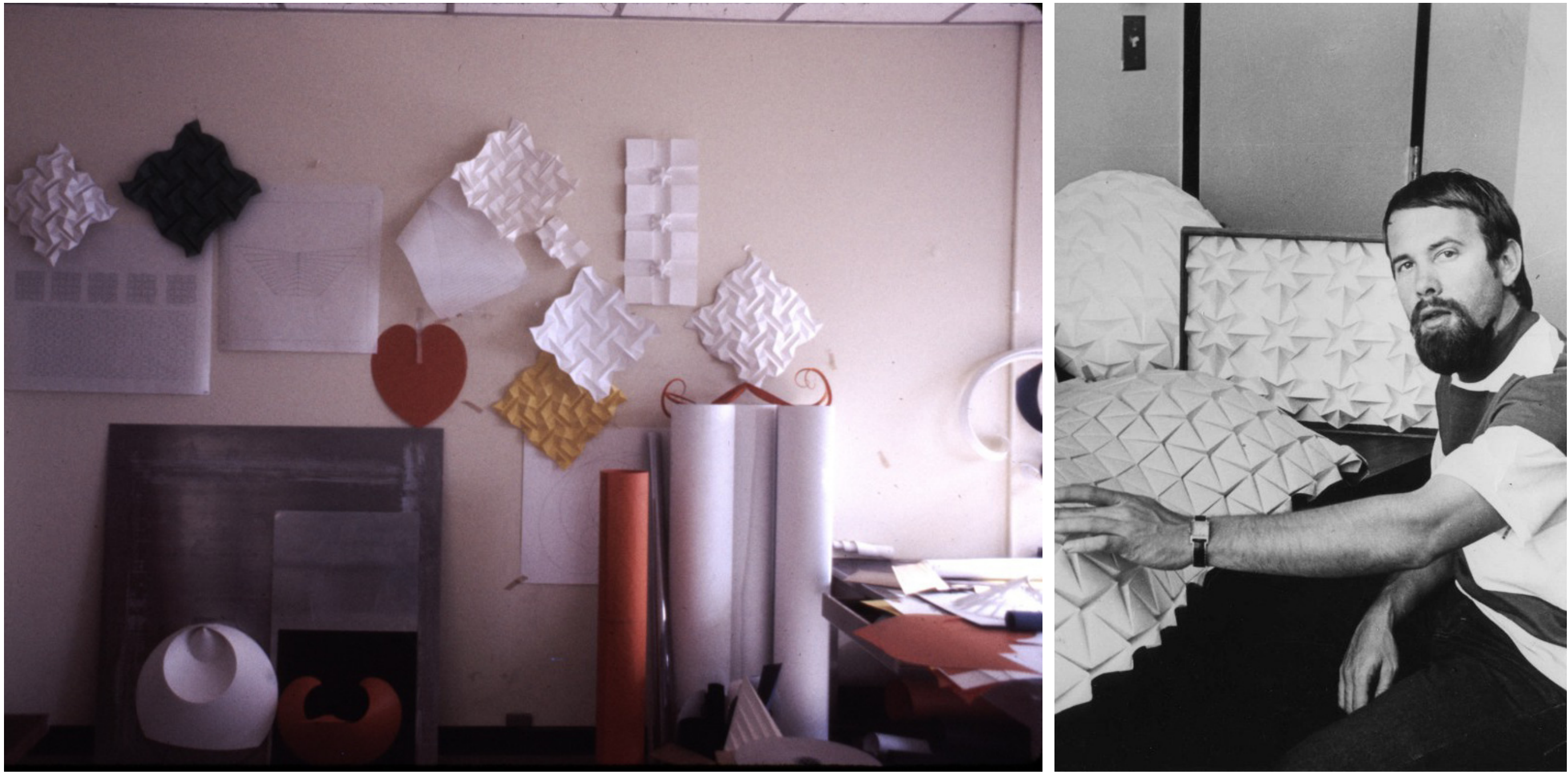**Fig 3.2.2** Lecture slides of impossible objects (1974, DAH [DAH])

3. David Albert Huffman (DAH)

3.1 DAH, a visual biography

Fig 3.2.7 Ron Resch and his SLC Paper Folding Lab, (left Nov. 1972; right, undated)

3. David Albert Huffman (DAH)

3.1 DAH, a visual biography
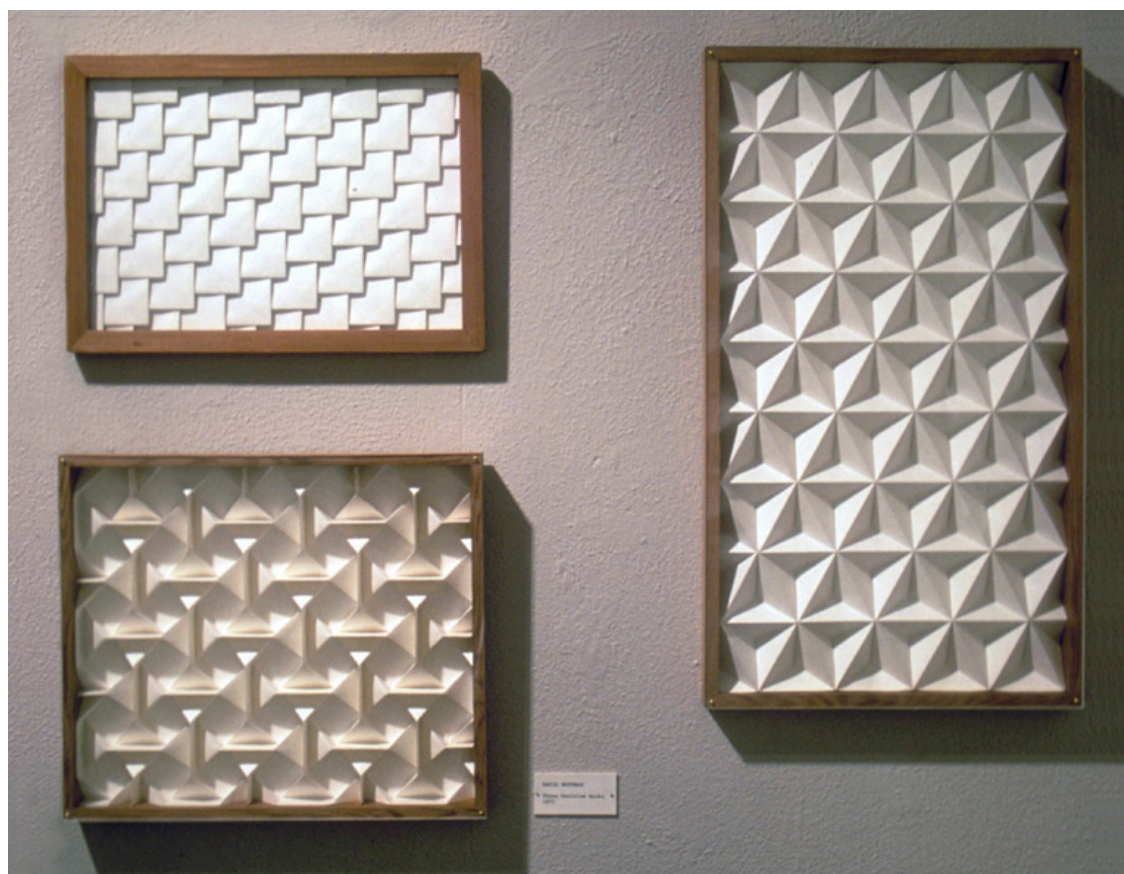
Fig 3.2.8 Exhibition at UCSC in 1978, Huffman's living room (undated, [TG])

3. David Albert Huffman (DAH)

3.1 DAH, a visual biography
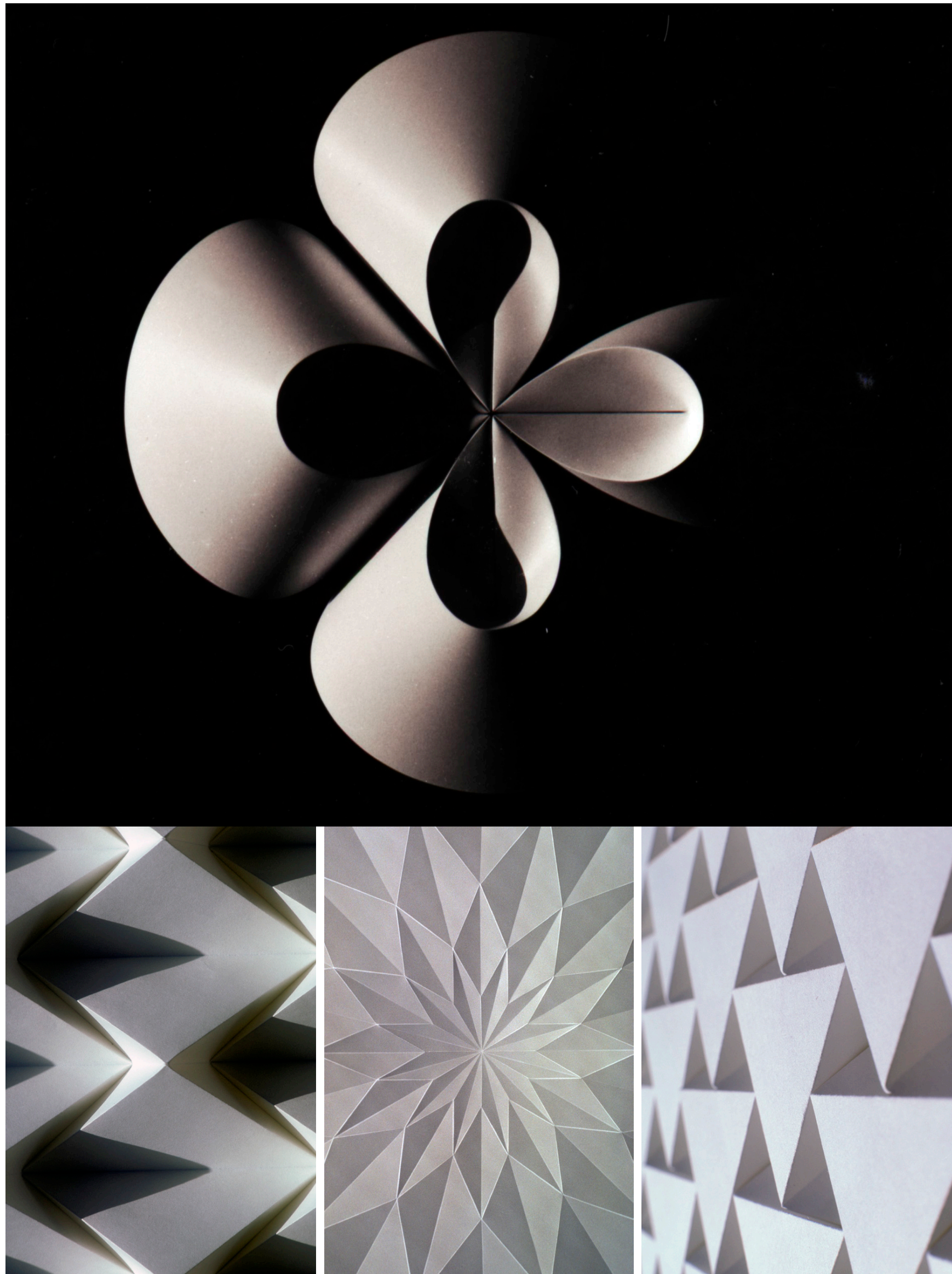
Fig 3.2.11 '4-lobed, cloverleaf design" (probably 1977 DAH [DAH])
Fig 3.2.12 'Detail photographs of straight crease tilings (1978, 1978, 1977, DAH [DAH])

3. David Albert Huffman (DAH)

3.3 DAH, mathematical models as tools
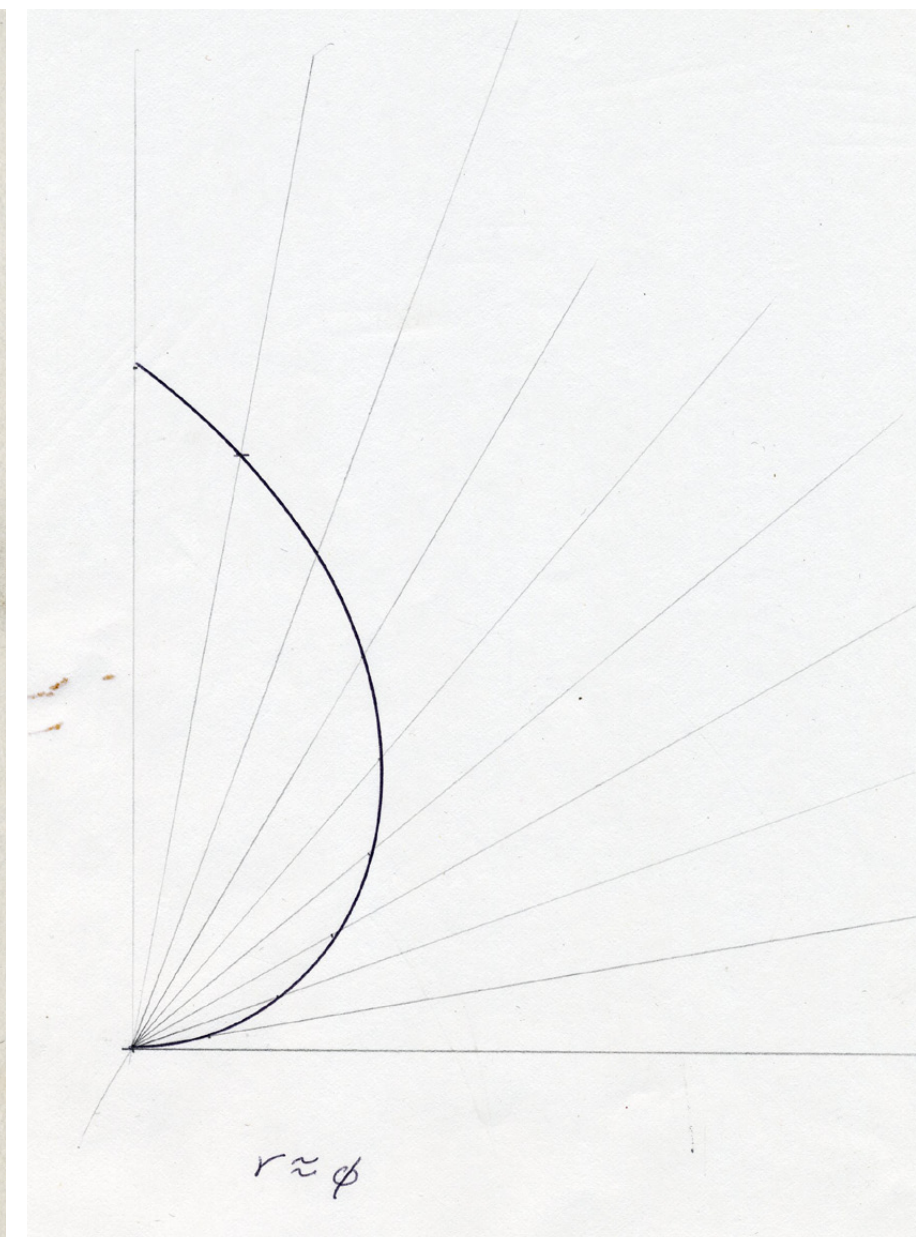
Catalog of curves
Curve Plotting
Discrete curves
Tiling
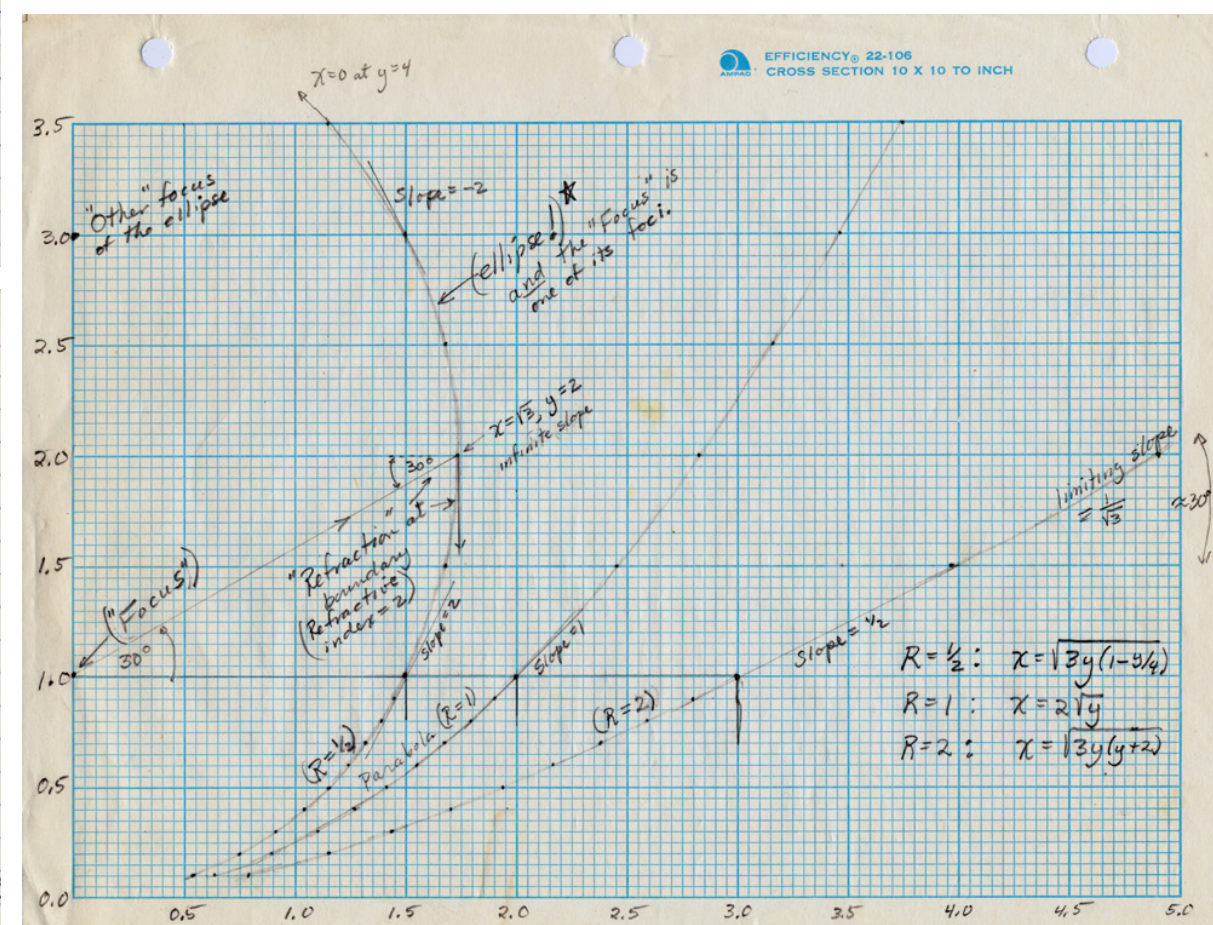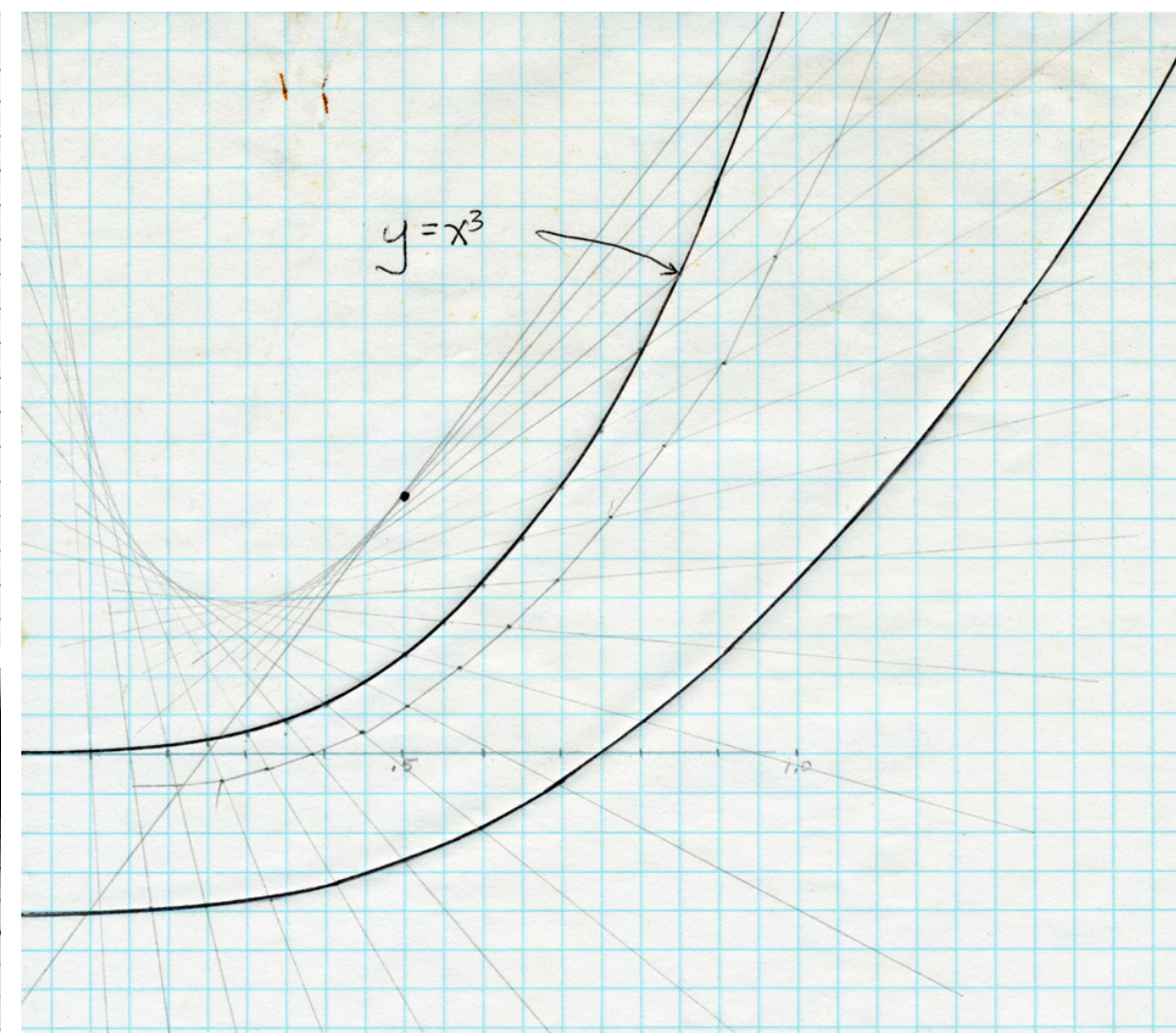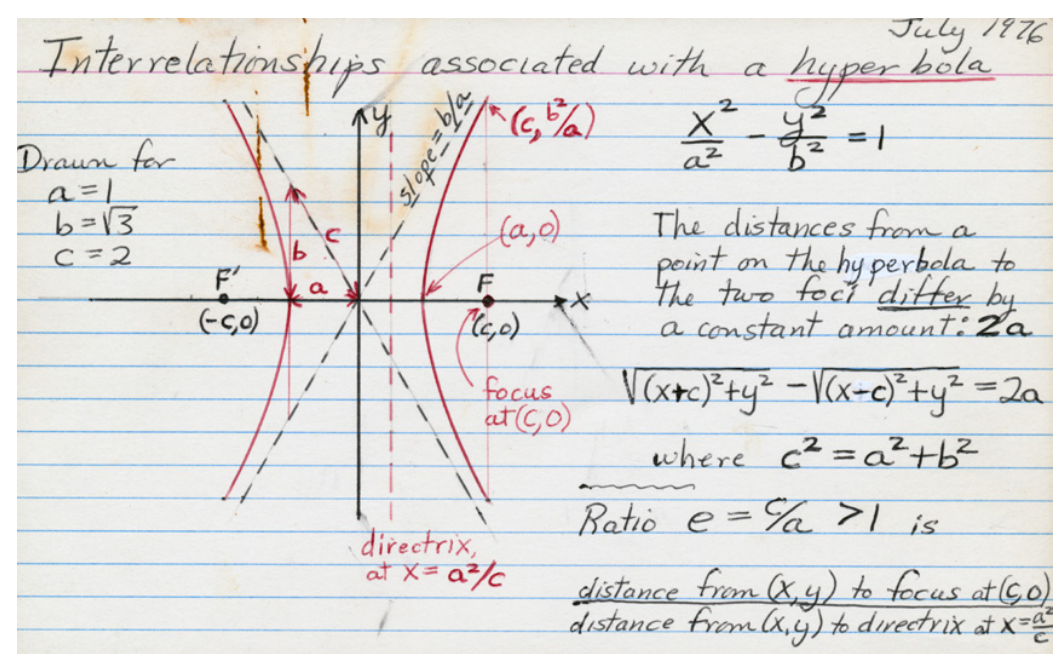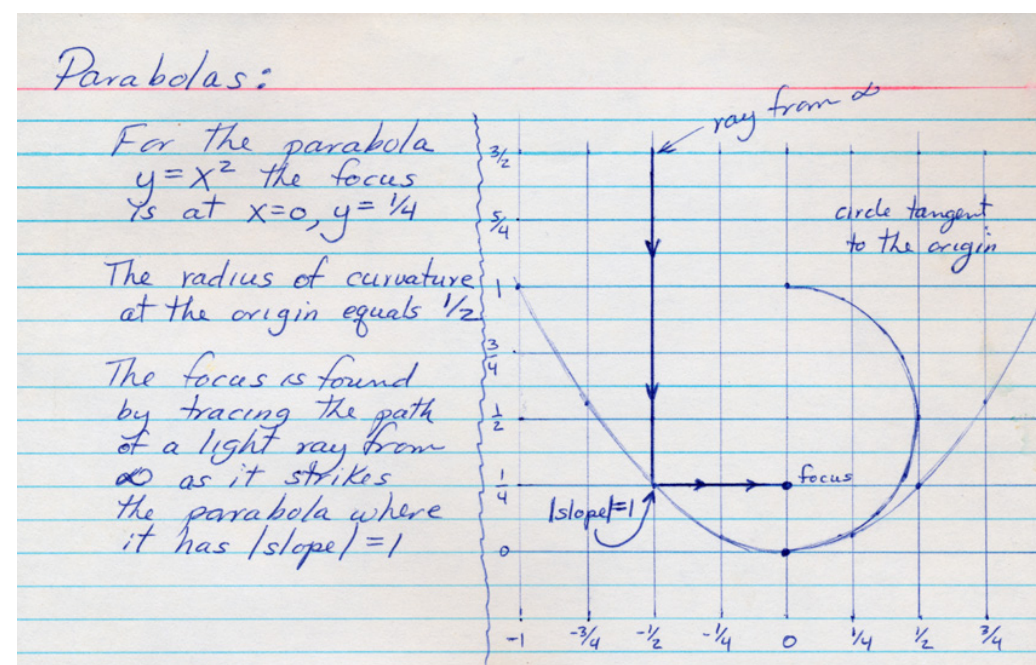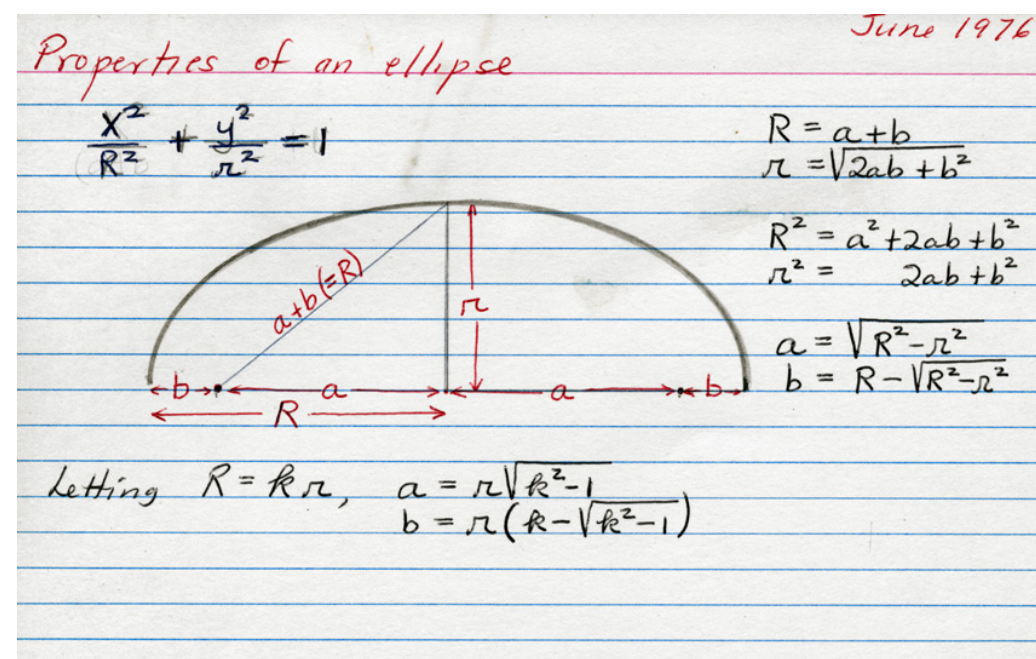
Fig 3.3.1 Conic sections, scaled parabola and a combination of curves (some from 1976, DAH [DK])

3. David Albert Huffman (DAH)
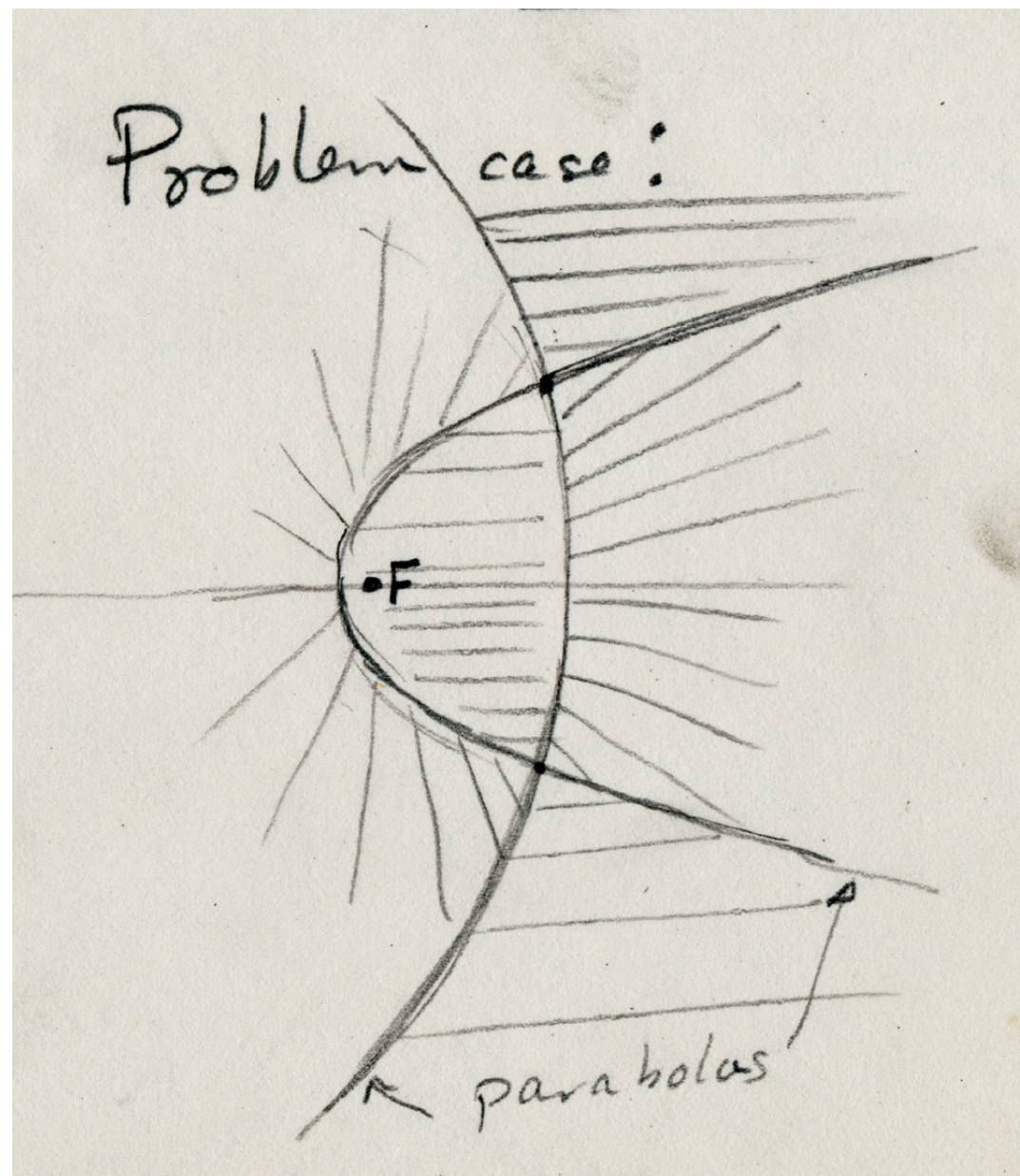
3.3 DAH, mathematical models as tools

Fig 3.3.2 Sketch (undated, DAH [DAH]), Drawing (undated, DAH [DAH])
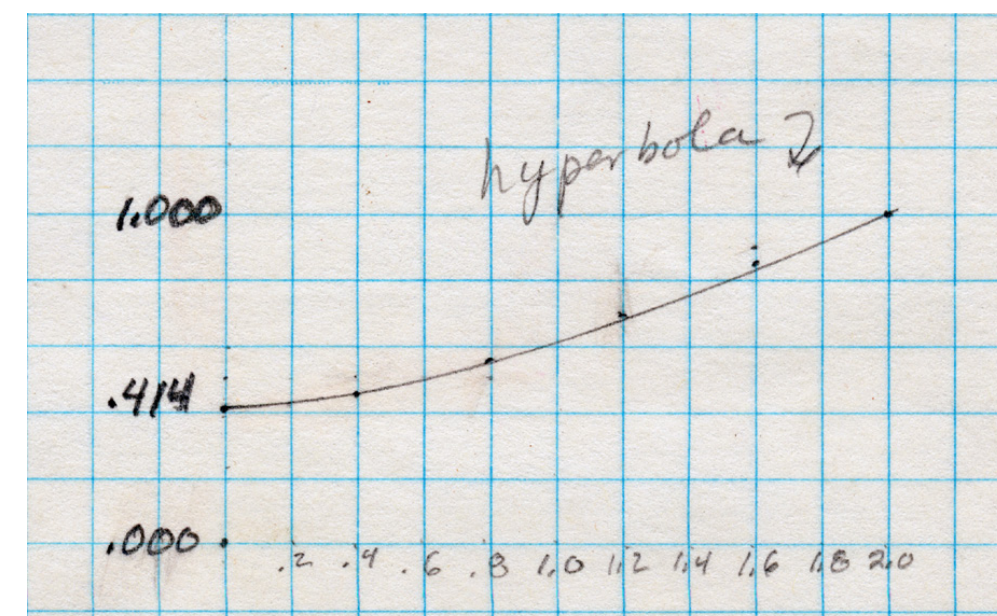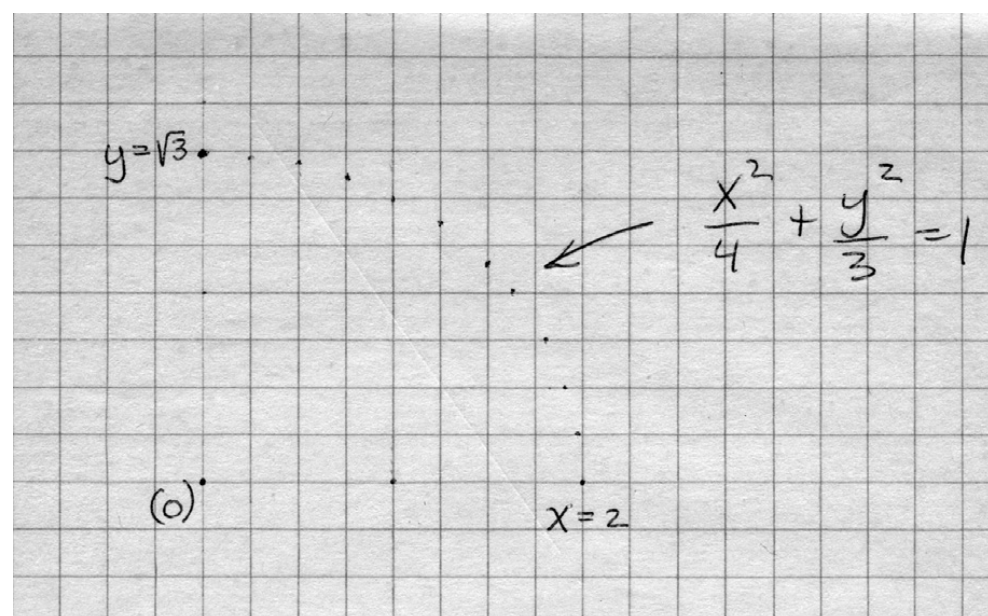
Fig 3.3.3 Sketches of curve plotting on graph paper

3. David Albert Huffman (DAH)

3.4 DAH and his tool drawer
The drafting table
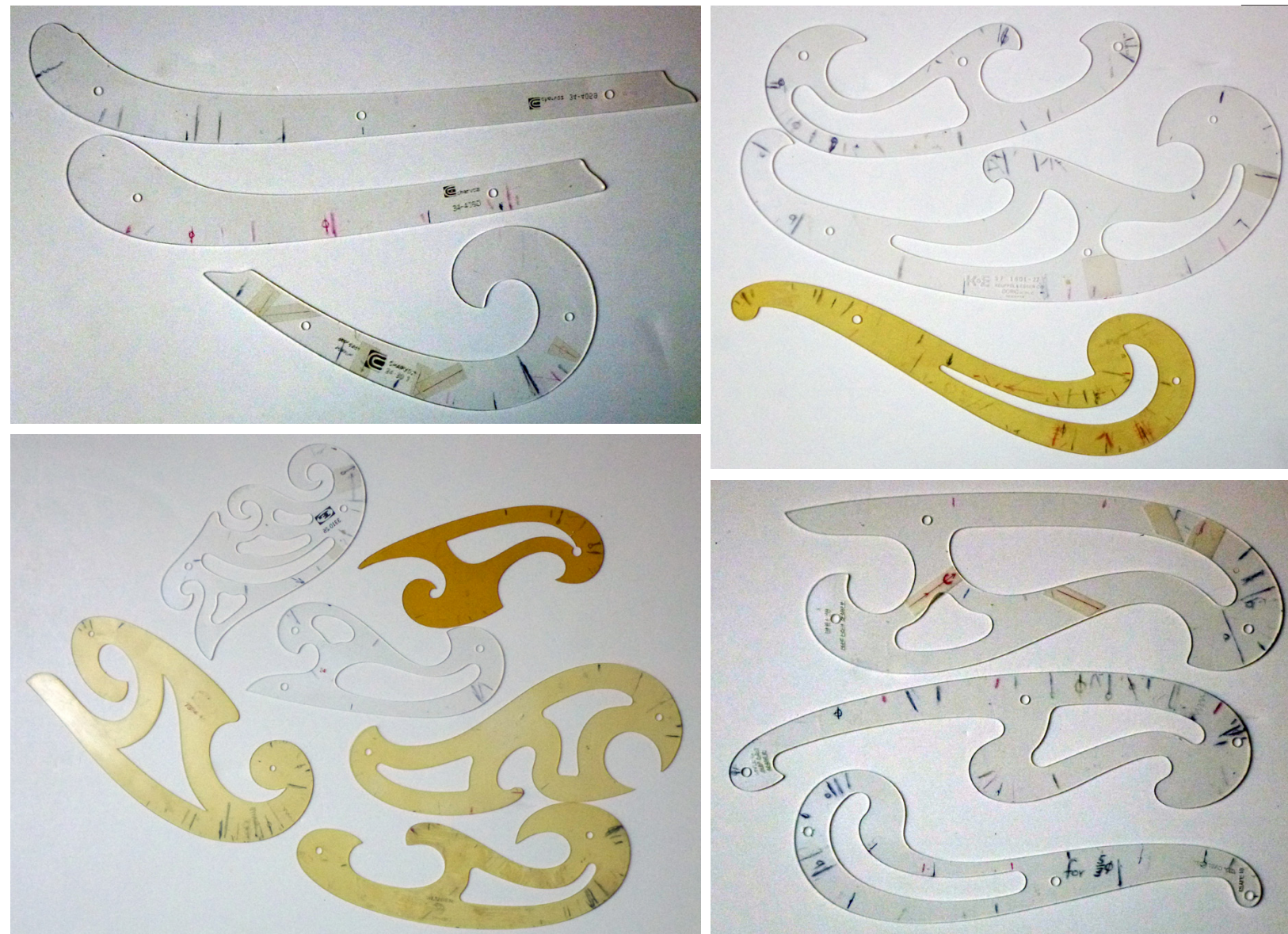French curves and custom templates
The ball burnisher
Folding aids

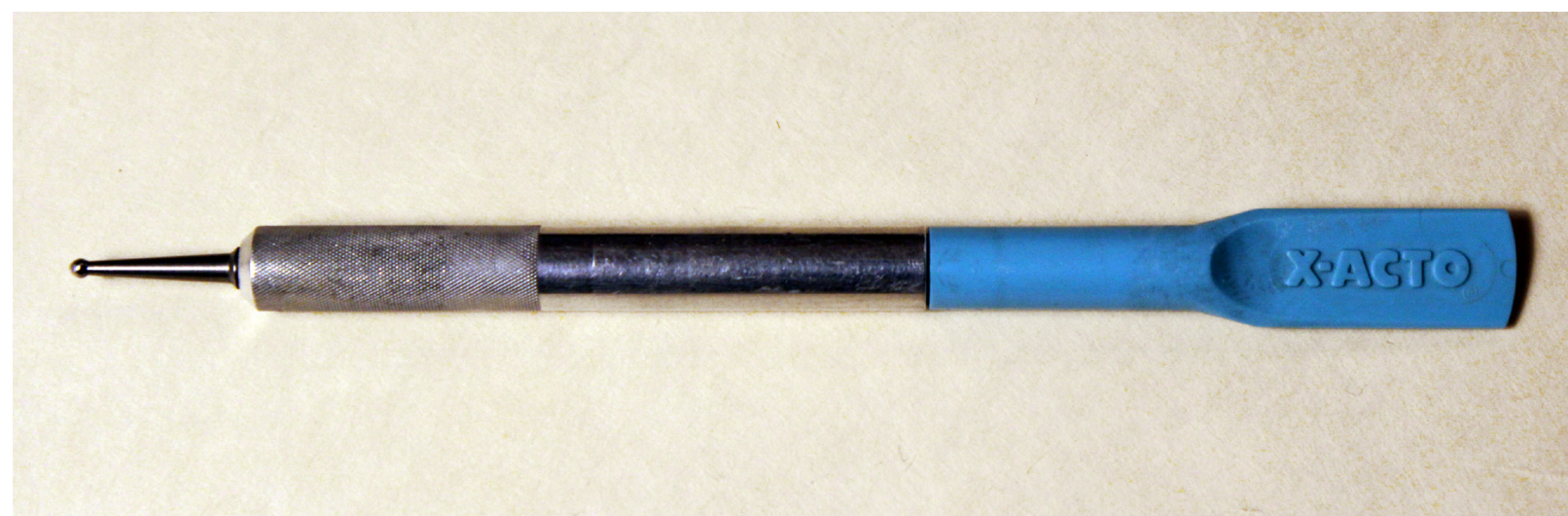**Fig 3.4.3** French curves sets with Huffman's markings [EAH]

Fig 3.4.5 X-Acto Letraset Craft Burnisher [DK]

Fig 3.4.6 Folding aids made of recycled margarine containers [EAH], Compass set and knives [EAH]

4. A taxonomy of DAH's curved crease paperfolding work

Archival material
The taxonomy
Definitions of crease pattern, ruling and gadget
Individual designs
The categories of the taxonomy
The tile of a crease pattern

4.1 Cylinder reflection

Cylinder reflection along sine curve or along parabola
Cylinder reflection along sine curve and tucking
Discrete cylinder reflection

4.2 Cone reflection

Cone reflection between two planes
Cone reflection with rotating axis
Cone reflection parallel to axis
Cone reflection and tucking
Cone reflection of general and partial cones

4.3 Gadgets with ellipses

Single ellipse
Gadgets with ellipses and tucking

4.4 Gadgets with parabolas

Gadgets with parabolas and pleating
Gadgets with parabolas and line segments
Gadgets with parabolas and line segments with a smooth transition
Gadgets with parabolas, line segments and circles
Gadgets with parabolas and line segments with inverted smooth transition
Gadgets with parabolic splines
Gadgets with parabolic splines with inclined axis
Gadgets with parabolic splines and circles
Gadgets with pleated parabolic splines
Gadgets with parabolic splines and line segments
Gadgets with parabolic splines and line segments for Donald Knuth
Gadgets with mixed splines and line segments

4.5 Gadgets with hyperbolas

4.6 Gadgets with parabolas and ellipses

Gadgets with parabolas, ellipses and line segments

4.7 Gadgets with parabolas, circles and ellipses

Gadgets with parabolas and circles
Gadgets with parabolas and ellipses

4.8 Gadgets with ellipses and hyperbolas

Gadgets with ellipses, hyperbolas and line segments
Gadgets with elliptic and hyperbolic splines

4.9 Gadgets with ellipses, parabolas and hyperbolas

4.10 Cone and cylinder gadget

4.11 Cyclic tilings with converging curves

Tilings with central vertex
Tilings with central gap or line segment
Tilings with central polygon

4.12 Tilings with loxodromic spirals

4.13 Sinks and vortexes

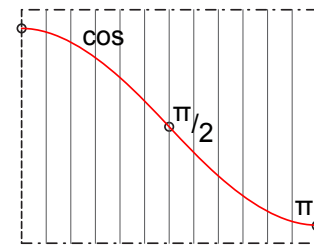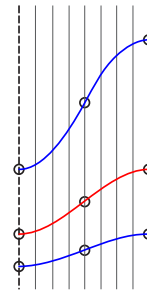# [I Reflection]

I Cylinder reflection

cos

π/2

π

Fig 4.1.4

Fig 4.1.6

I Cone reflection

A

Fig 4.2.3

c

A

Fig 4.2.16

A

Fig 4.2.47

I Cylinder reflection and tucking

sin

Fig 4.1.16

sin

Fig 4.1.21

sin
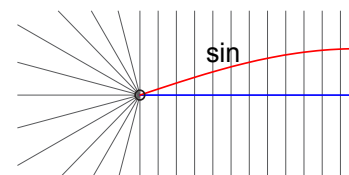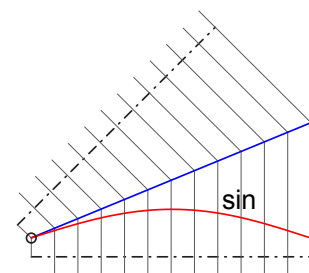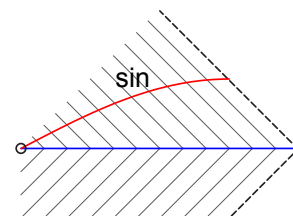
Fig 4.1.26

I Cone reflection and tucking

c
c
c
c

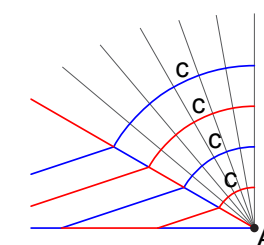A

Fig 4.2.51
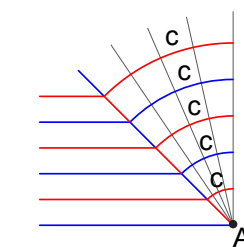
c
c
c
c

A

Fig 4.2.53

c
c
c
c
c

A
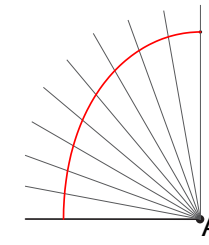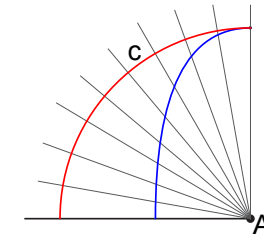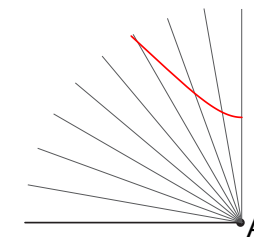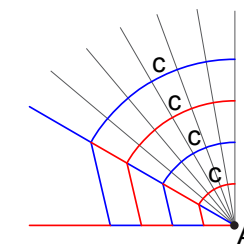
Fig 4.2.55

Fig 4.4 I Reflection

[II Refraction Gadgets]

II Gadgets with ellipses
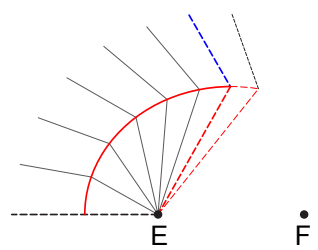
Fig 4.3.2 left
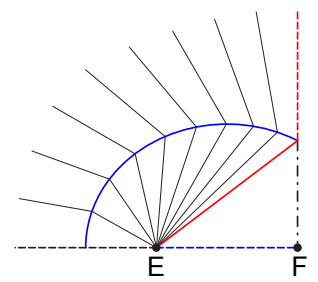
Fig 4.3.2 right
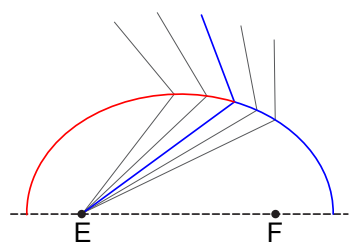
Fig 4.3.3

III Gadgets with parabolas
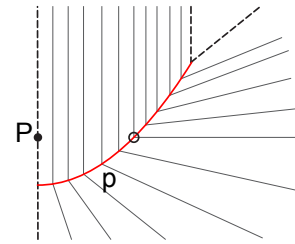
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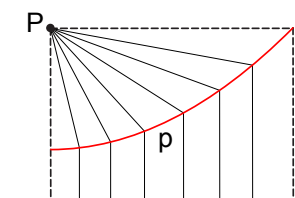
Fig 4.4.2 right
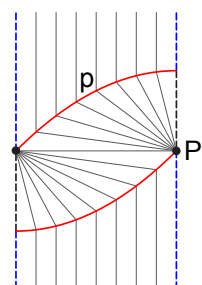
Fig 4.4.28
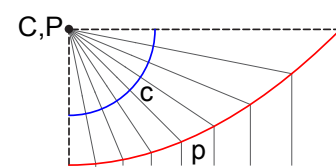
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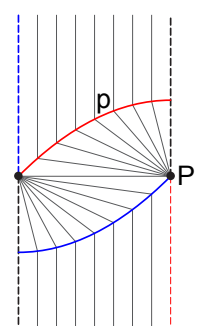
Fig 4.4.54

Gadgets with quadratic splines
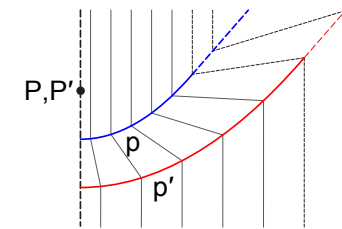
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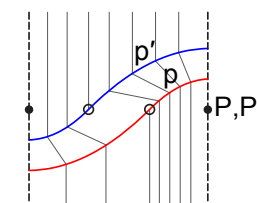
Fig 4.4.68
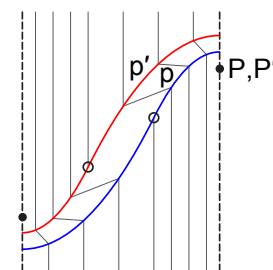
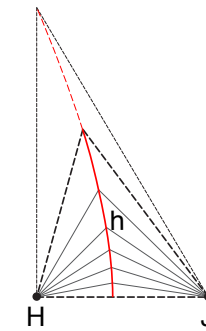Fig 4.4.87
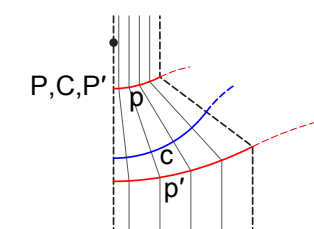
Fig 4.4.102
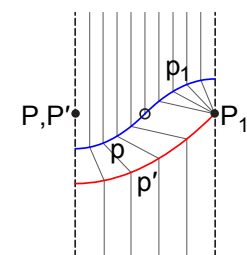
Fig 4.4.115
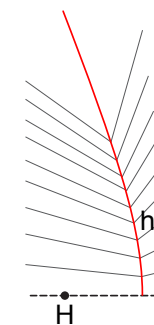
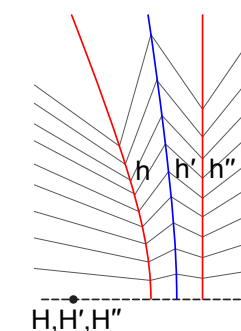**Gadgets with hyperbolas**
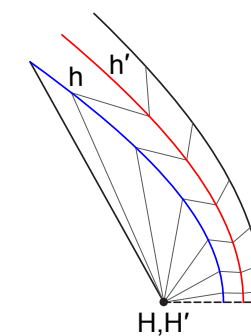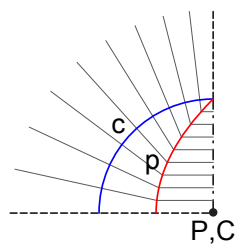

Fig 4.5.2 left

Fig 4.5.2 right

Fig 4.5.3

Fig 4.5.6

Fig 4.5 II Refraction gadgets

Fig 4.6 II Refraction gadgets

[II Refraction Gadgets], [III Forced Rulings], [IV Converging Curves]
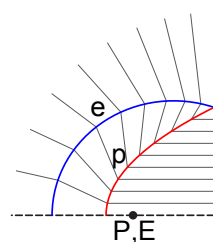
Gadgets with combined curve types
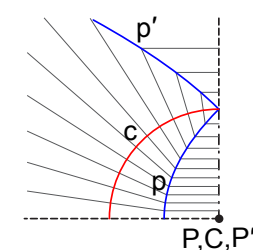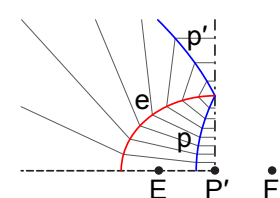

Fig 4.6.3 left

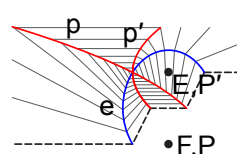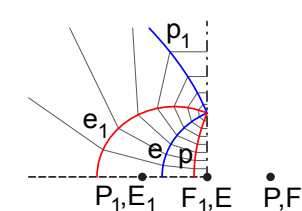Fig 4.6.3 right

Fig 4.7.2

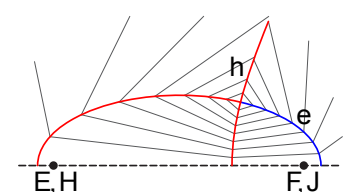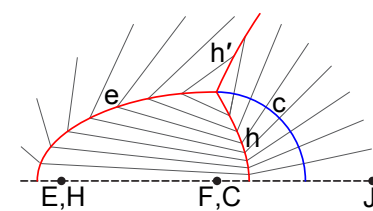Fig 4.7.15
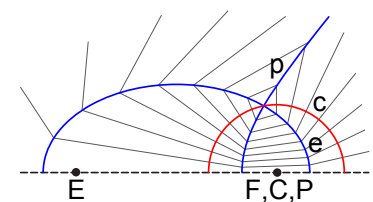
Fig 4.7.20

Fig 4.7.22

Gadgets with combined curve types
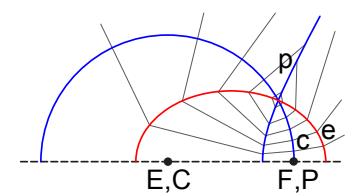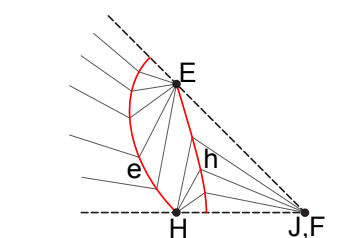
Fig 4.8.2

Fig 4.8.4

Fig 4.8.8

Fig 4.8.17

Fig 4.8.18
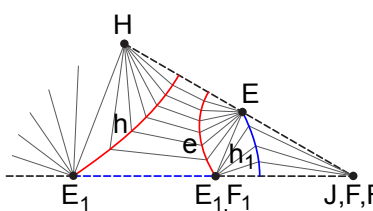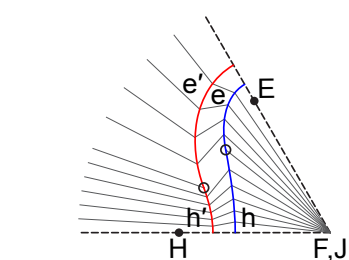
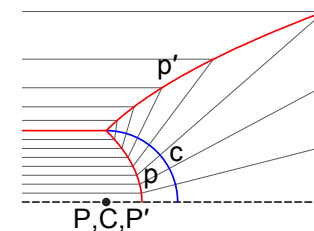Fig 4.8.24

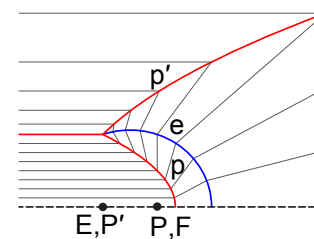Fig 4.8.26

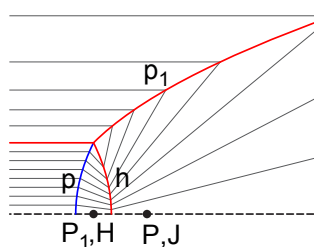**II Gadgets with combined curve types**
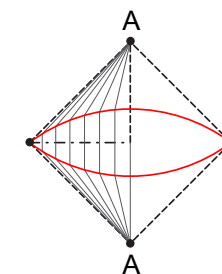
Fig 4.9.2

Fig 4.9.4

Fig 4.9.8

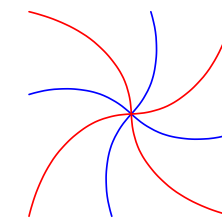**III Forced Rulings**

Fig 4.10.3
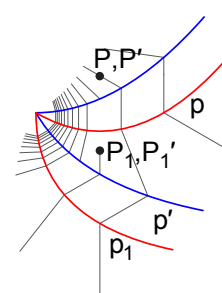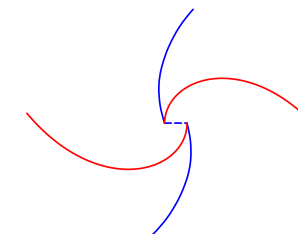
IV Converging curves
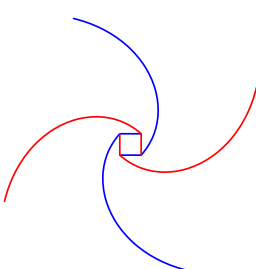
Fig 4.11.1

Fig 4.11.8

(Fig 4.11.16) Fig 4.11.19

Fig 4.11.26
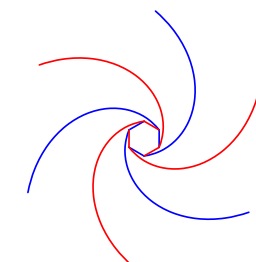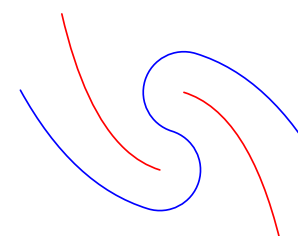
Fig 4.11.28 (Fig 4.11.30)

Fig 4.13.11

Fig 4.7 II Refraction gadgets

Fig 4.8 II Refraction gadgets, III Forced Rulings, IV Converging Curves

4. A taxonomy of DAH's curved crease paperfolding work

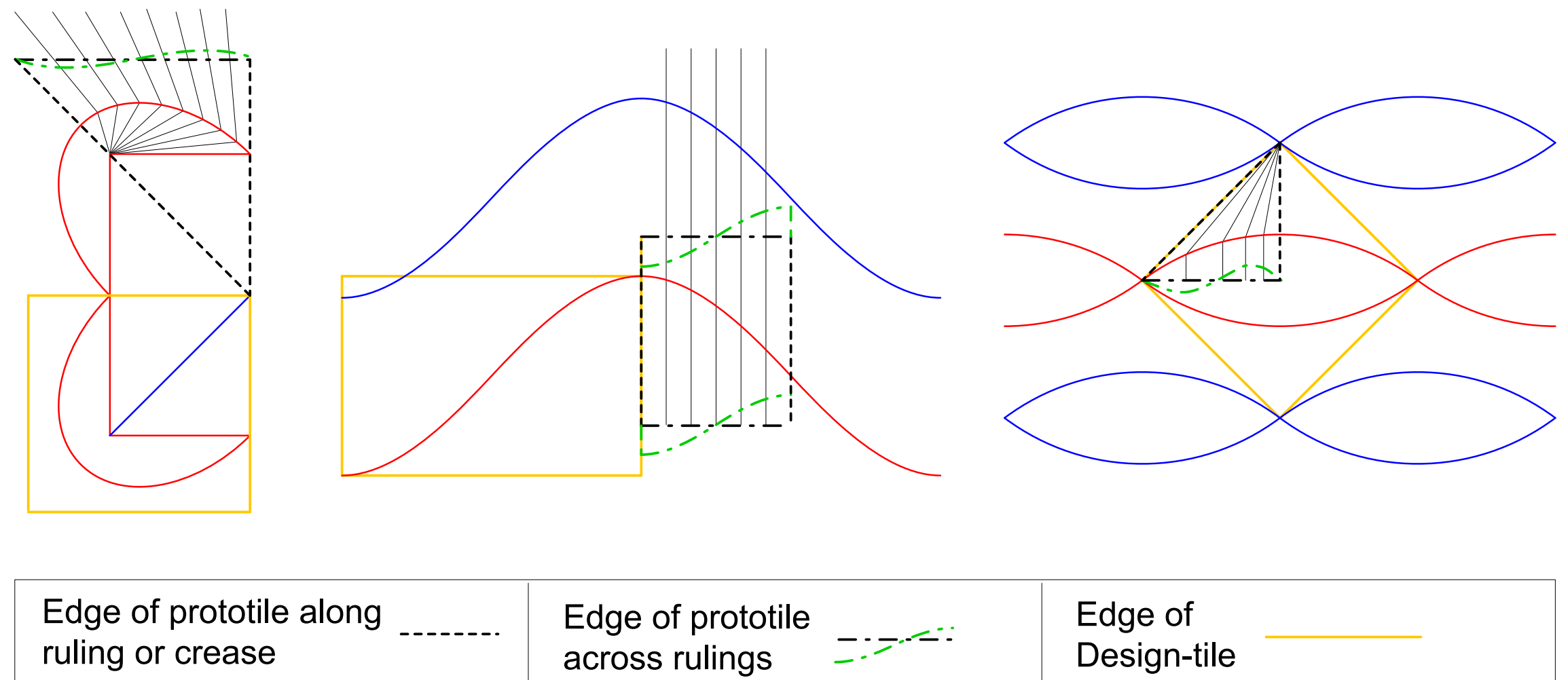| Edge of prototile along ruling or crease | - - - - - - - | Edge of prototile across rulings | - · - · - · | Edge of Design-tile | —— |

Fig 4.9 Prototiles and design tiles [DK]

4. A taxonomy of DAH's curved crease paperfolding work
[I Reflection]

4.1 Cylinder reflection

Cyl. reflection along sine curve or parabola
Cyl. reflection along sine curve and tucking
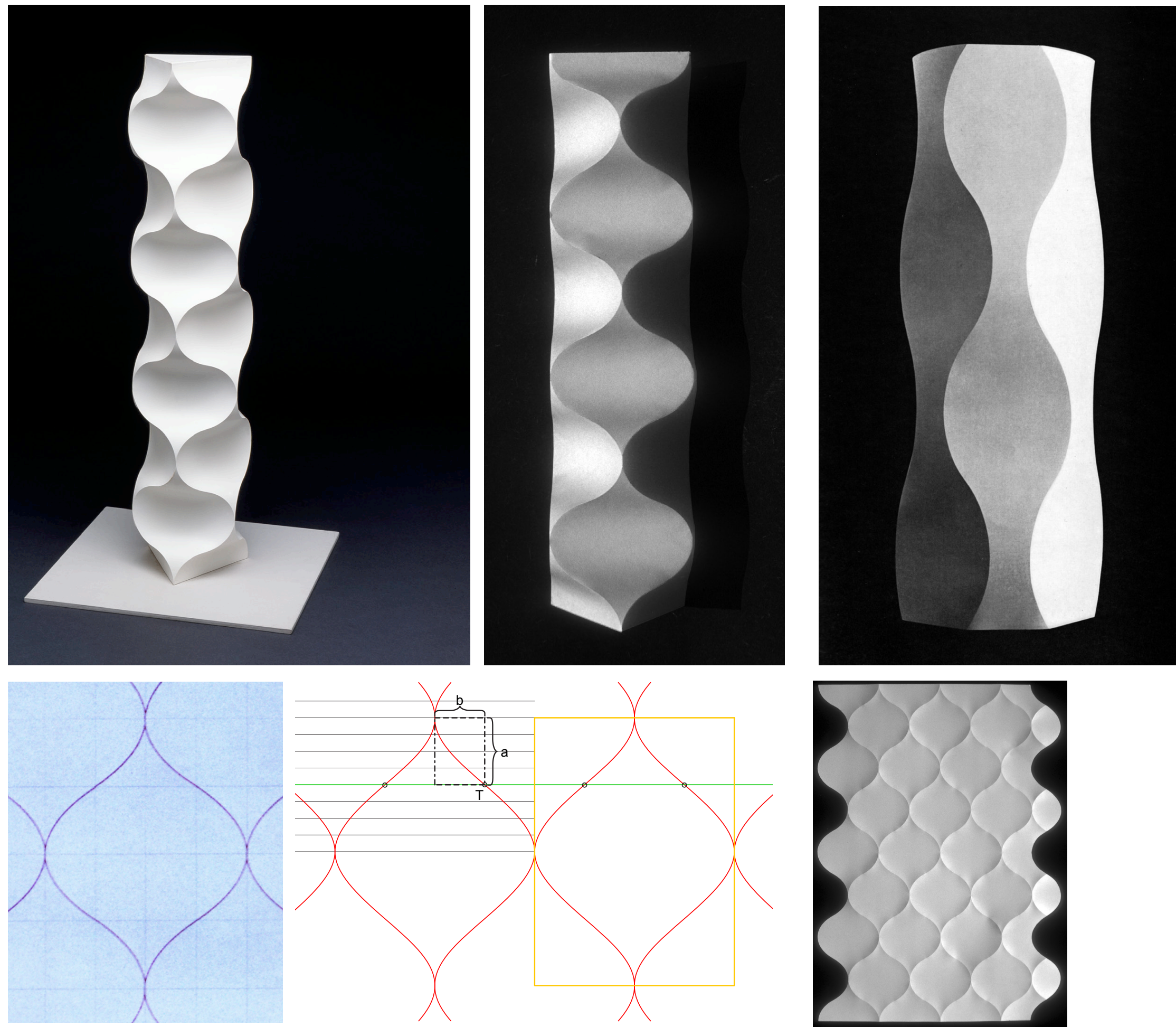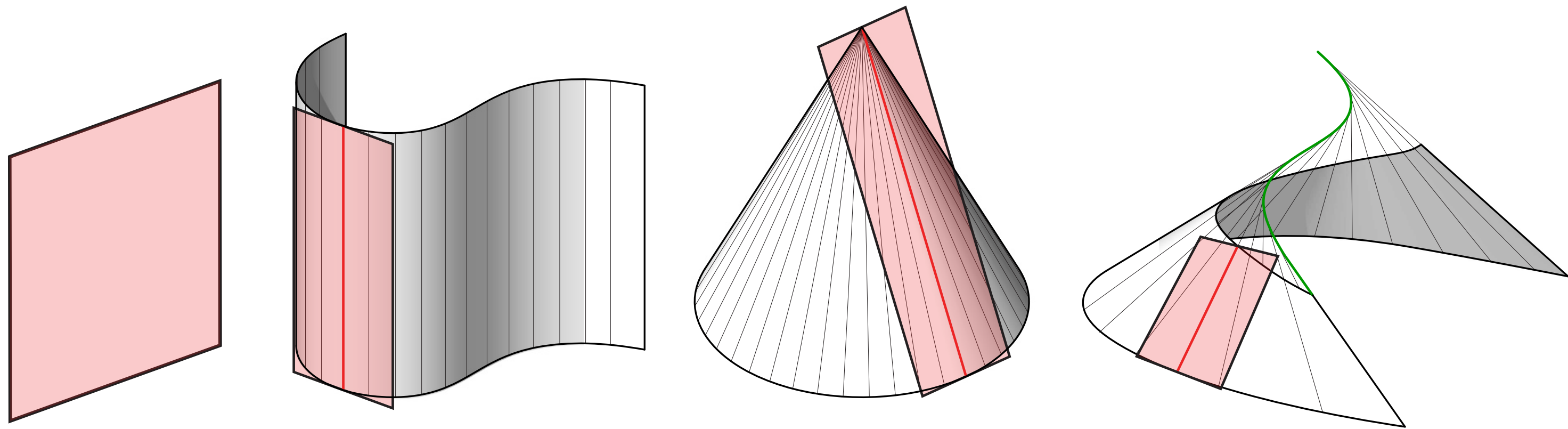Discrete cylinder reflection

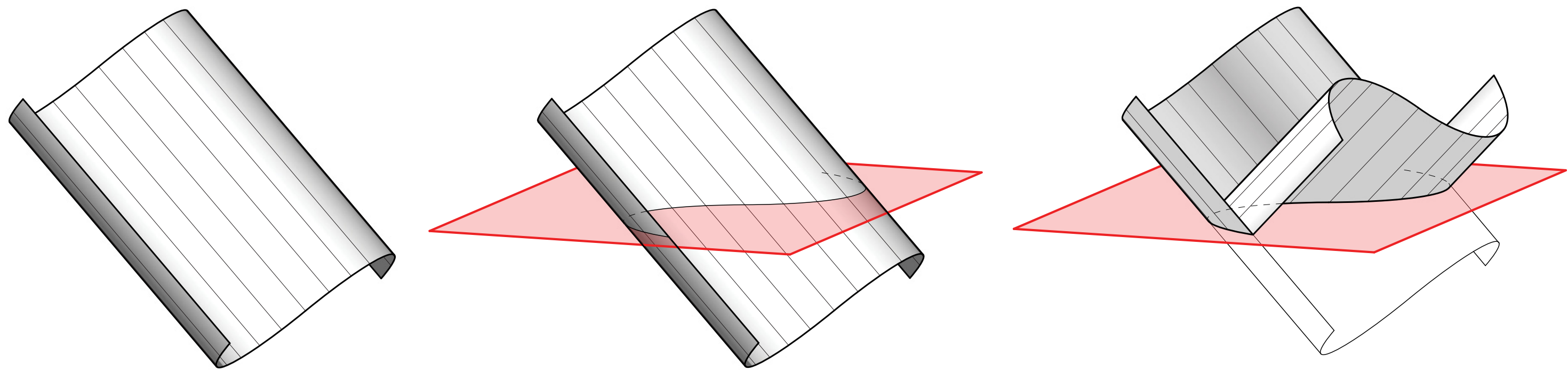**Fig 4.1.10** 'Horizontally fluted column' (1977, DAH [TG]), Paper model
Fig 4.1.12 Model by Ogawa, Vinyl model in flat state (1977, DAH [DAH])
Fig 4.1.11 Vinyl model (1977, DAH [DAH]), Crease pattern [DK]
Fig 4.1.1 Cylinder reflection through a plane

Developable surface: plane, general cylinder, general cone, tangent surface

Reflection of a general cylinder through a plane

4. A taxonomy of DAH's curved crease paperfolding work
[I Reflection]

4.2 Cone reflection
Cone reflection between two planes
Cone reflection with rotating axis
Cone reflection parallel to axis
Cone reflection and tucking
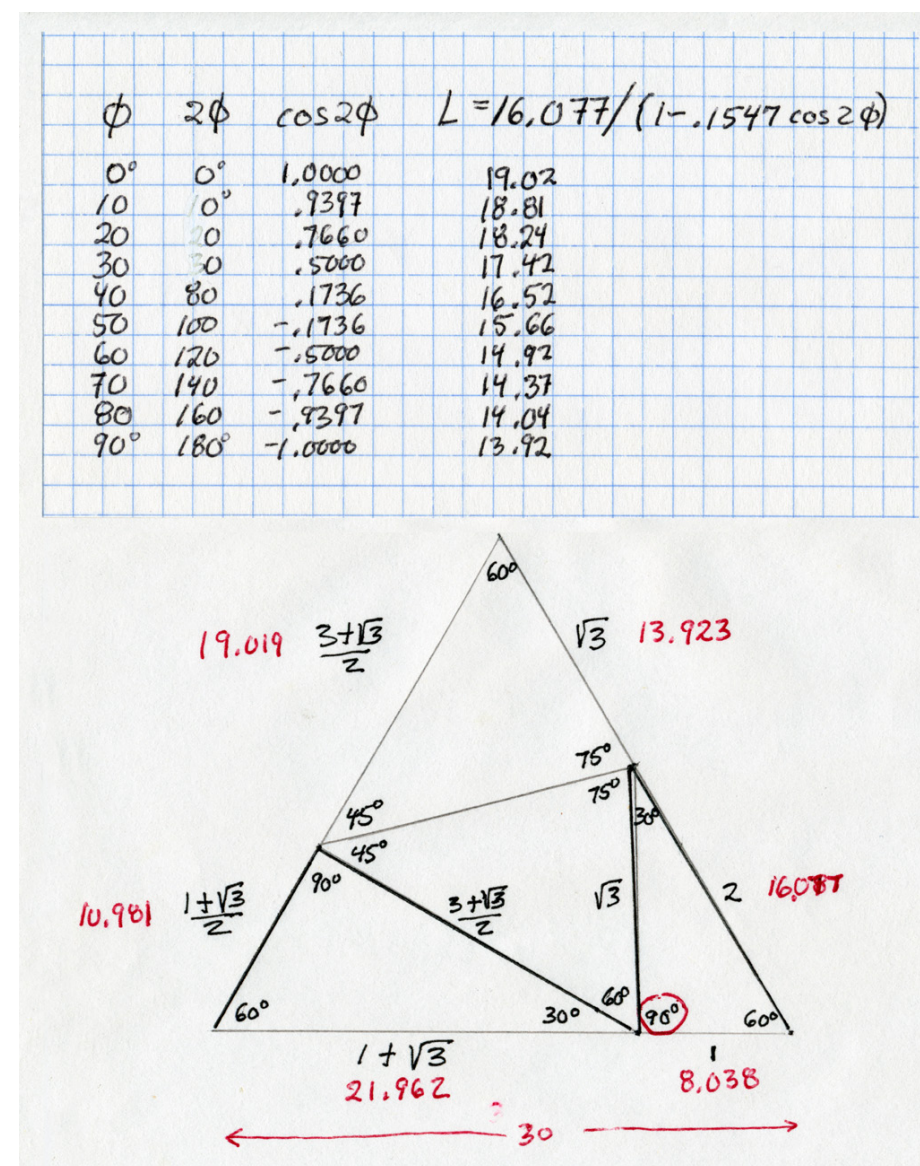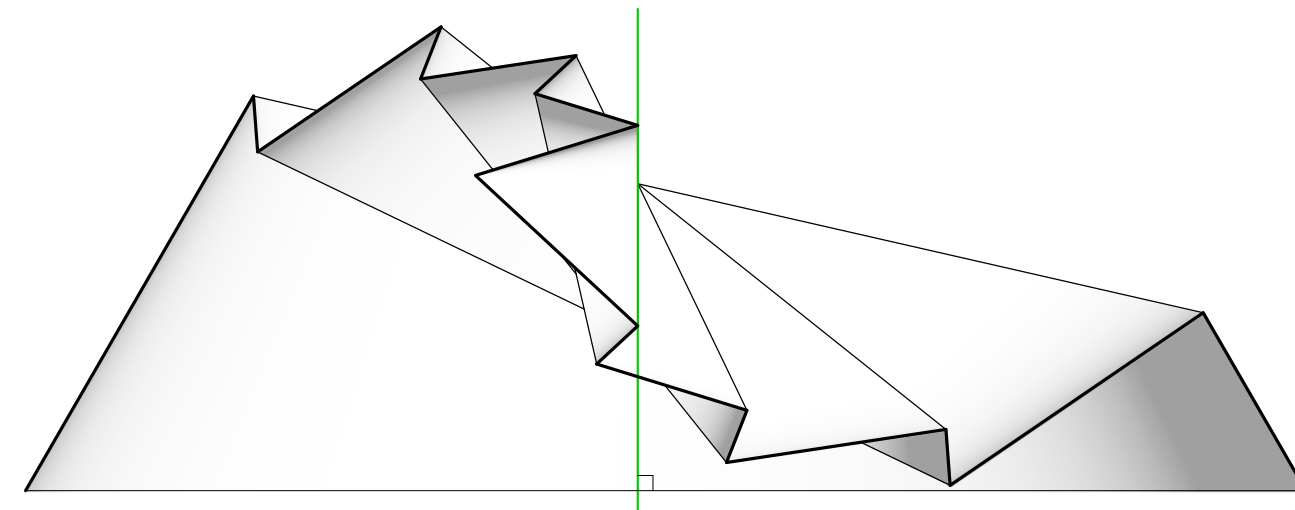Cone reflection of general and partial cones

$$L = 16.077/(1 - .1547\cos 2\phi)$$

ϕ	2ϕ	$\cos 2\phi$	L
0°	0°	1.0000	19.02
10	0°	.9397	18.81
20	0	.7660	18.24
30	0	.5000	17.42
40	80	.1736	16.52
50	100	-.1736	15.66
60	120	-.5000	14.92
70	140	-.7660	14.37
80	160	-.9397	14.04
90°	180°	-1.0000	13.92

Fig 4.2.1 Section and polar coordinates (undated, DAH
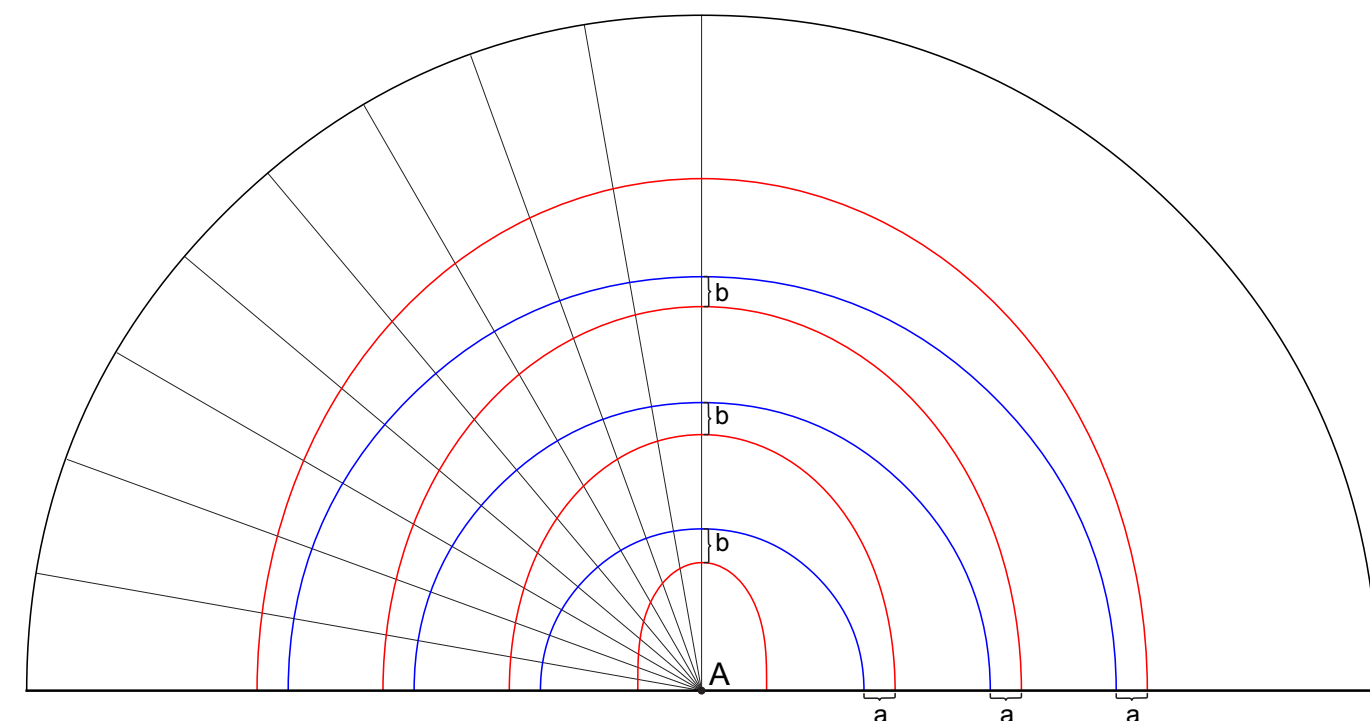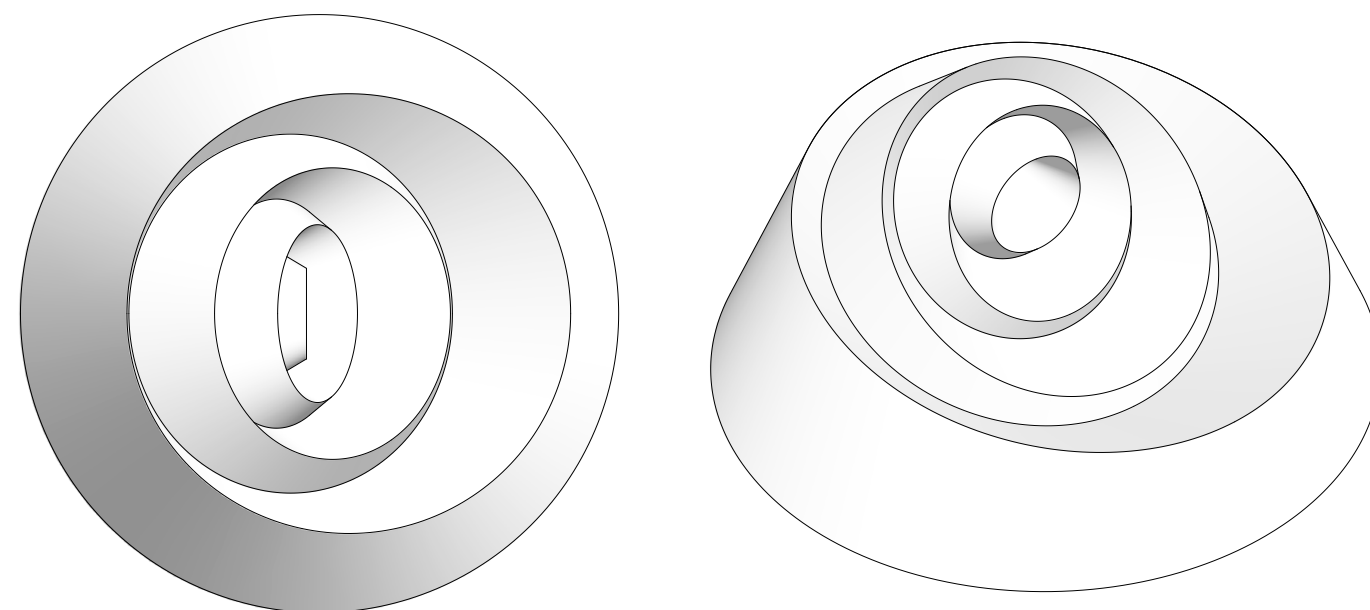
Fig 4.2.42 Folded section [UP]
Fig 4.2.43 3d model [UP]
Fig 4.2.45 Drawing (1978, DAH [DAH])
Fig 4.2.44 Crease pattern [DK]

4. A taxonomy of DAH's curved crease paperfolding work

[II Refraction Gadgets]

4.3 Gadgets with ellipses
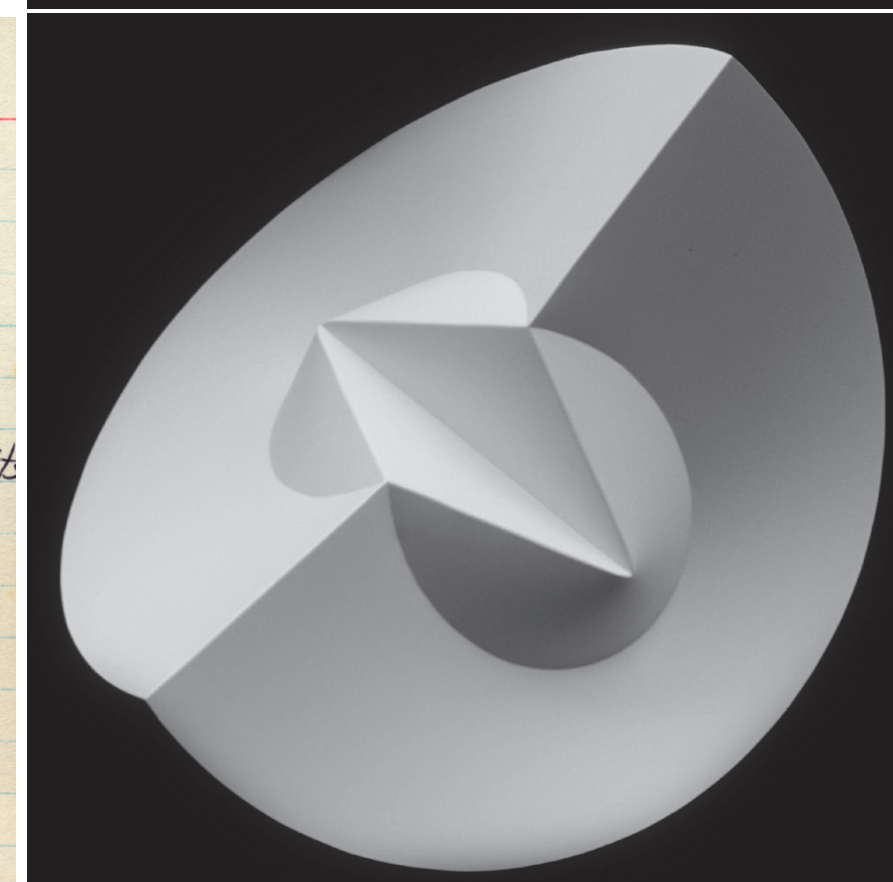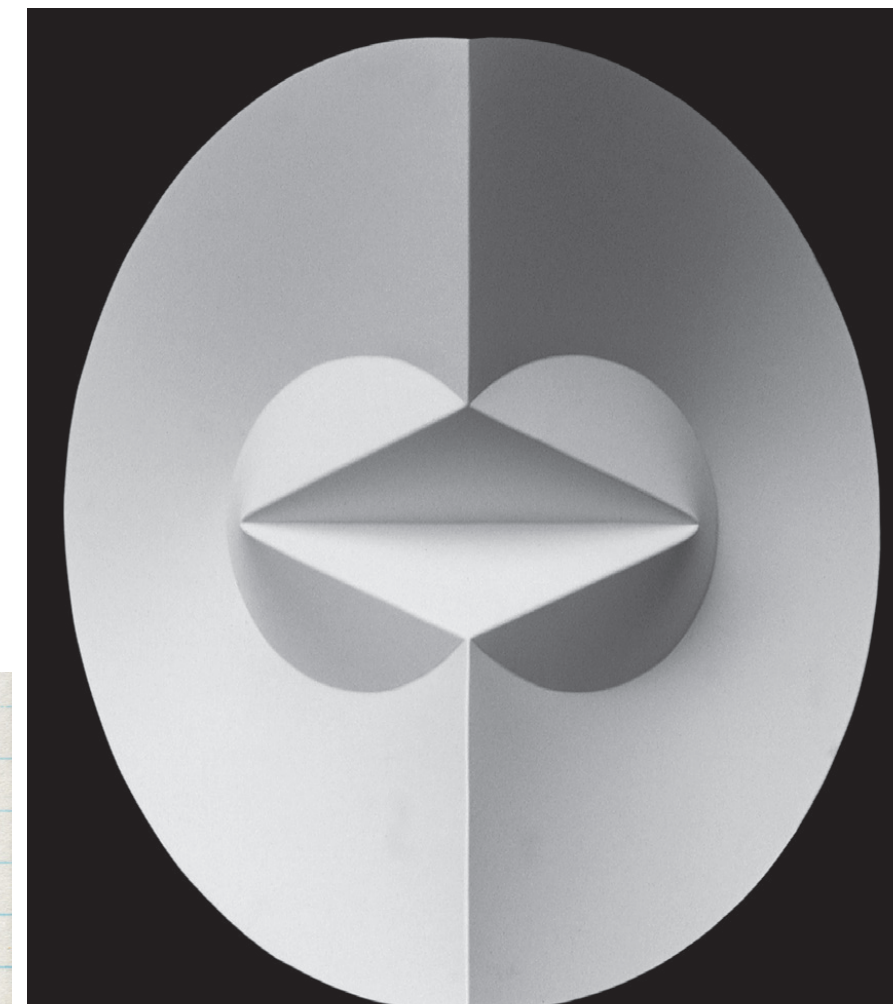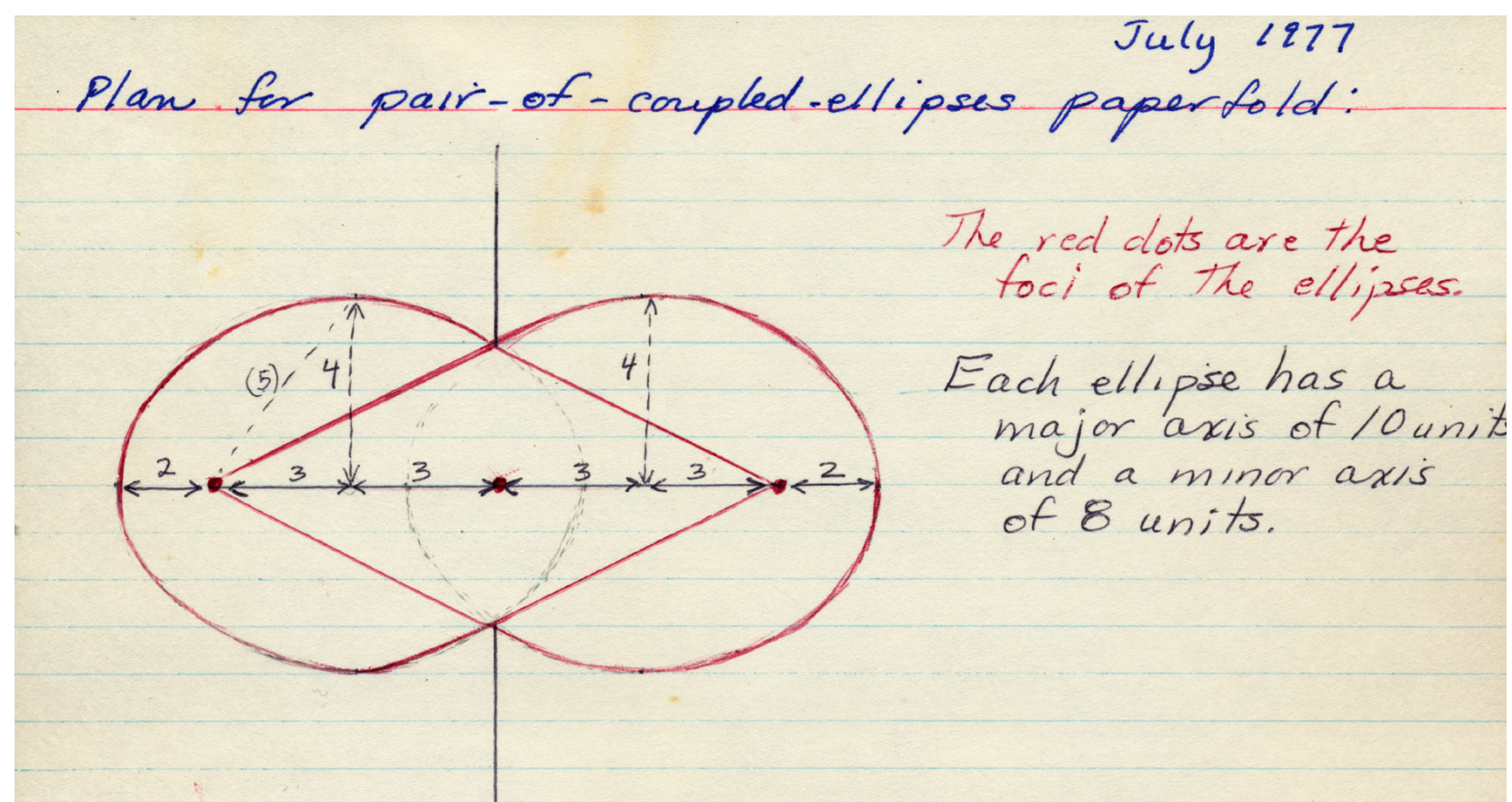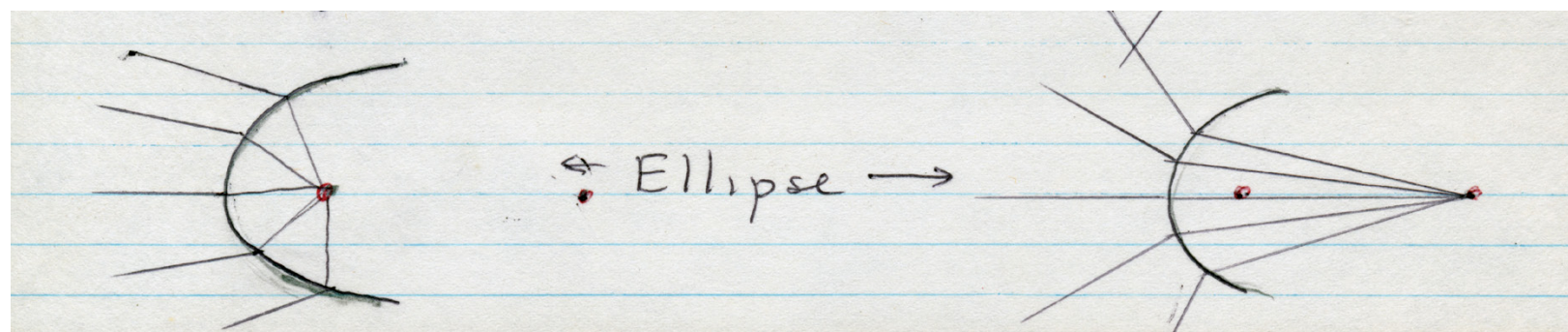Single ellipse
Gadgets with ellipses and tucking

July 1977

Plan for pair-of-coupled-ellipses paperfold:

The red dots are the foci of the ellipses.

Each ellipse has a major axis of 10 units and a minor axis of 8 units.
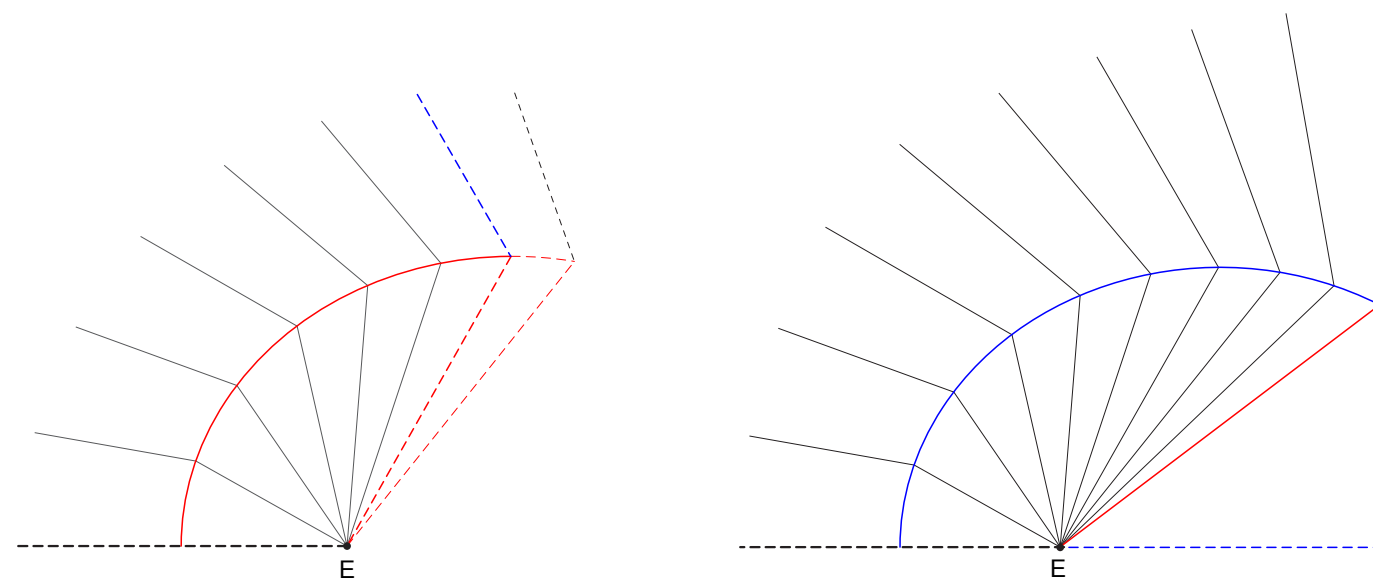
Fig 4.3.1 Index card (undated, DAH [DK])
Fig 4.3.2 Gadget with ellipses [DK]
Fig 4.3.5 Index card (1977, DAH [DK])
Fig 4.3.6 'Pair-of-coupled ellipses' (undated, [DAH] [EAH]), Identical model [EAH])

4. A taxonomy of DAH's curved crease paperfolding work

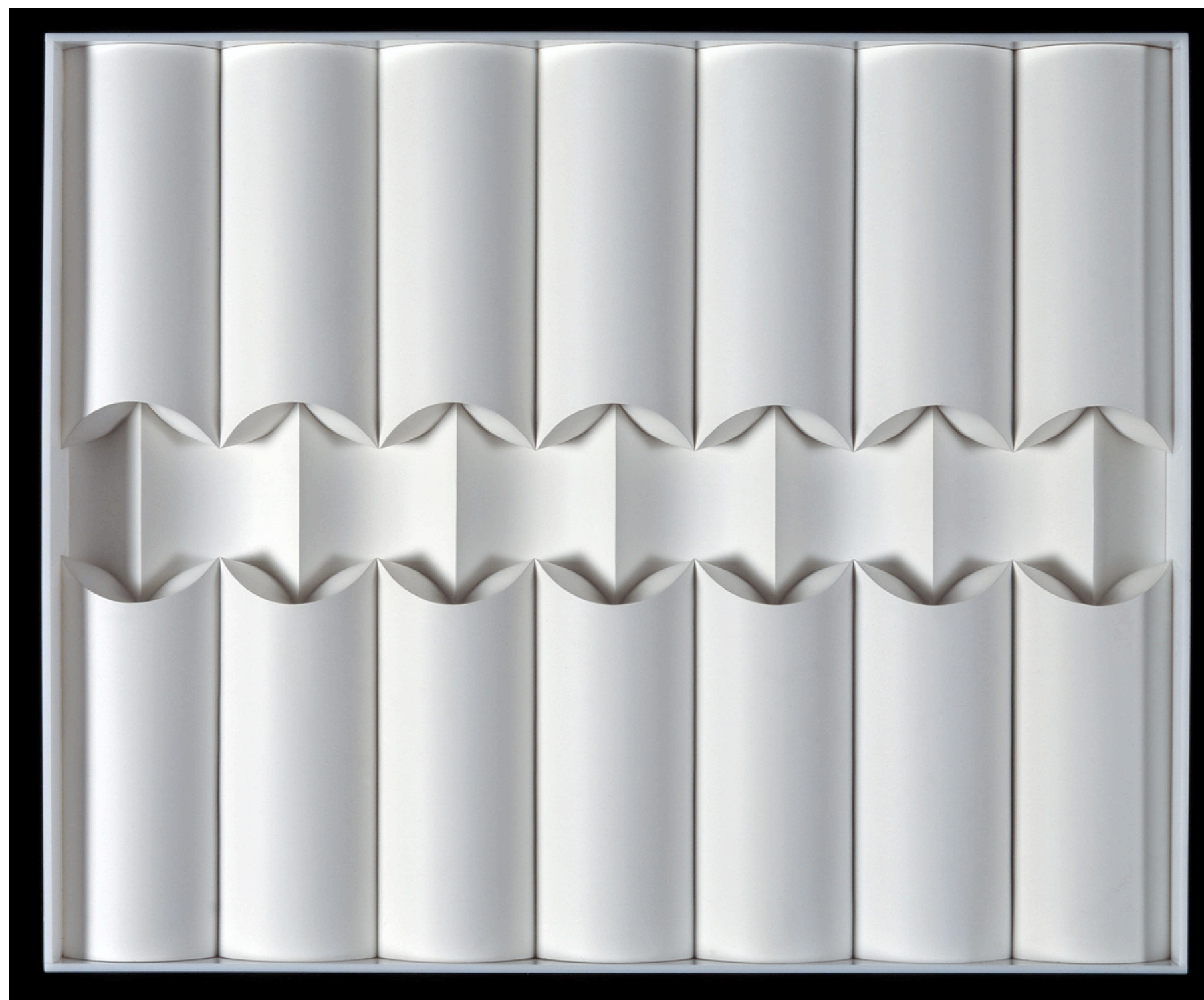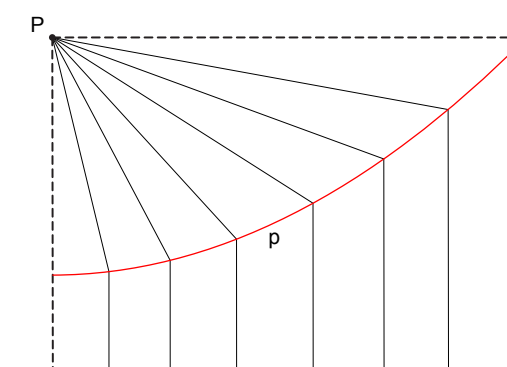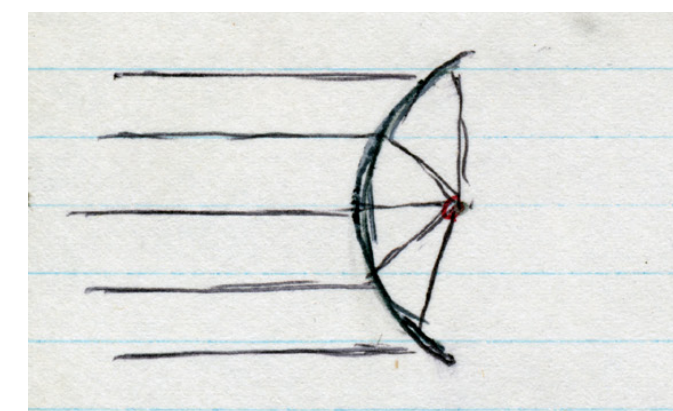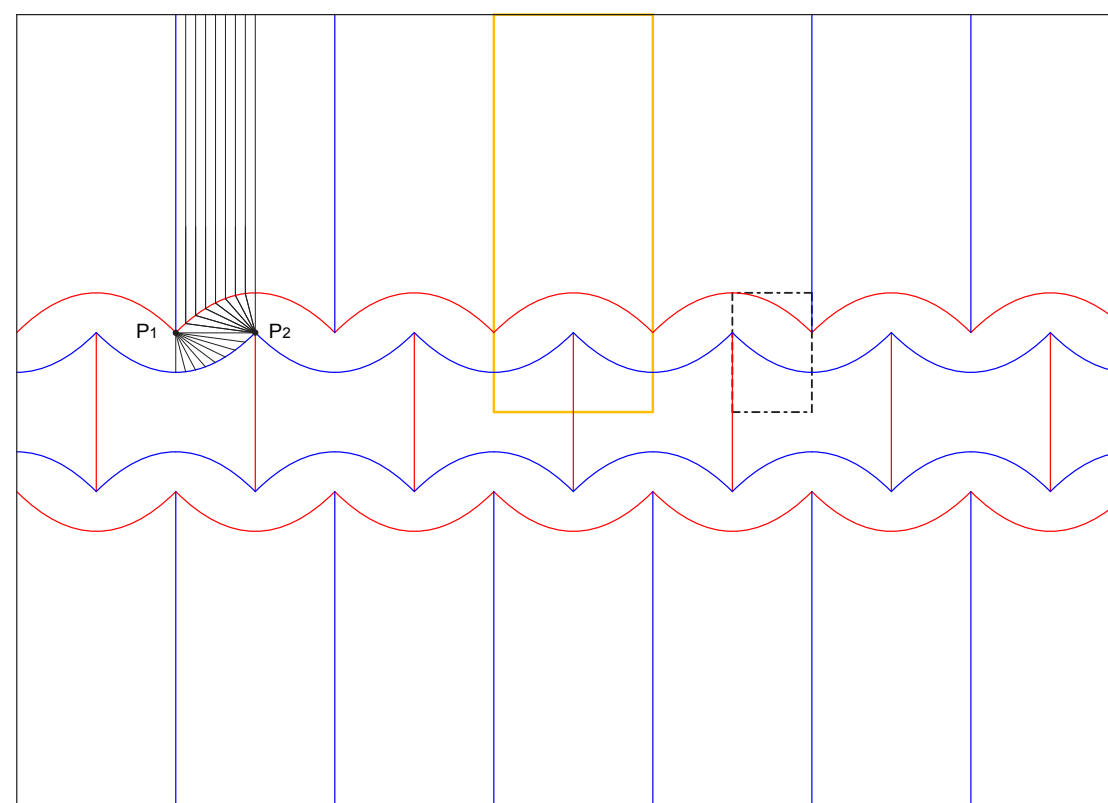[II Refraction Gadgets]

4.4 Gadgets with parabolas

Fig 4.4.44 'Seven interrupted semi cylinders' (1977, DAH [TG])

4. A taxonomy of DAH's curved crease paperfolding work

[II Refraction Gadgets]

4.4 Gadgets with parabolas

Fig 4.4.96 Vinyl model (1998, DAH [TG])
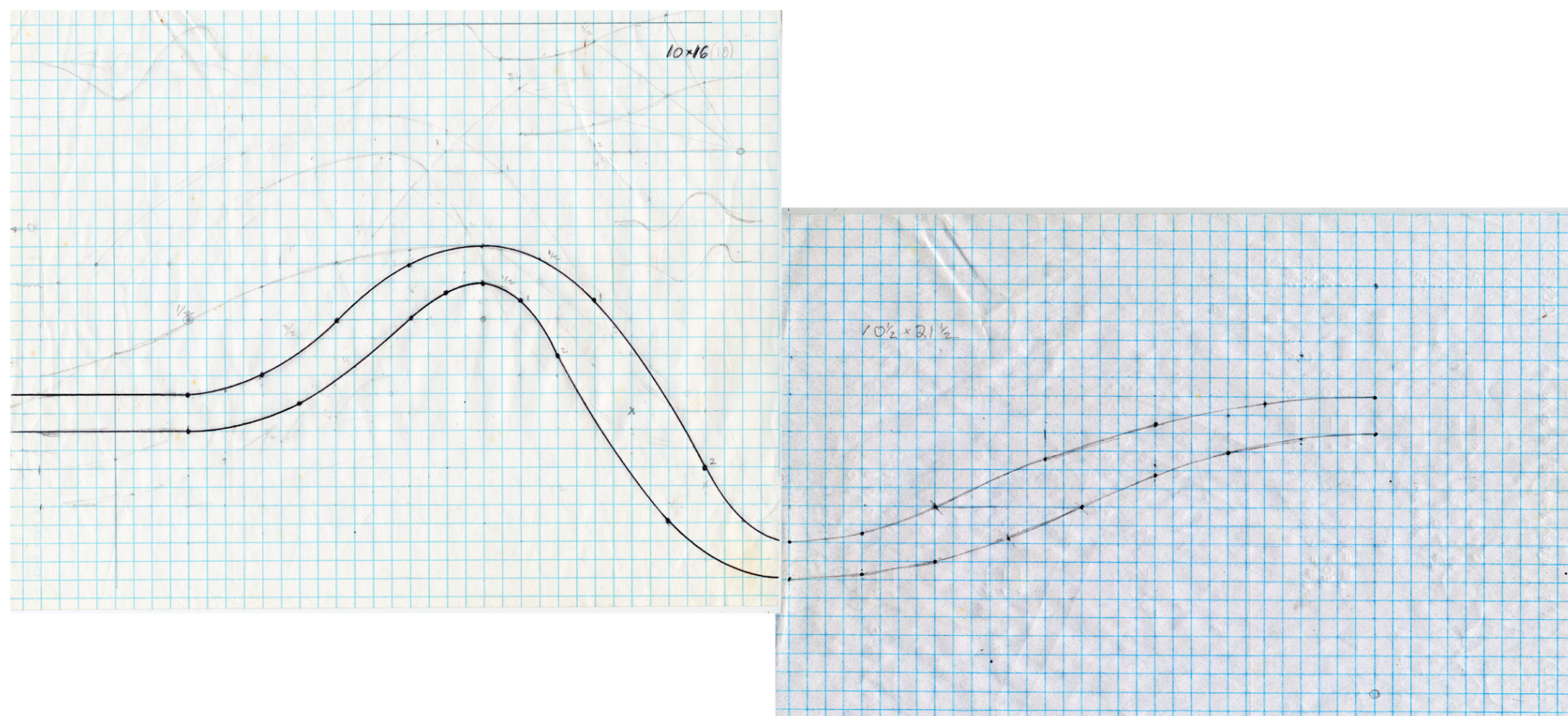

Fig 4.4.97 Drawings (undated, DAH [DK])

4. A taxonomy of DAH's curved crease paperfolding work

[II Refraction Gadgets]

4.4 Gadgets with parabolas

$a' = 1/2$
$a = 2/3$
$b = 1/6$

$a' = 1/2$
$a = 3/4$
$b = 1/4$

$a' = 1/2$
$a = 1$
$b = 1/2$

$a' = 3/4$
$a = 1$
$b = 1/4$

$a' = 1/2$
$a = 1$
$b = 1/2$

Fig 4.4.131 Paper model (undated, DAH [DK]), Simulated model [AH]
Fig 4..2 Gadget table [DK]
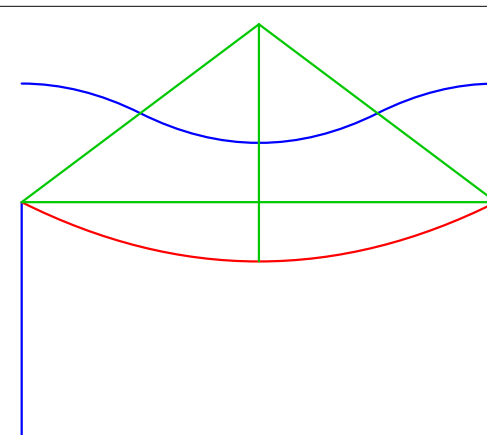**Fig 4.4.132** Crease pattern [DK]

4. A taxonomy of DAH's curved crease paperfolding work

[II Refraction Gadgets]

4.5 Gadgets with hyperbolas

Fig 4.5.3 Paper model (undated, [DAH] [DK]), Crease pattern [DK]

Fig 4.5.4 Simulation, top and bottom view [JH]

4. A taxonomy of DAH's curved crease paperfolding work
[II Refraction Gadgets]

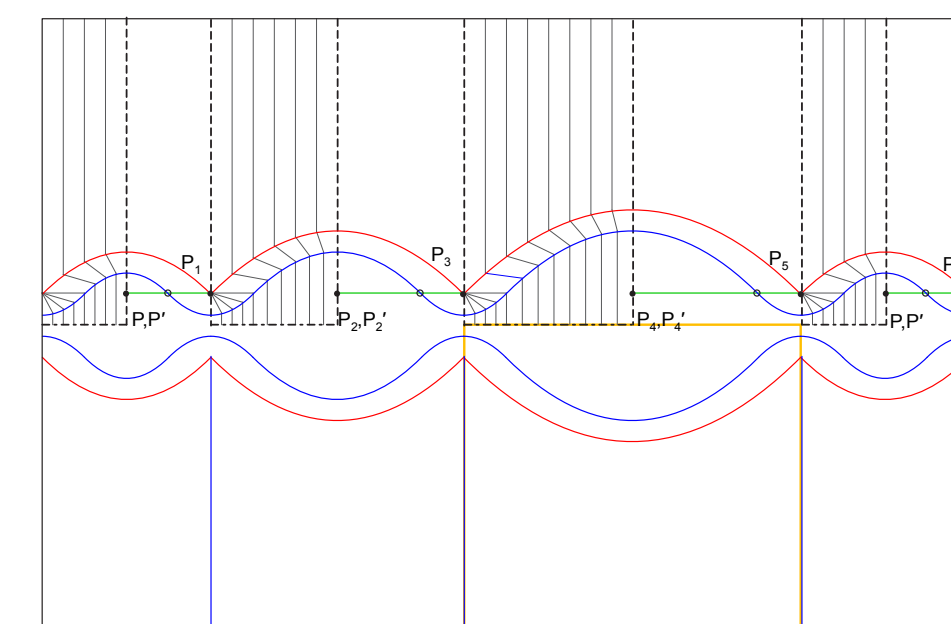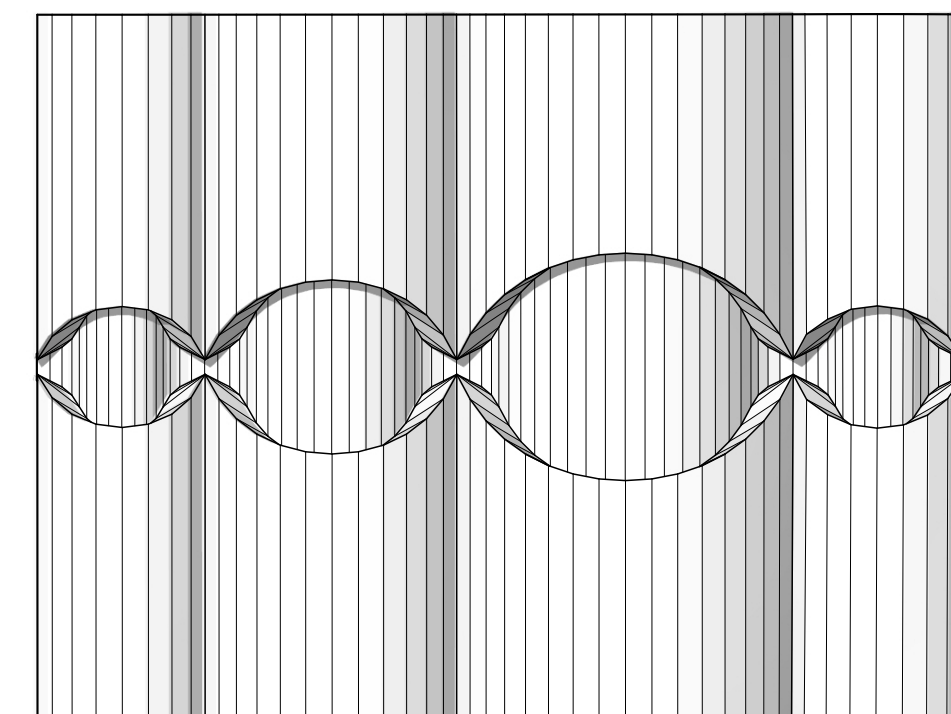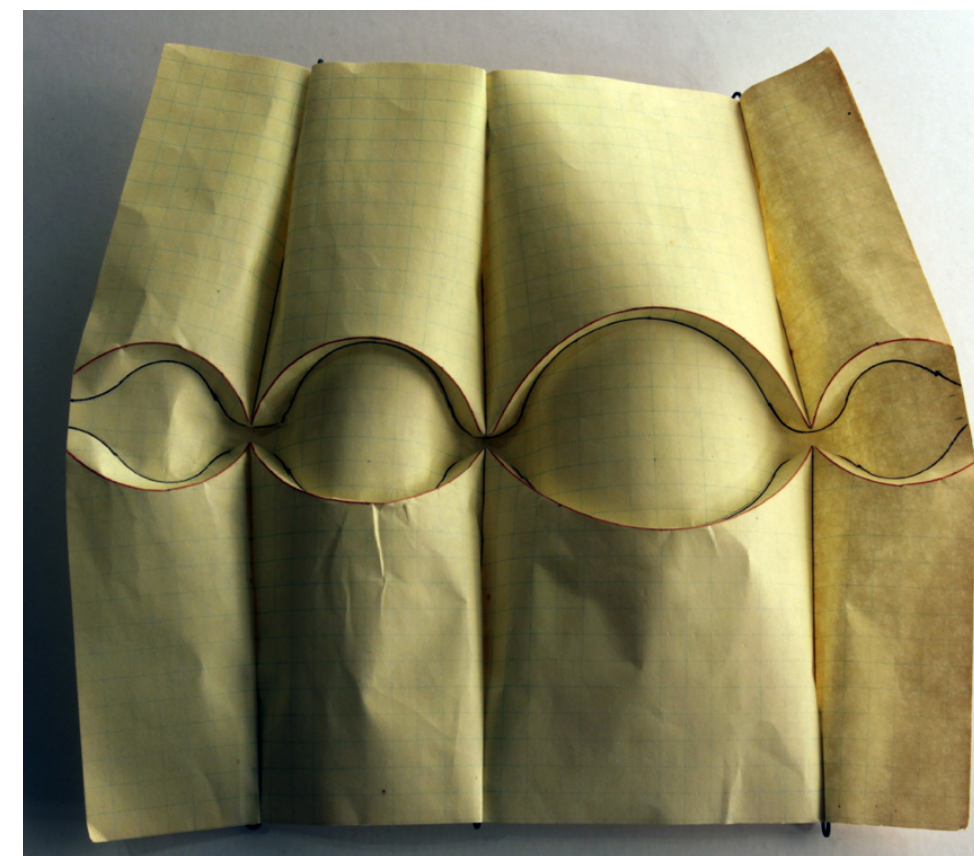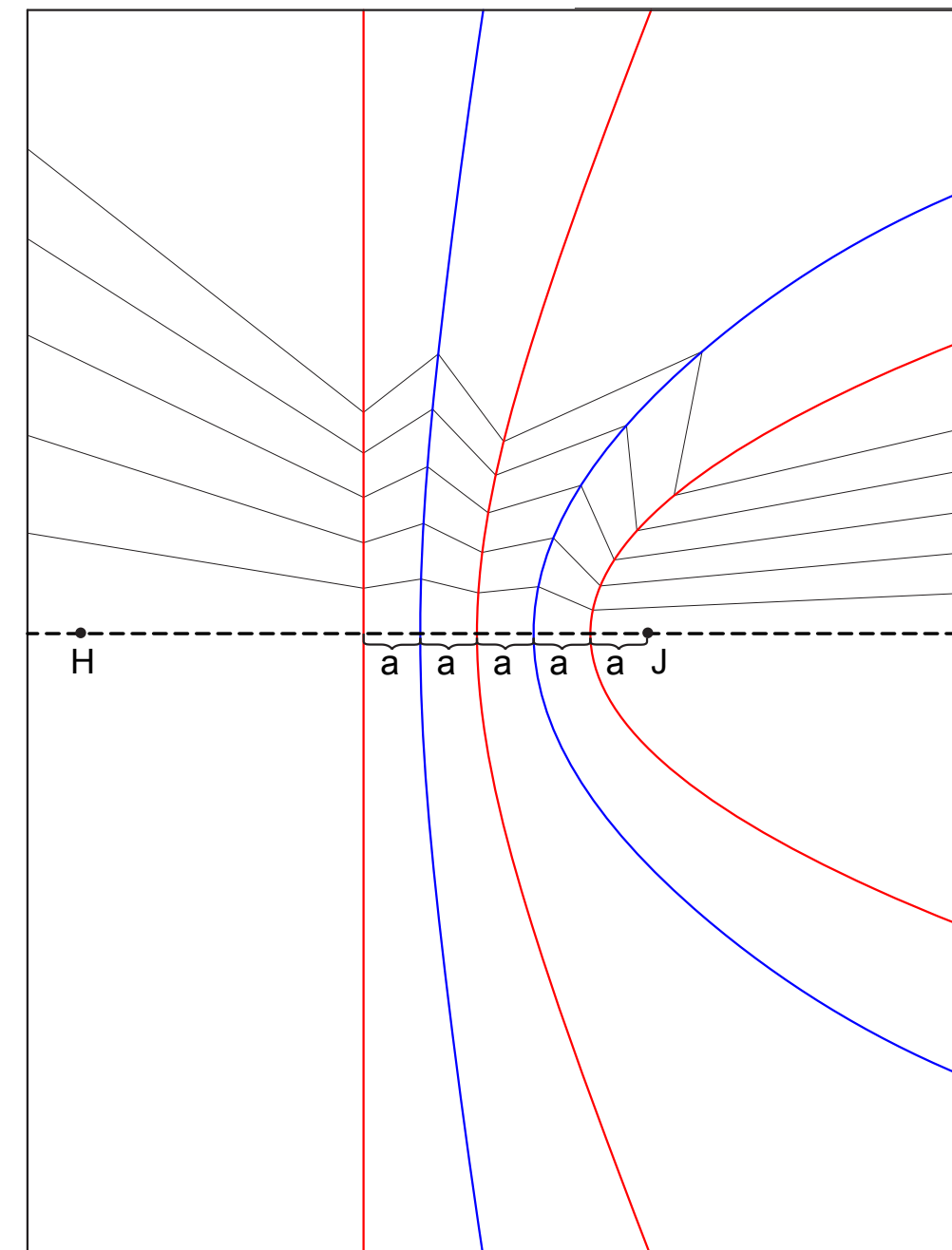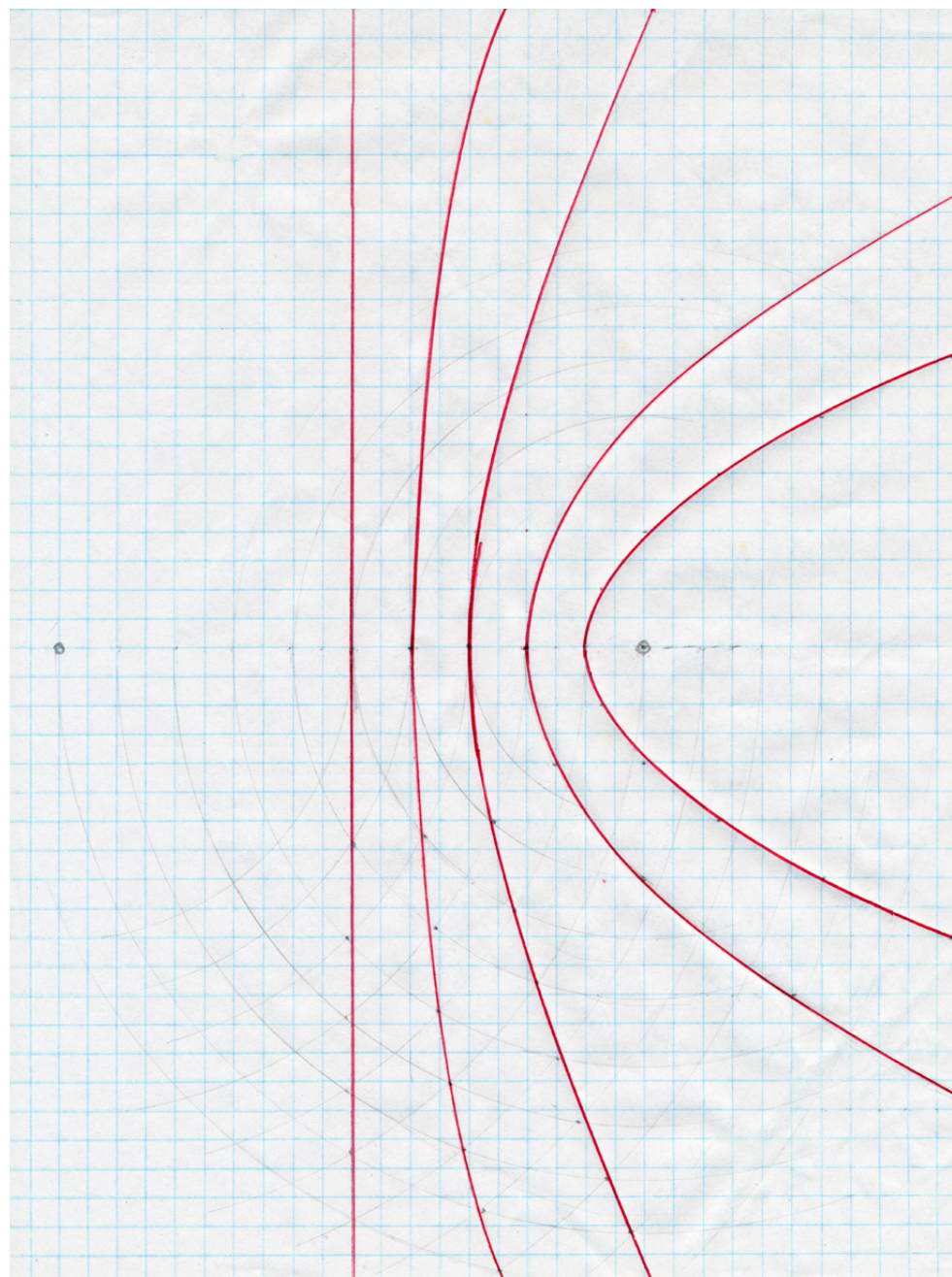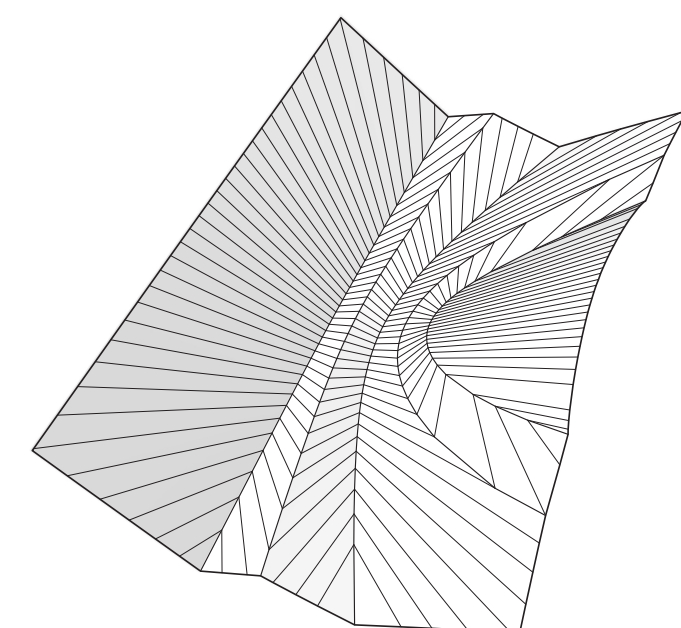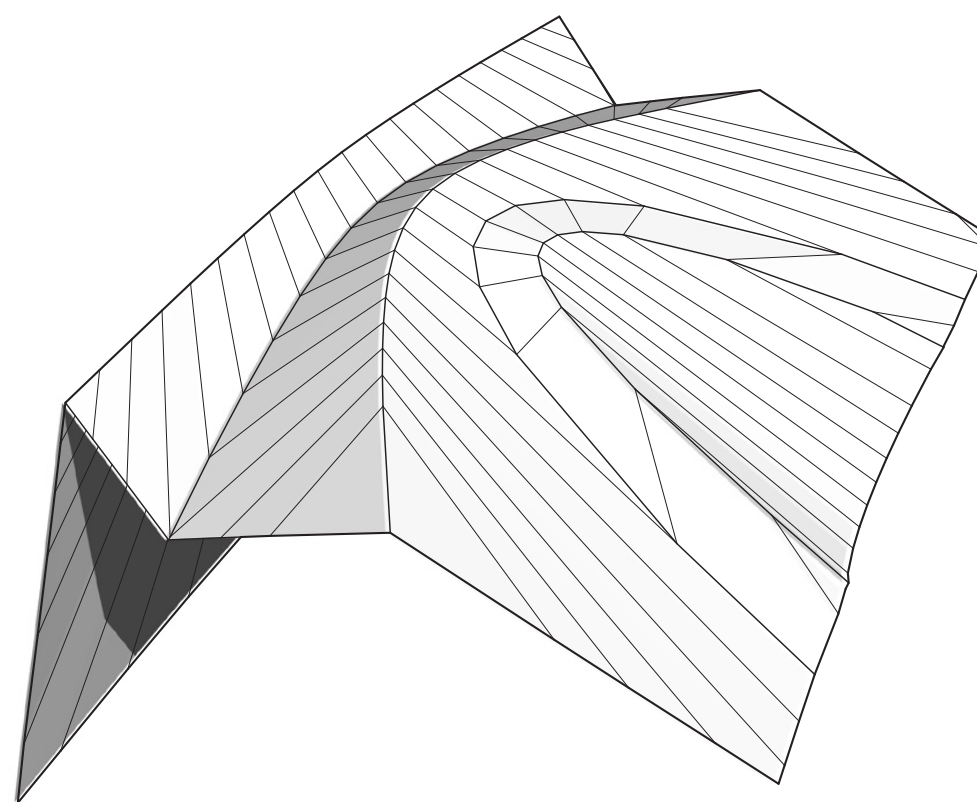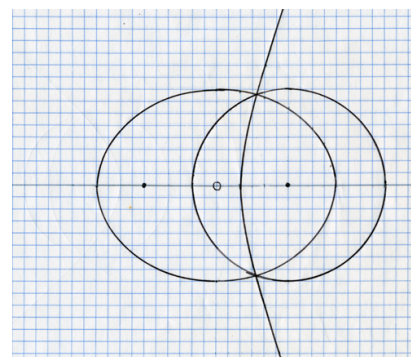### 4.6 Gadgets with parabolas and ellipses
Gadgets with parabolas, ellipses and line segments

a = 4   [1]
r = 8   [2]
f = 12 [3]
d = -2 [-0.5]
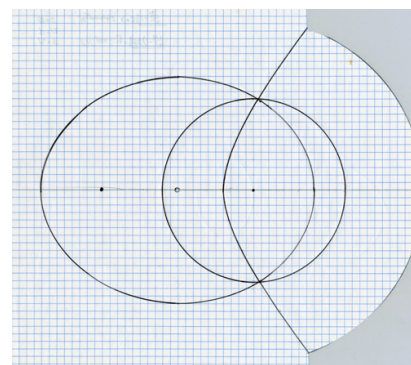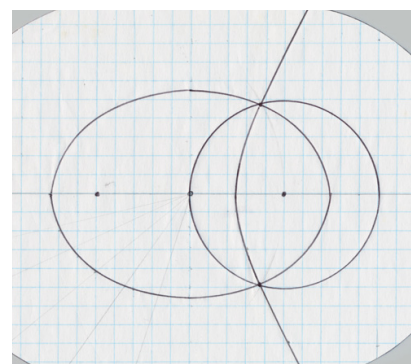
a = 4   [1]
r = 12 [3]
f = 20 [5]
d = -2 [-0.5]

a = 2.5 [1]
r = 5   [2]
f = 10 [4]
d= 0   [0]

a = 4   [1]
r = 8   [2]
f = 24 [3]
d = 4   [1]

a = 3   [1]
r = 6   [2]
f = 24 [3]
d = 6   [2]

a = 4   [1]
r = 6   [2.5]
f = 48 [12]
d = 18 [4.5]

a = 3   [1]
r = 6   [2]
f = 24 [3]
d = 6   [2]
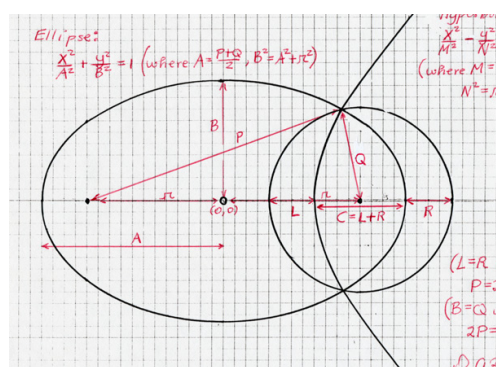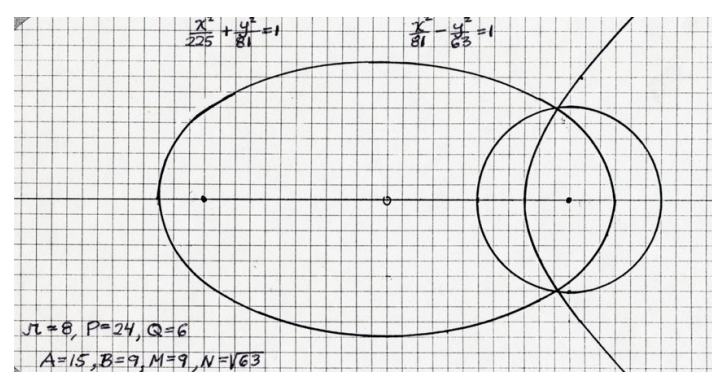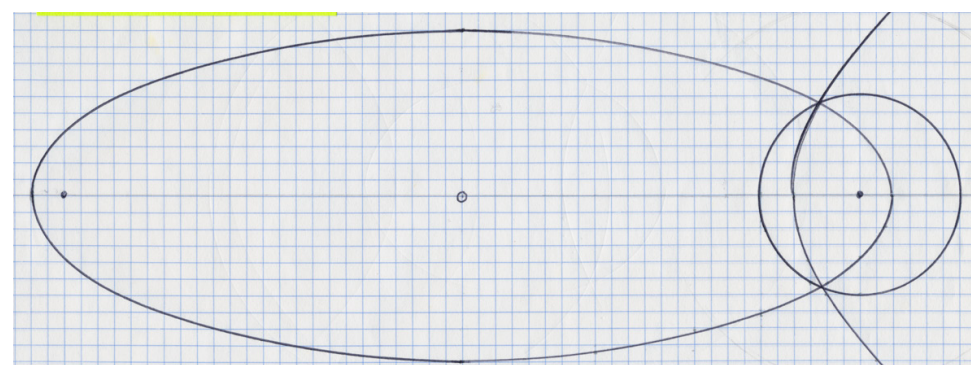
Fig 4.8.9 Vinyl model (undated, DAH [DK])
Fig 4.8.10 Photo copy of drawing (undated, DAH [DK]), Crease pattern [DK]
Fig 4.8.17 Paper models with parameter values (most undated, DAH [DK])

4. A taxonomy of DAH's curved crease paperfolding work
[II Refraction Gadgets]

4.8 Gadgets with ellipses and hyperbolas
Gadgets with ellipses, hyperbolas and line segments
Gadgets with elliptic and hyperbolic splines

Fig 4.8.25 Sketch (undated, DAH, DK)

Fig 4.8.26 Crease pattern [DK]

4. A taxonomy of DAH's curved crease paperfolding work
[III Forced Ruling]

4.10 Cone and cylinder gadget
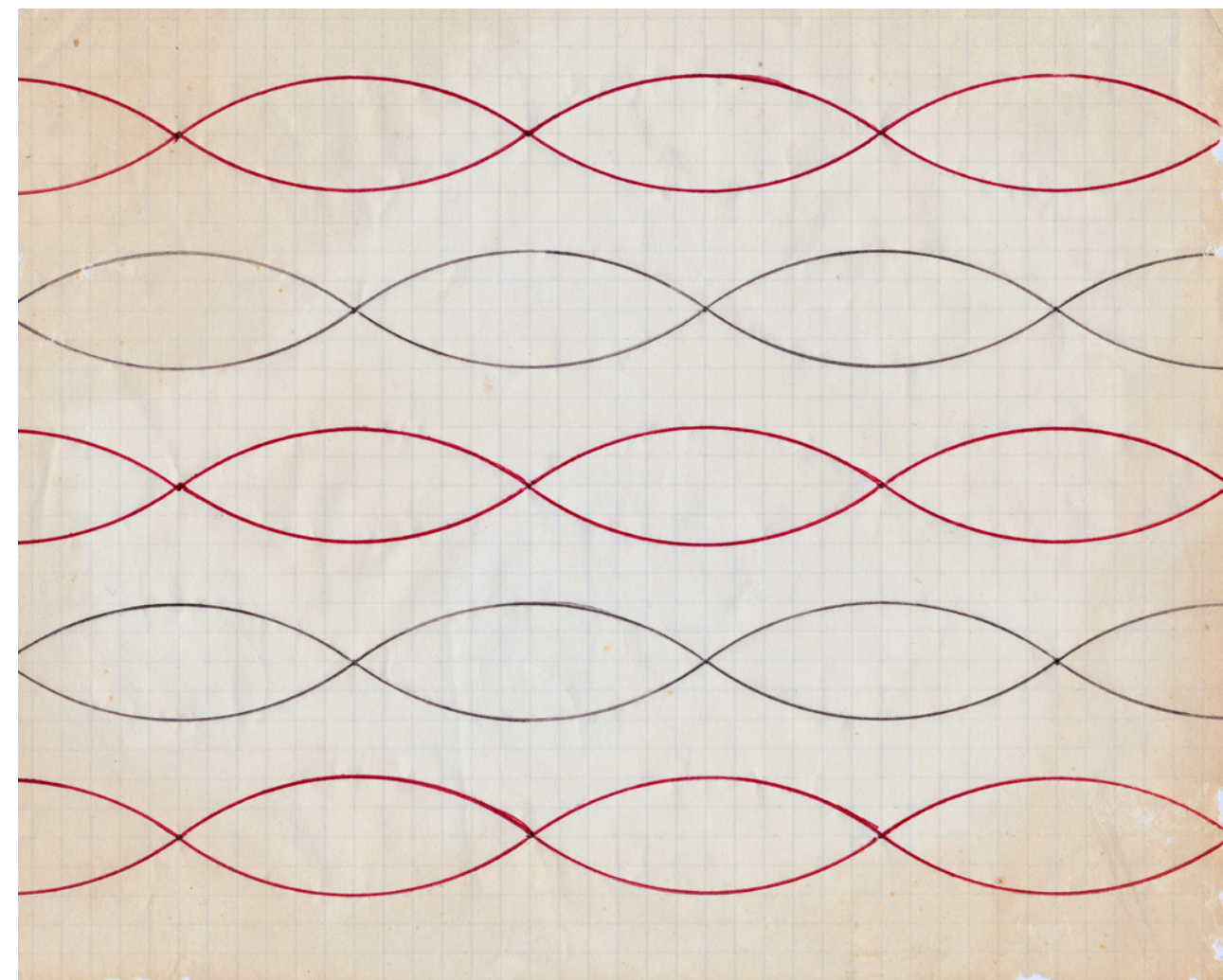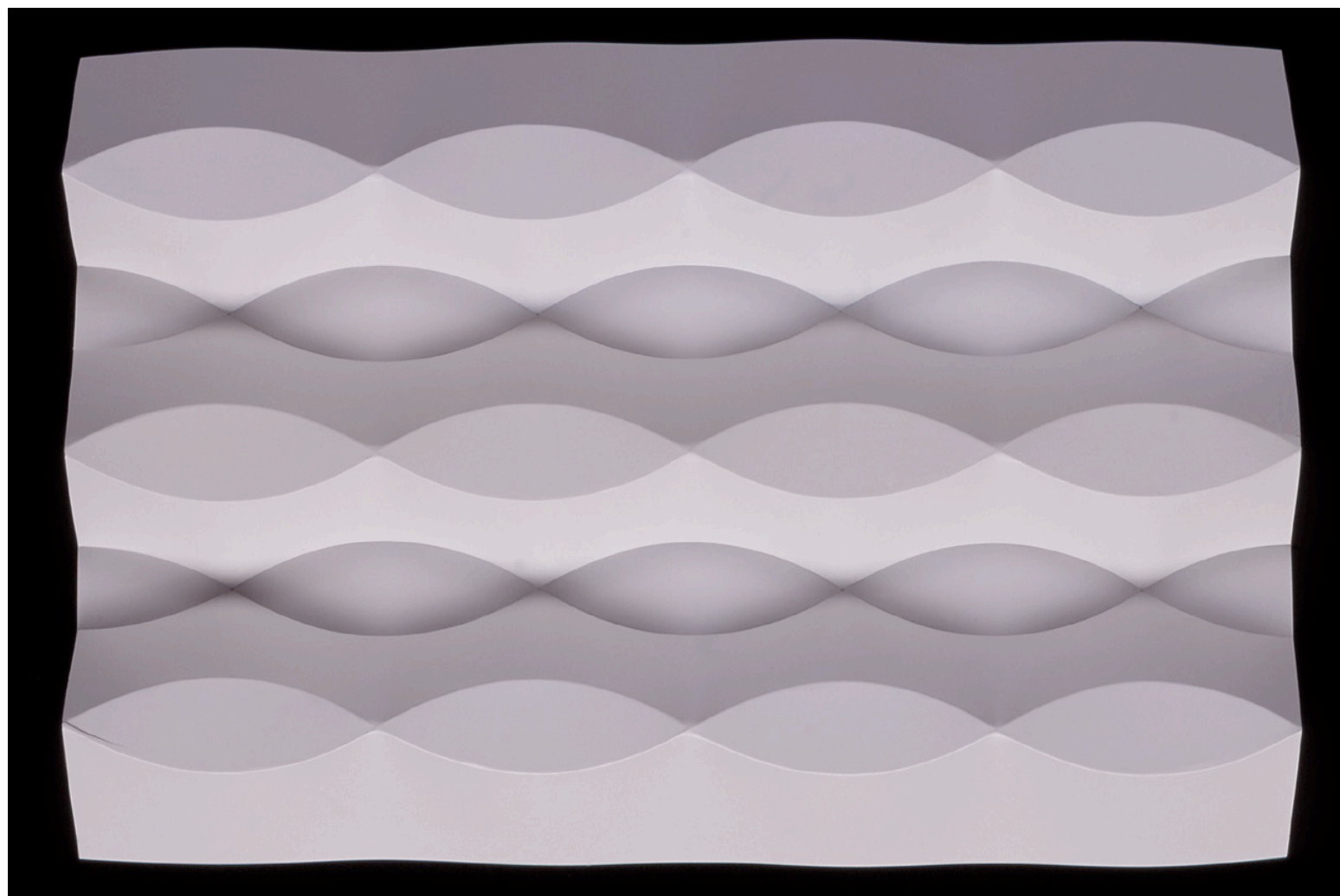
Fig 4.10.1 Vinyl model (1993, DAH [TG])

Fig 4.10.2 Sketch (undated, DAH [DK])

Fig 4.10.3 Crease pattern [DK]

Fig 4.10.4 Simulated model [AH]

4. A taxonomy of DAH's curved crease paperfolding work
[IV Converging Curves]

4.11 Cyclic tilings with converging curves
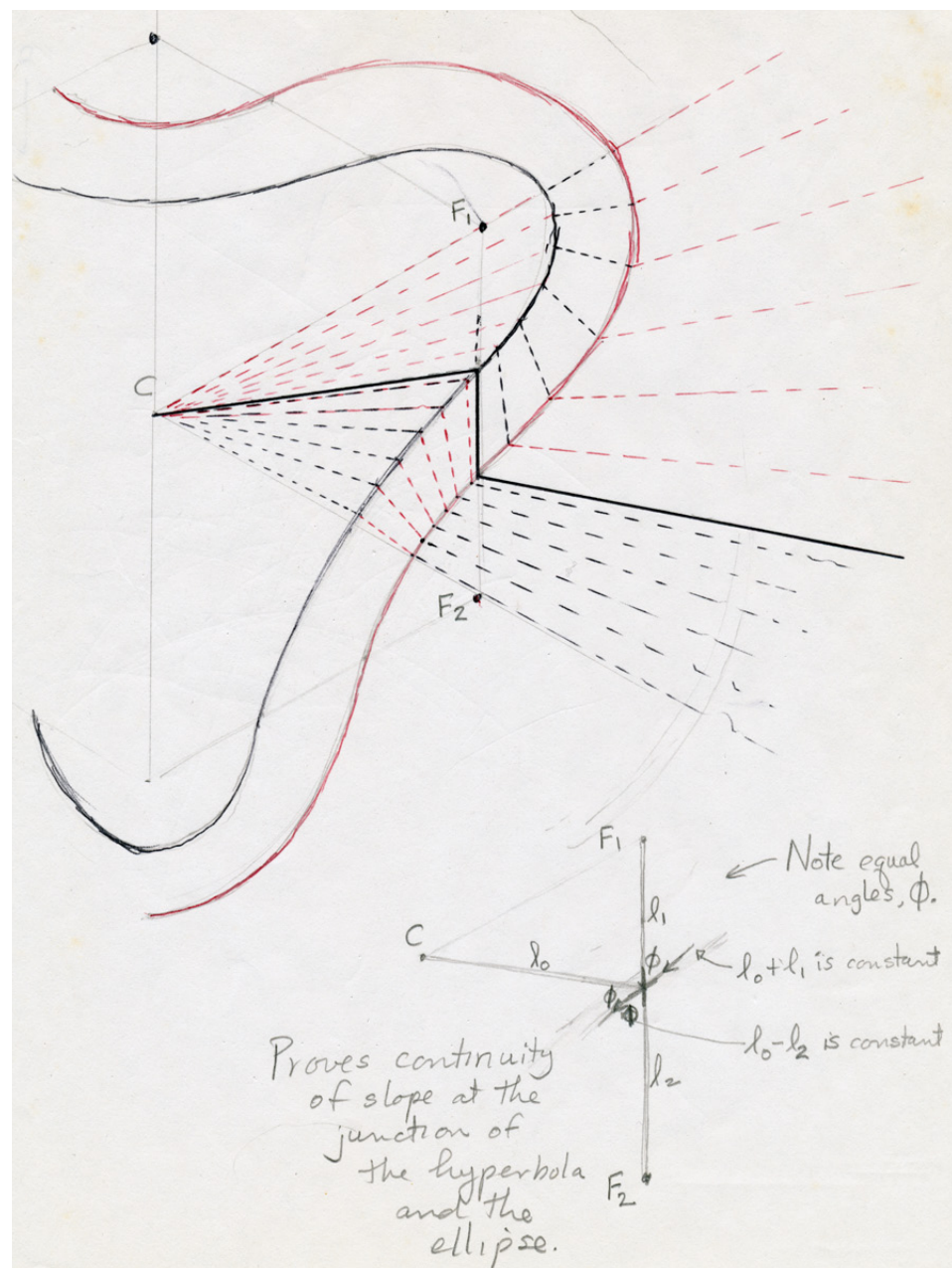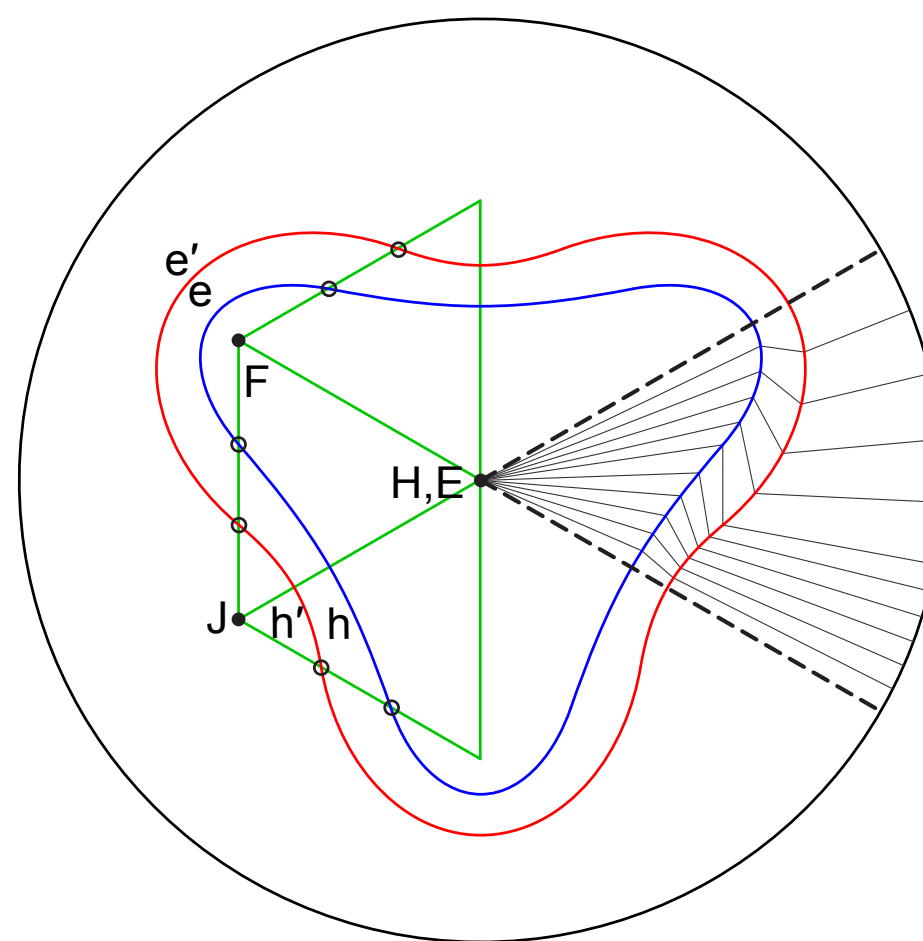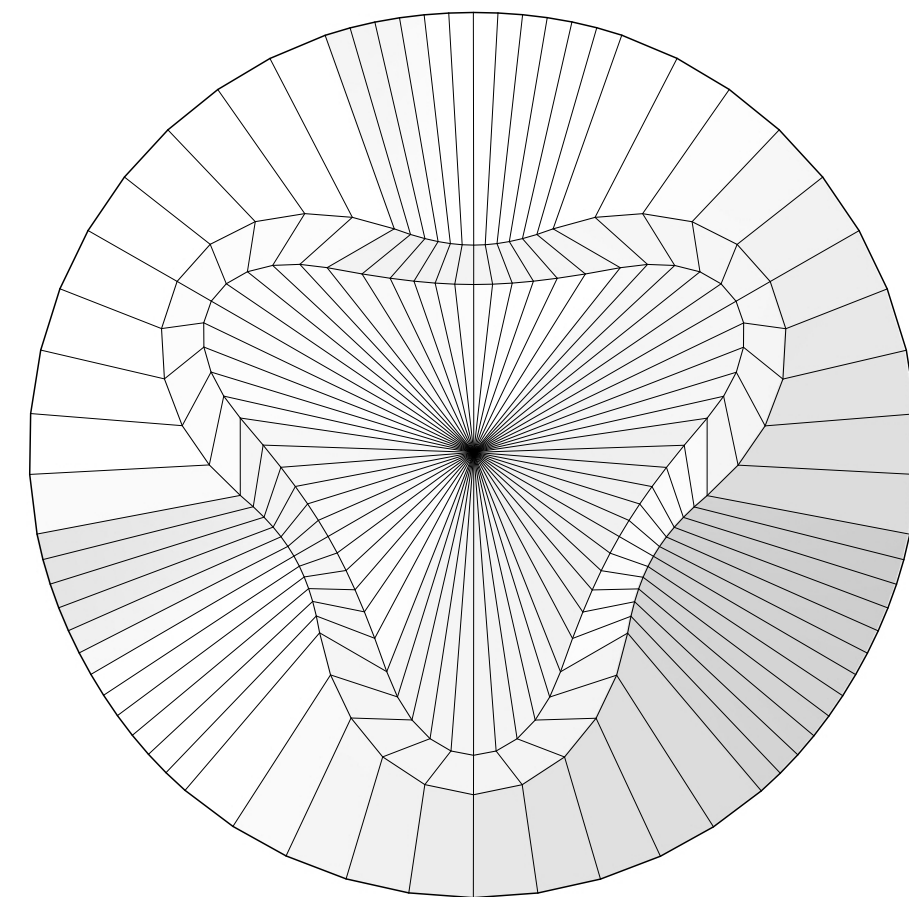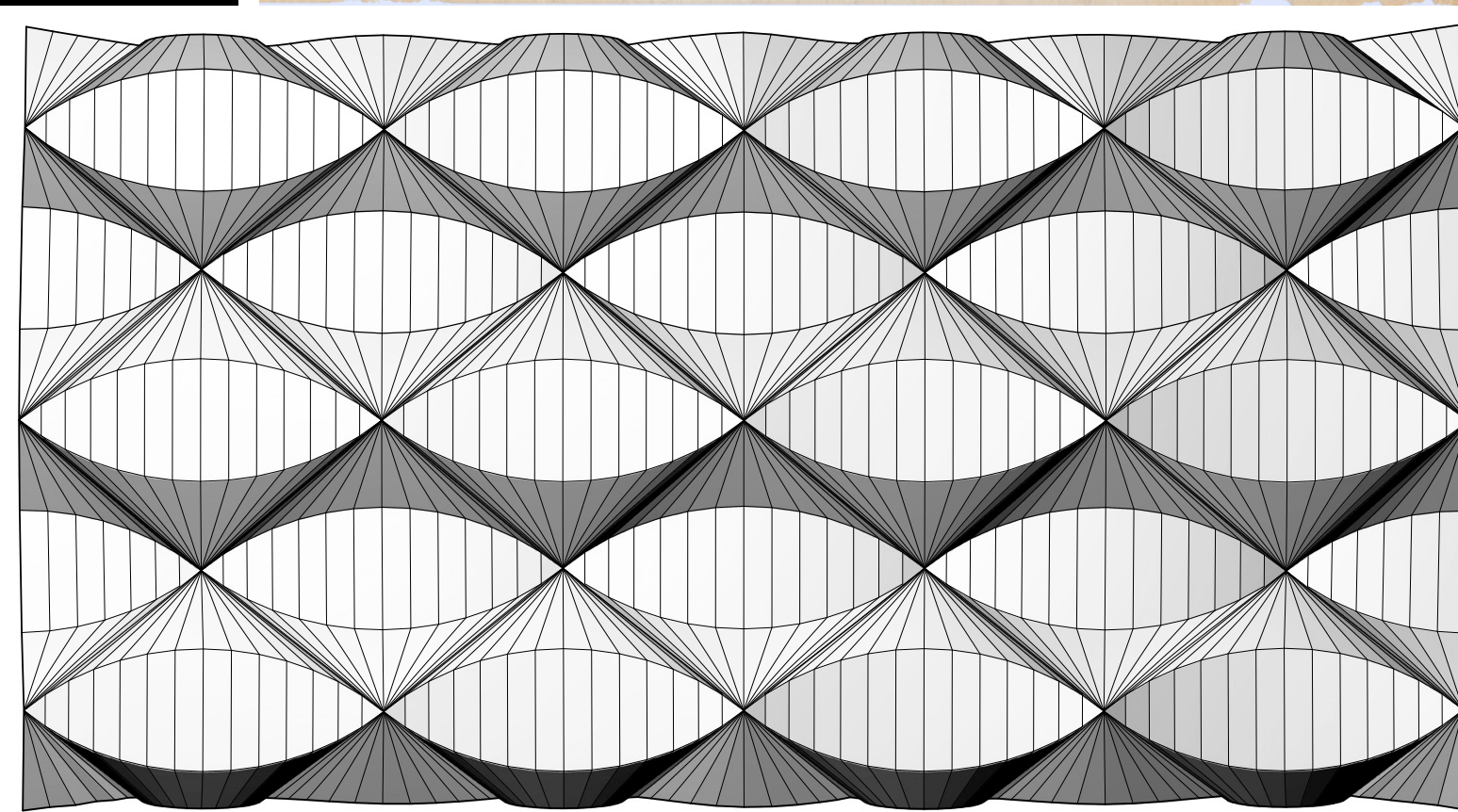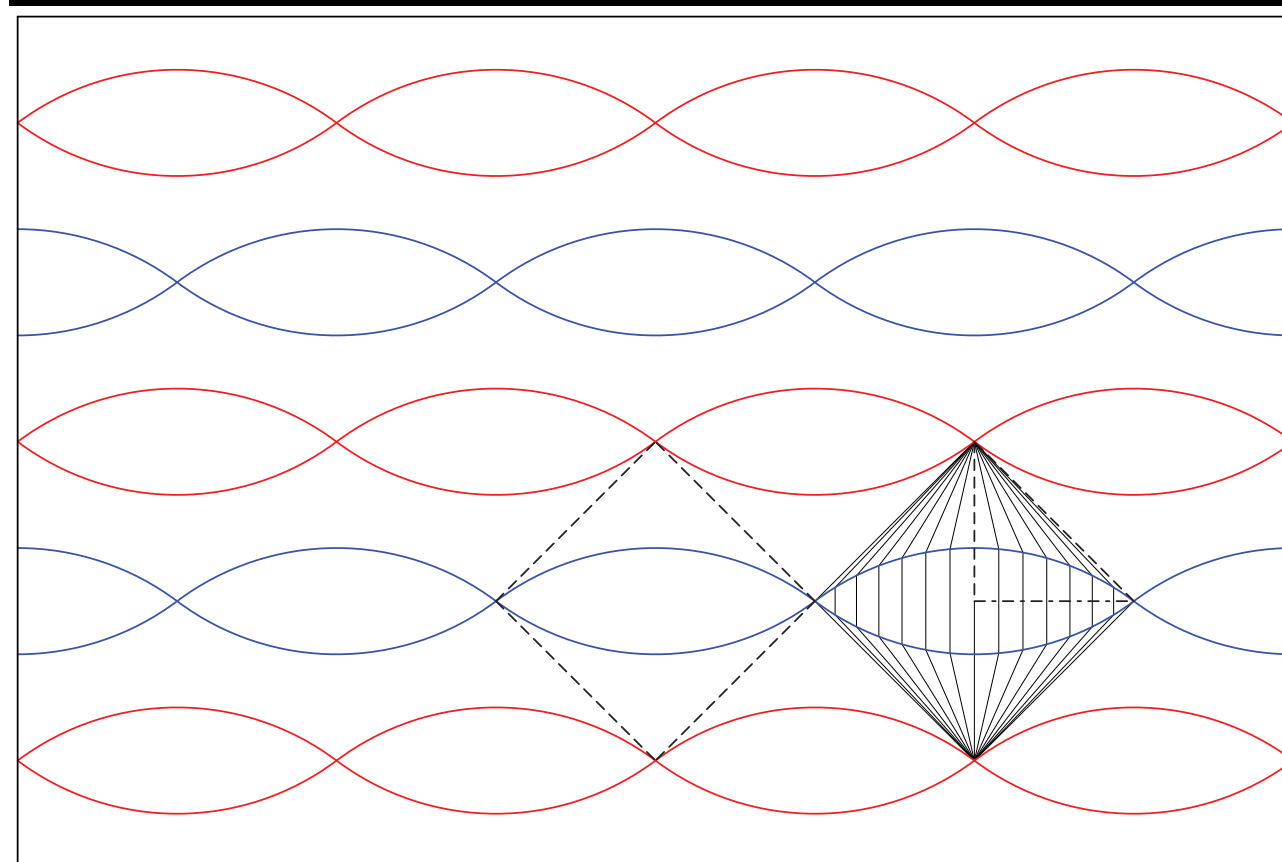Tilings with central vertex
Tilings with central gap or line segment
Tilings with central polygon

Pursuit curve: $y = \frac{1}{3}\left[2 - (2+x)\sqrt{1-x}\right]$

"Tangent" spiral:
$$R = \tan\left(\frac{\theta}{R} + \frac{\pi}{2}\right)$$

The spiral starts at ($\theta = -R\pi/2$, $R=0$) and turns through an angle of $R\pi/2$ (ccw) as its radius →∞. As the spiral expands it approaches the line $y = -R$; going toward $x = +\infty$

Example: $R = 3$, $R = \tan\left(\frac{\theta}{3} + \frac{\pi}{2}\right)$
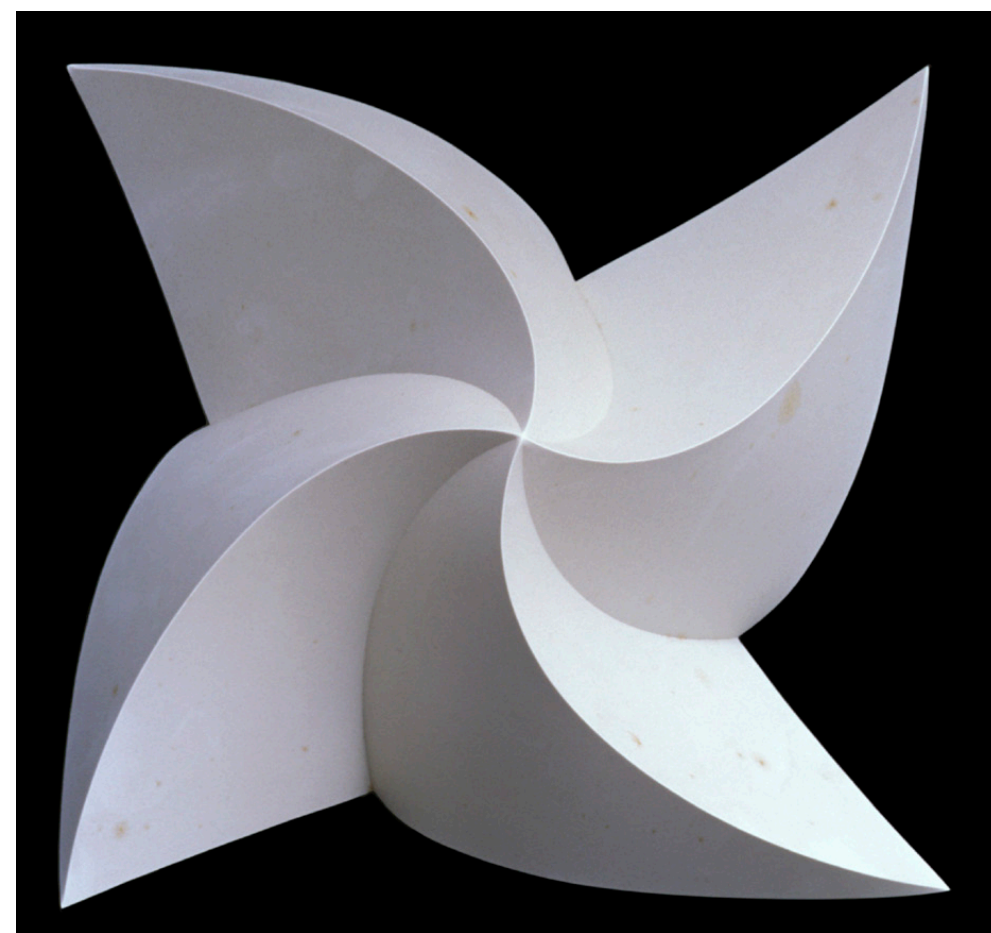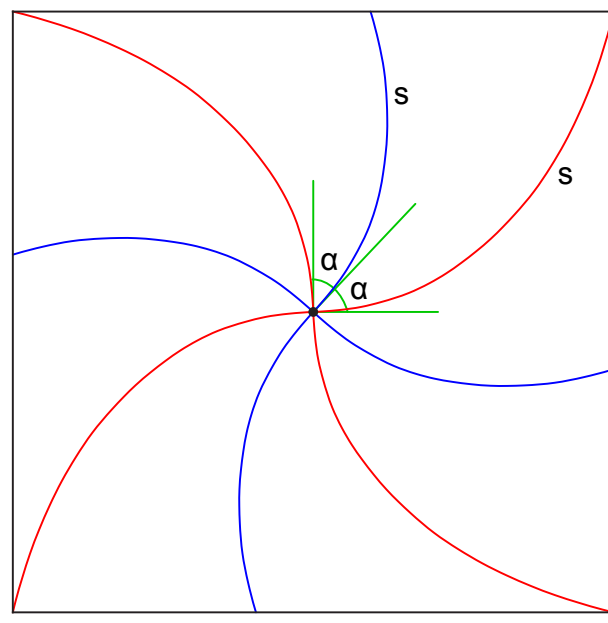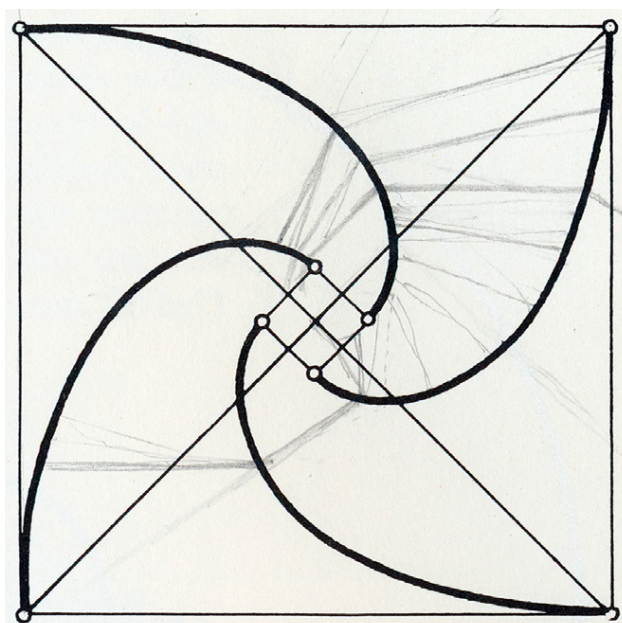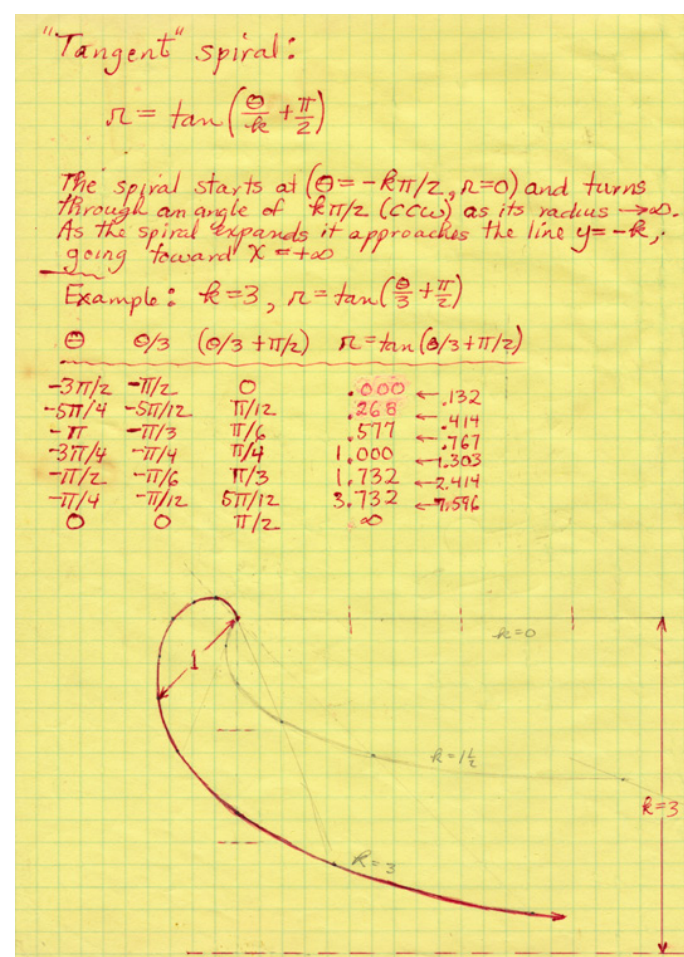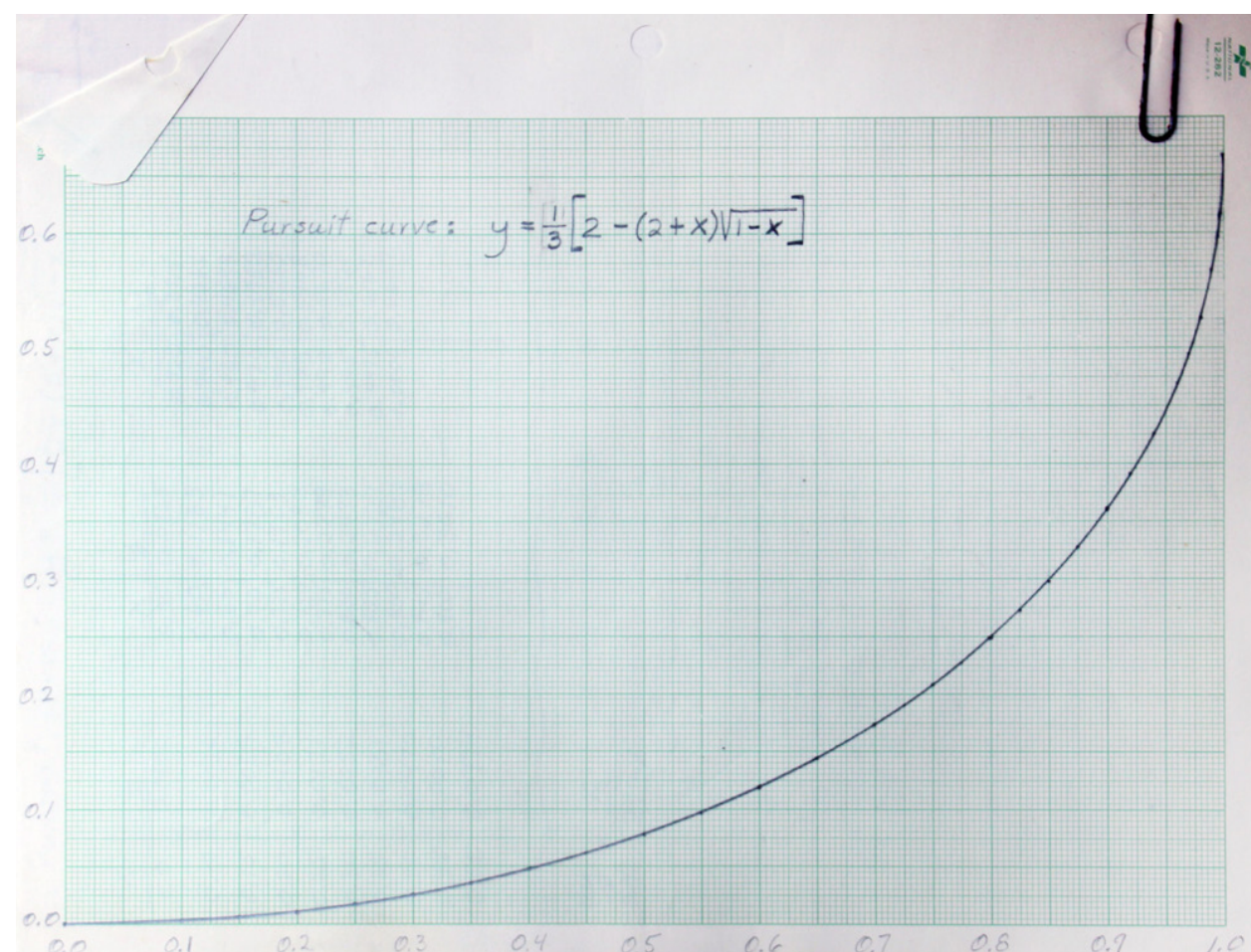
Fig 4.11.2 Notes and drawings (undated, DAH [DK])
Fig 4.11.3 Vinyl model (undated, DAH [EAH]), Paper reconstruction [AH]
Fig 4.11.4 Sketch on figure (undated, DAH [DK]), Crease pattern [DK]

4. A taxonomy of DAH's curved crease paperfolding work
[IV Converging Curves]

4.12 Tilings with loxodromic spirals

Fig 4.12.4 Paper model (undated, DAH [DK])
Fig 4.12.2 Paper models (undated, DAH [DK])
Fig 4.12.1 Vinyl model (undated, DAH [TG]), Huffman in front of 'the peanut' (1991 [M.Mulbry])

4. A taxonomy of DAH's curved crease paperfolding work
[IV Converging Curves]

4.12 Tilings with loxodromic spirals

First set of circles

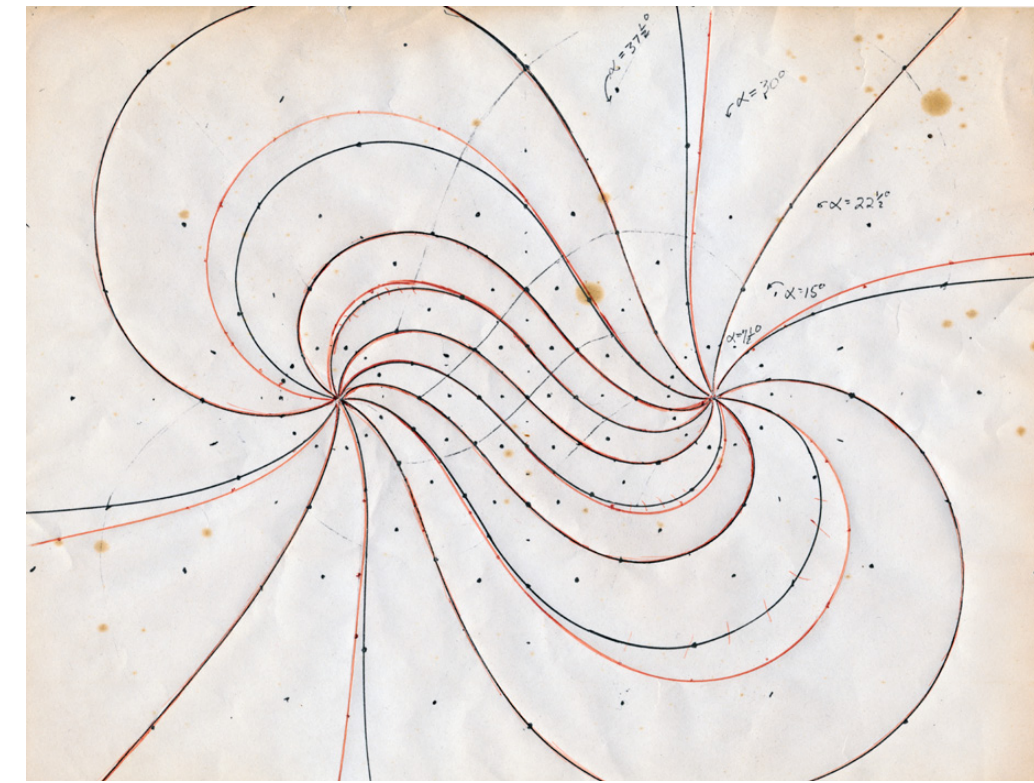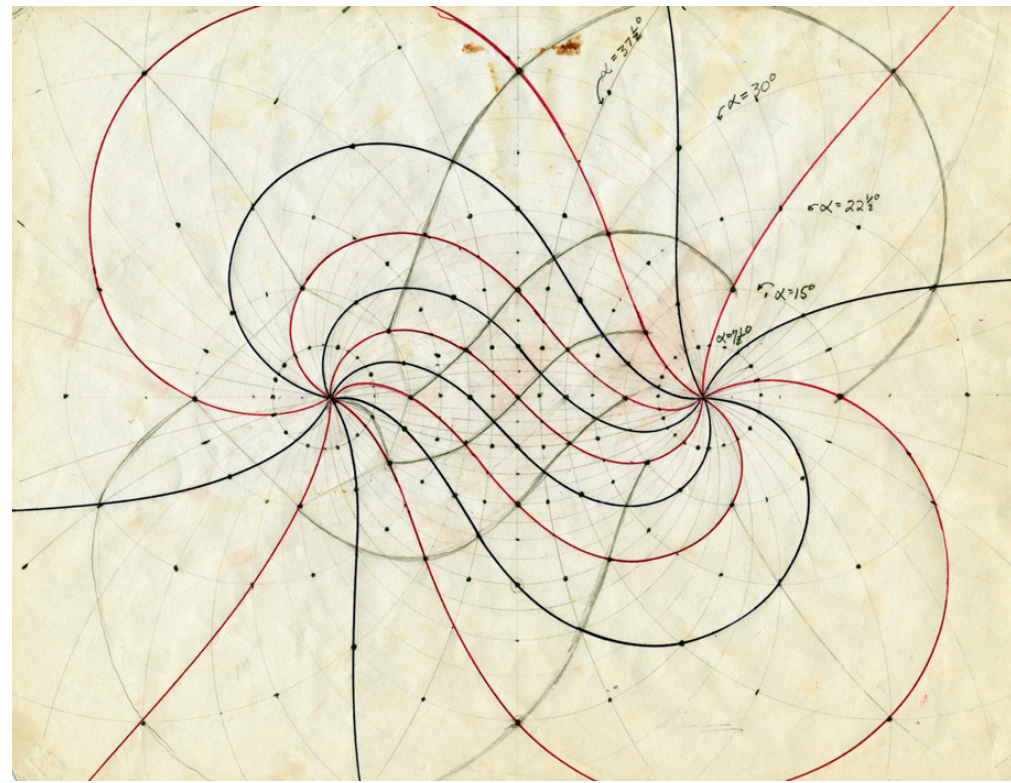| α	X-intercepts				center at	
0	5.00000		5.00000		5.00000	
3¾	5.70140	22½/32	4.38490	12¼/32	5.04315	1½/32
7½	6.51615	16½/32	3.83665	26½/32	5.17640	5.5/32
11¼	7.48305	15½/32	3.34090	11/32	5.41195	13¼/32
15	8.66025	21/32	2.88675	20½/32	5.77350	24¾/32
18¾	10.13900	4½/32	2.46575	15/32	6.30235	9¾/32
22½	12.07105	2¼/32	2.07105	2¼/32	7.07105	2.25/32
26¼	14.72955	23¼/32	1.69725	22¼/32	8.21340	6¾/32
30	18.66025	21/32	1.33975	11/32	10.00000	
33¾	25.13670	4¼/32	.99455	32/32	13.06565	3/32
37½	37.97875	31/32	.65825	21/32	19.31850	10/32
41¼	76.28525	9/32	.32770	10½/32	38.30650	9¾/32
45	∞		.00000		∞	

Second set of circles

| Θ	y-intercepts				center at	
180	∞		∞		∞	
165	37.97875	31/32	-.65825	21/32	18.66025	21/32
150	18.66025	21/32	-1.33975	11/32	8.66025	21/32
135	12.07105	2.25/32	-2.07105	2.25/32	5.00000	
120	8.66025	21/32	-2.88675	28.5/32	2.88675	28.5/32
105	6.51615	16.5/32	-3.83665	26.5/32	1.33975	11/32
90	5.00000		-5.00000		0.00000	
75	3.83665	26.5/32	-6.51615	16.5/32	-1.33975	11/32
60	2.88675	28.5/32	-8.66025	21/32	-2.88675	28.5/32
45	2.07105	2.25/32	-12.07105	2.25/32	-5.00000	
30	1.33975	11/32	-18.66025	21/32	-8.66025	21/32
15	.65825	21/32	-37.97875	31/32	-18.66025	21/32
0	0		∞		-∞	
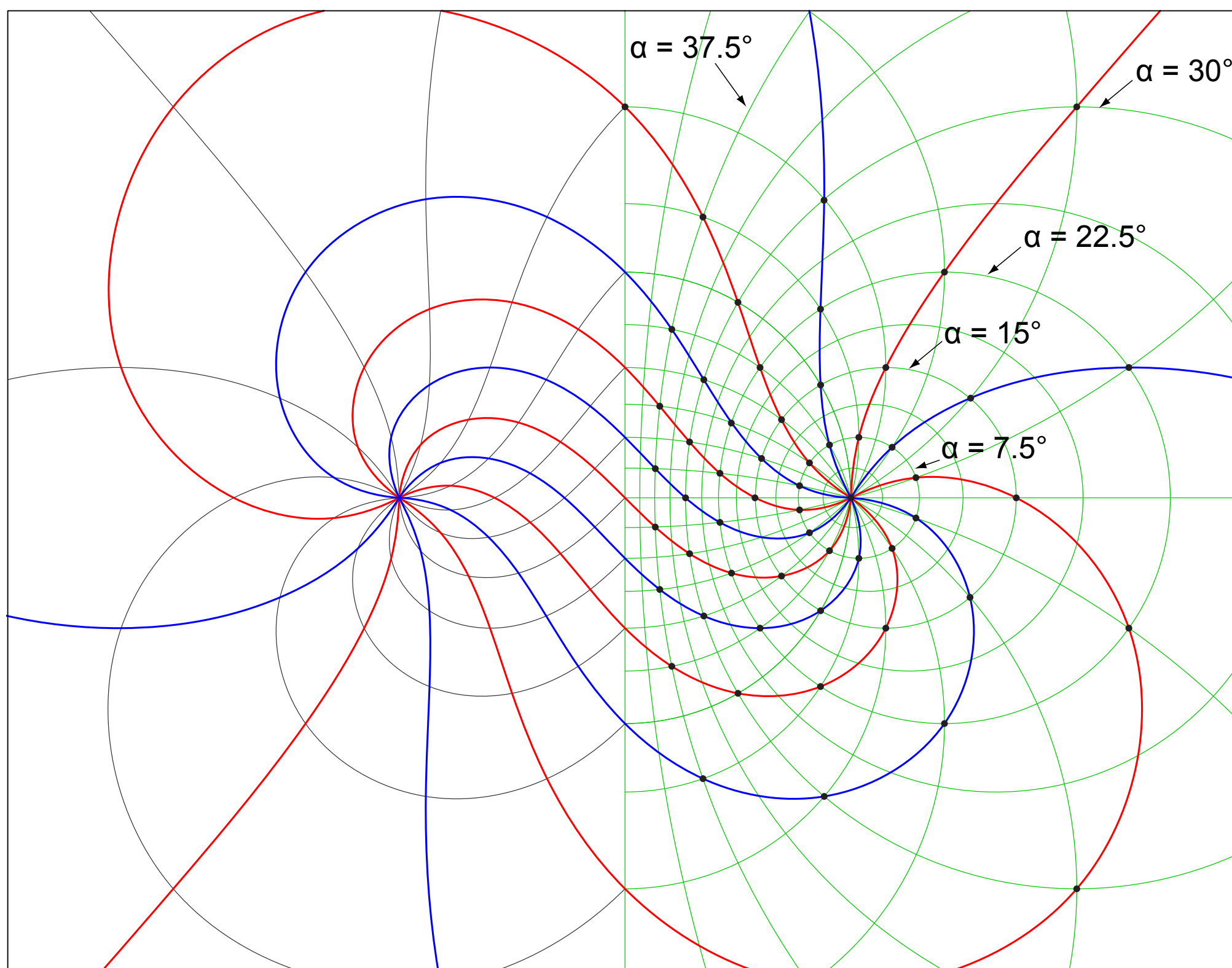
Fig 4.12.5 Index cards with coordinate tables (undated, DAH [DK])
Fig 4.12.3 Crease pattern [DK]

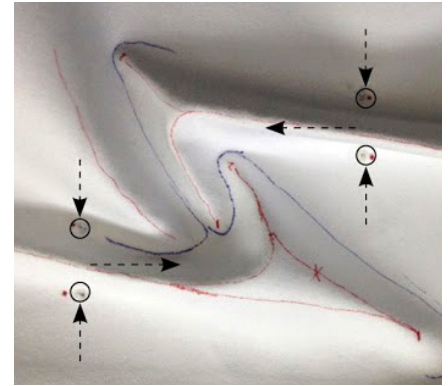# Designing with curved creases - digital and analog constraints
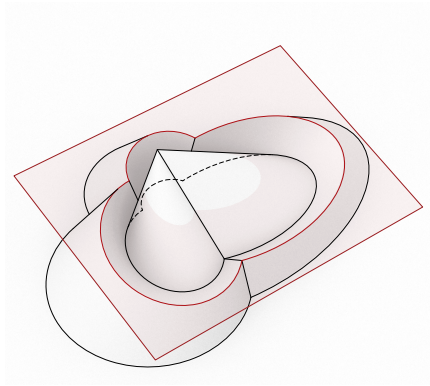
Developable Surface Reflection
(direct digital representation)
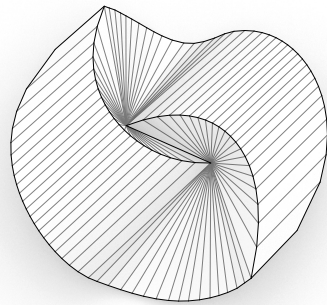
Refraction Gadgets
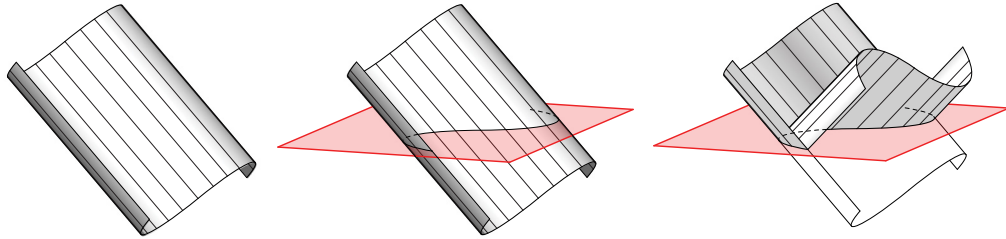(indirect digital representation)

Drawing Open and Closed Curves
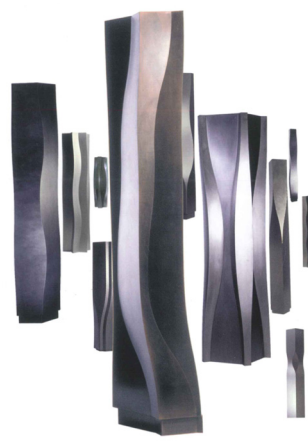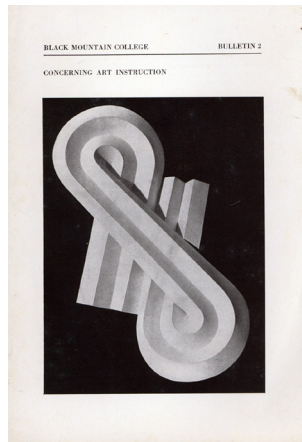(direct analog representation)

Sculpting and Post-rationalizing Surfaces
(indirect analog representation)
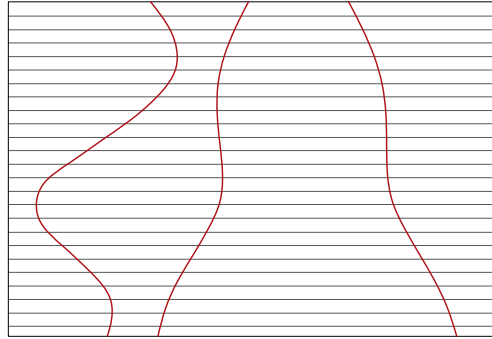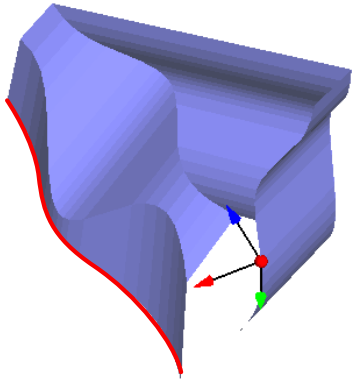
Defining Design Approaches and their Design Representation

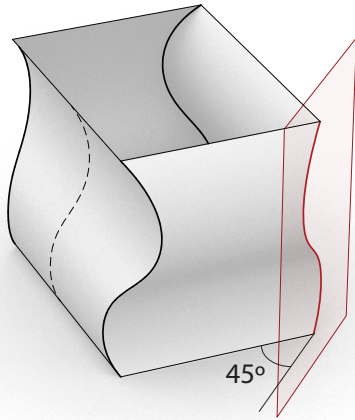# 2 Developable Surface Reflection

*Mirror reflection of general cylinder (DK)*

Work by students of Josef Albers, Black Mountain College (1950s [photographer unknown]),
Bulletin Cover 1944 with model made in Albers' class, Black Mountain College (1944)
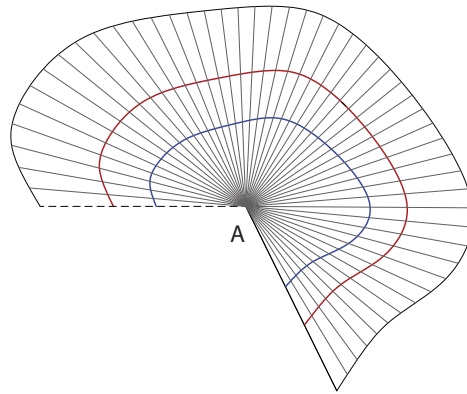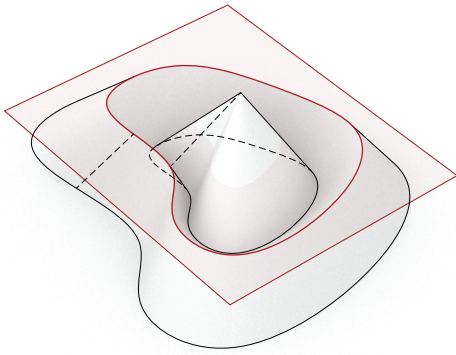*Metal column covers and wall panel (Haresh Lalvani)*

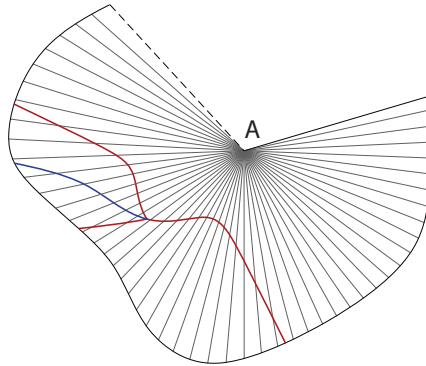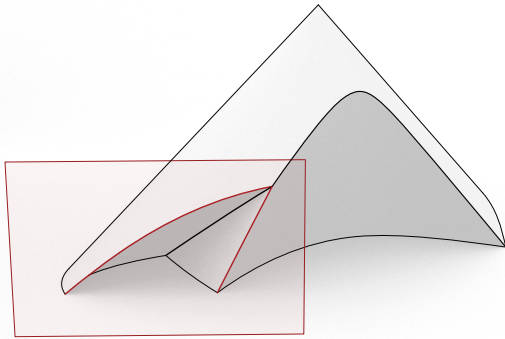*3d model in ORI-REF, crease pattern (Ashley Hickman)*

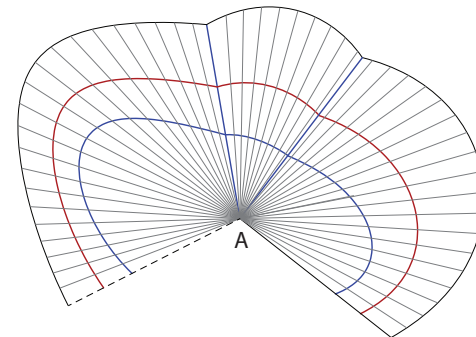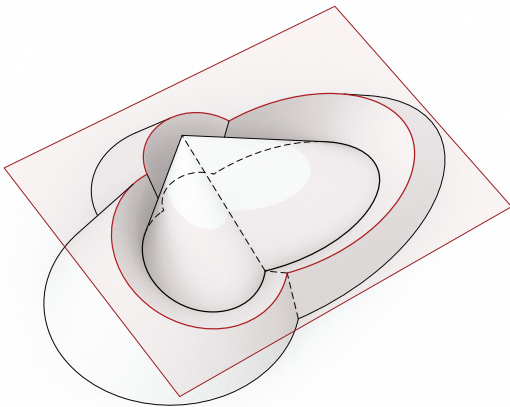*3d model in Rhno3d with reflection plane, crease pattern (Kalliopi Oikonomou)*
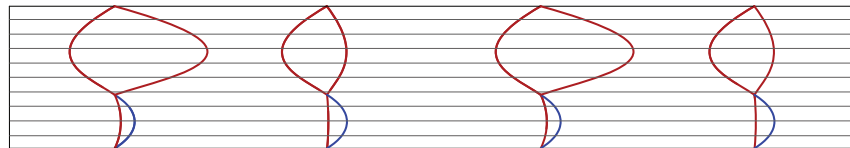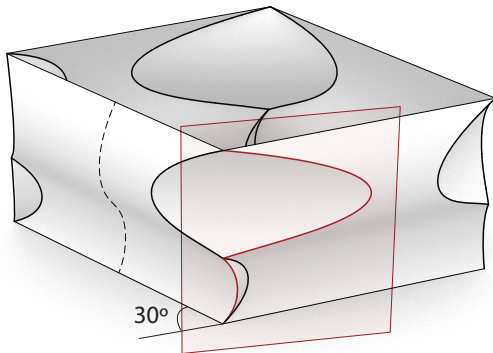
3d model with two reflection planes per corner, crease pattern (DK)

3d model of cone with two horizontal reflections, crease pattern (Ashley Hickman)
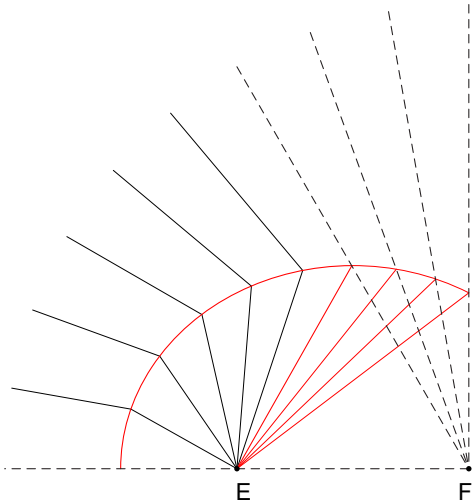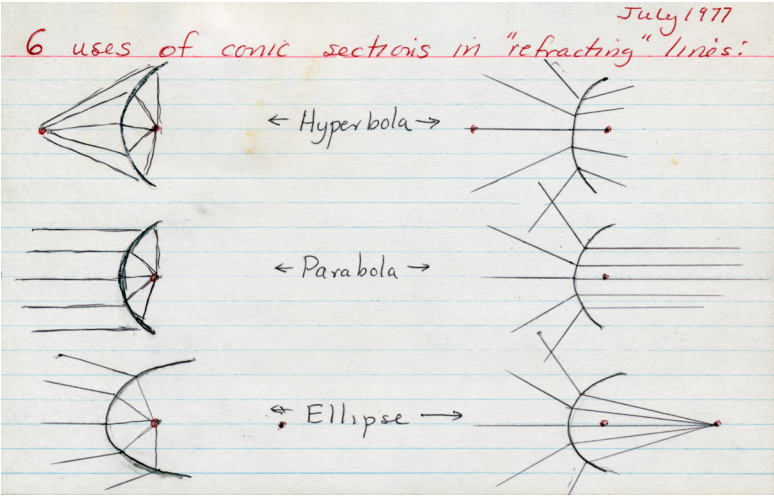

3d model of cone with two vertical reflections, crease pattern (Kalliopi Oikonomou)

3d models of cone reflections with two reflection, crease pattern (Kalliopi Oikonomou)

Refraction Gadgets

Refractive properties of conic sections (1977, DAH, David A. Huffman archive)
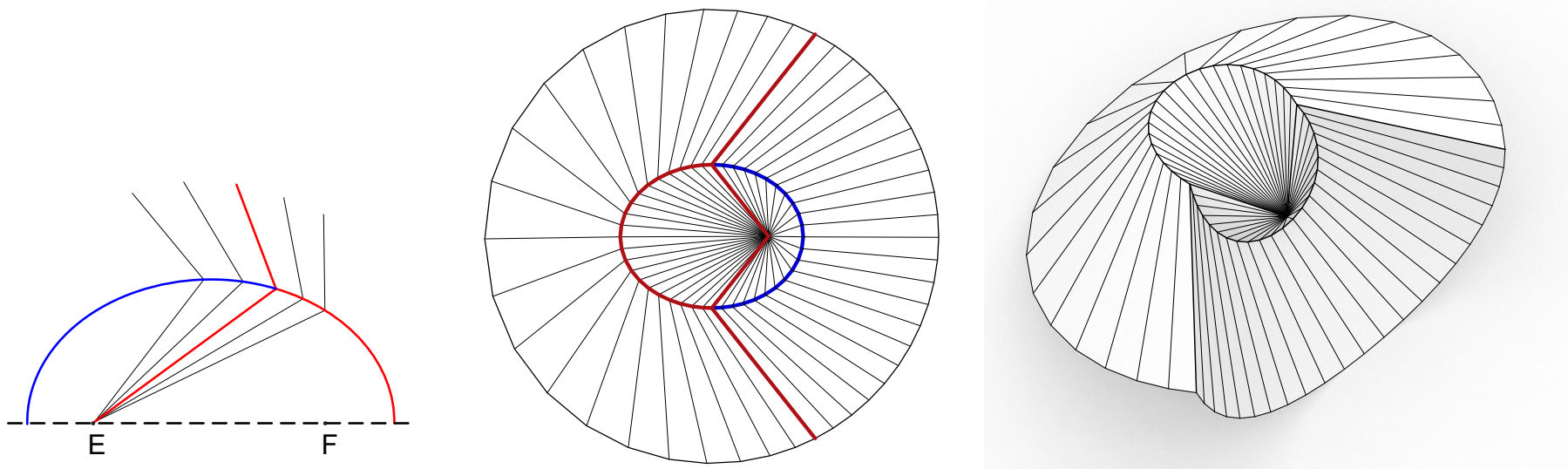*Sketch (undated, DAH, David A. Huffman archive), Gadget (DK)*

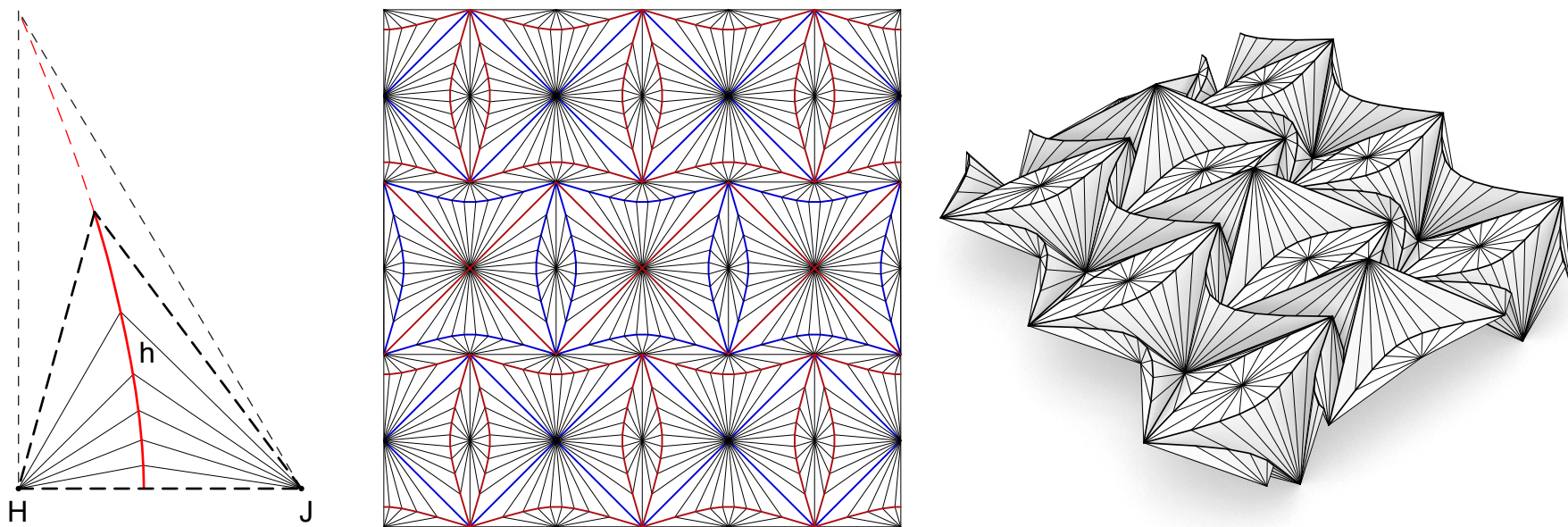*Figure 2.2.4 Ellipse gadget, crease pattern, simulation (Kalliopi Oikonomou)*

Figure 2.2.6 Simulations of gadgets with parabolas and hyperbolas, crease patterns (Kalliopi Oikonomou)
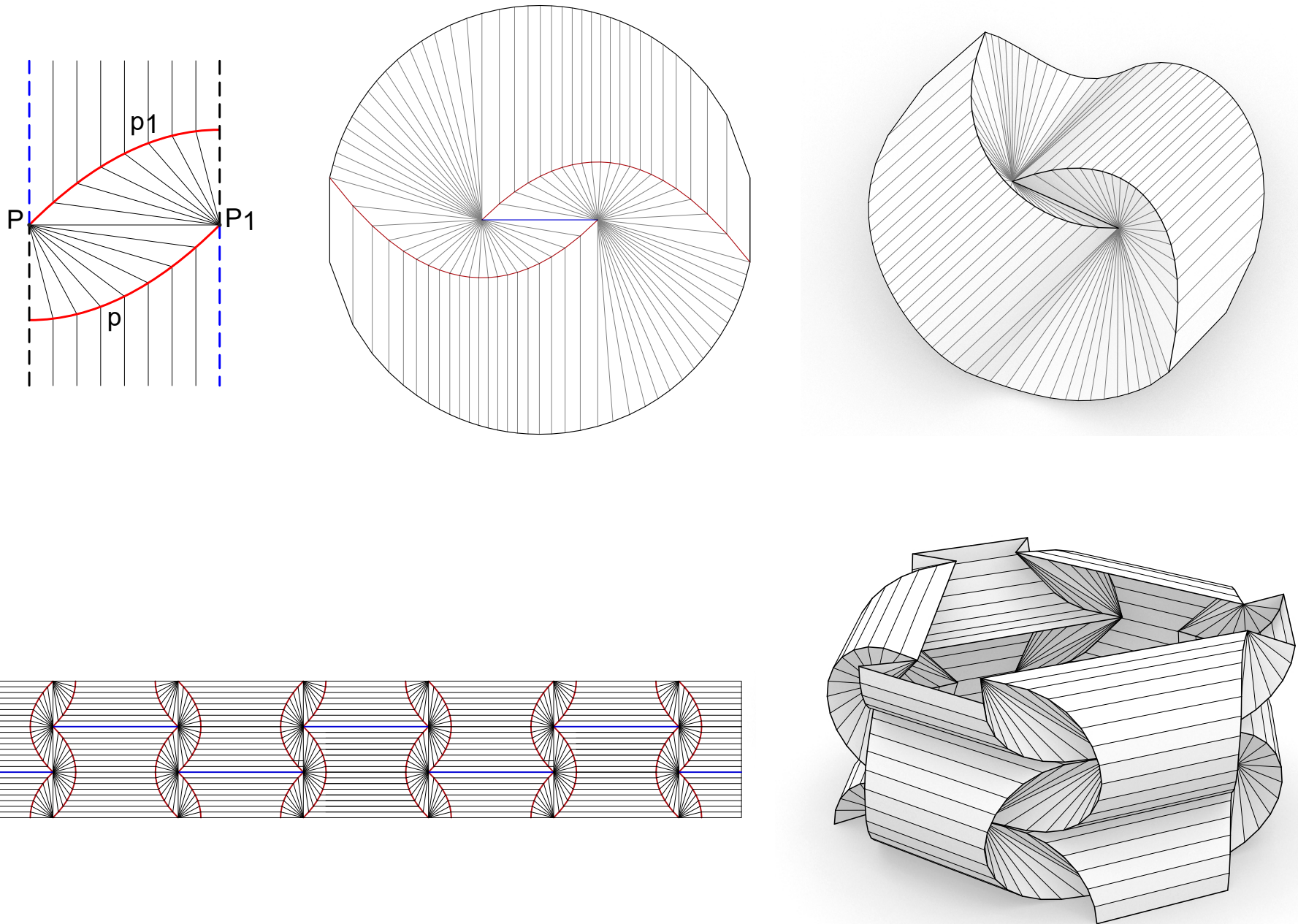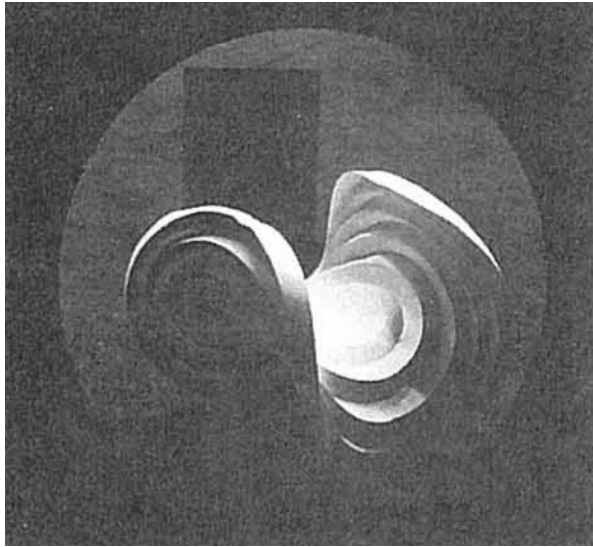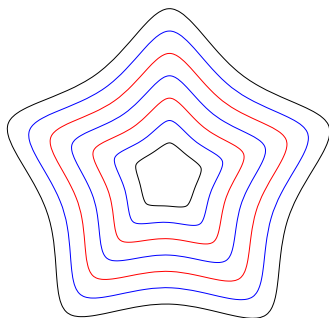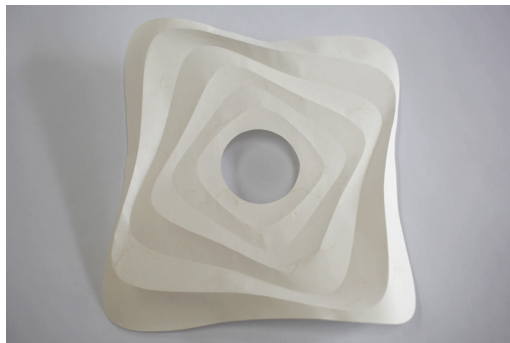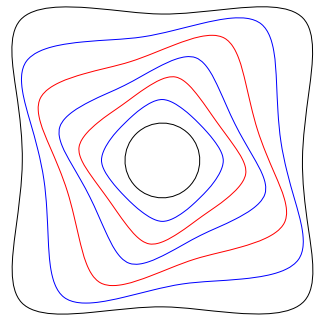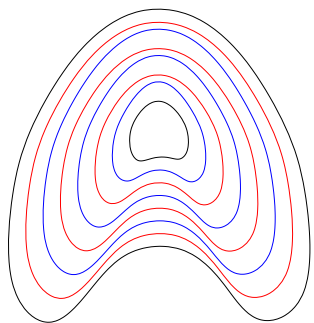
Figure 2.2.5 Simulations of gadgets with parabolas and hyperbolas, crease patterns (Kalliopi Oikonomou)
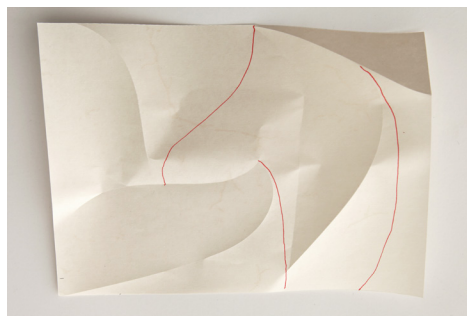
Drawing Closed and Open Curves

Bauhaus model, extended discs (Josef Albers Foundation), Horse head (Roy Iwaki)

Designs with self-closing curves with varying degrees of curvature (Aleksandra Chechel)
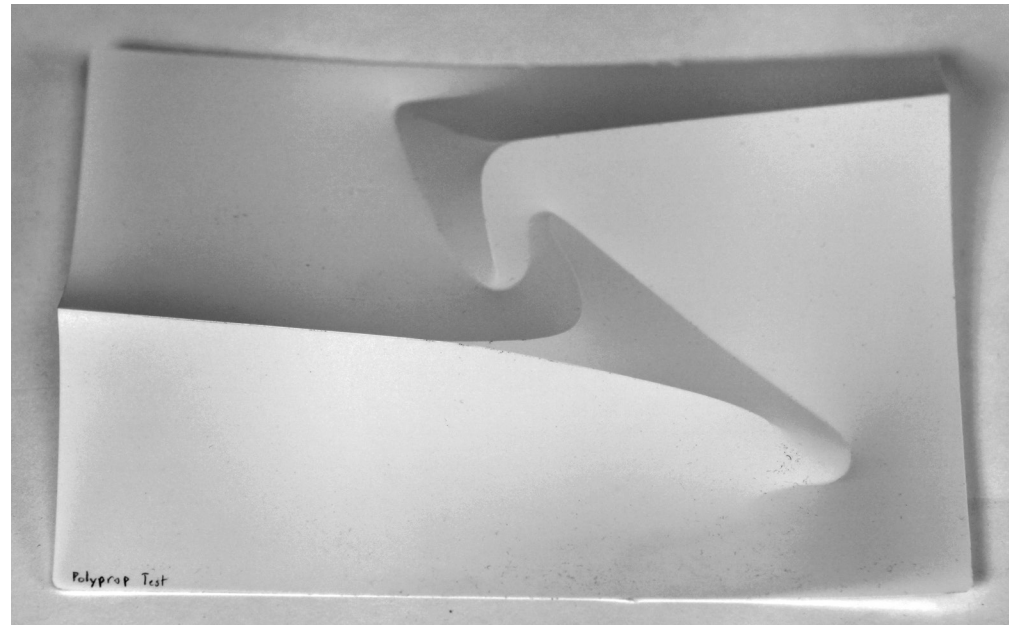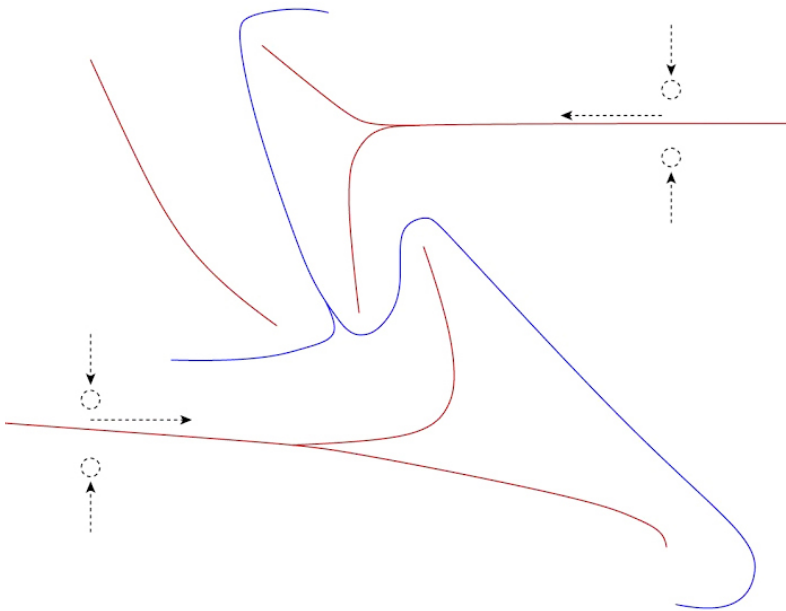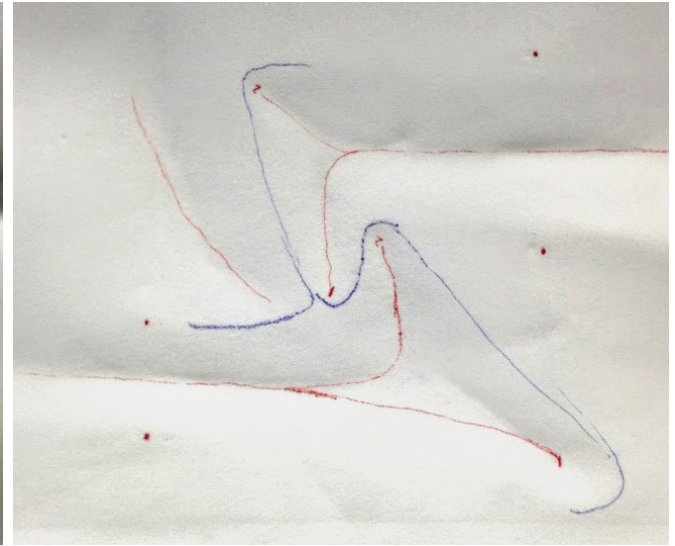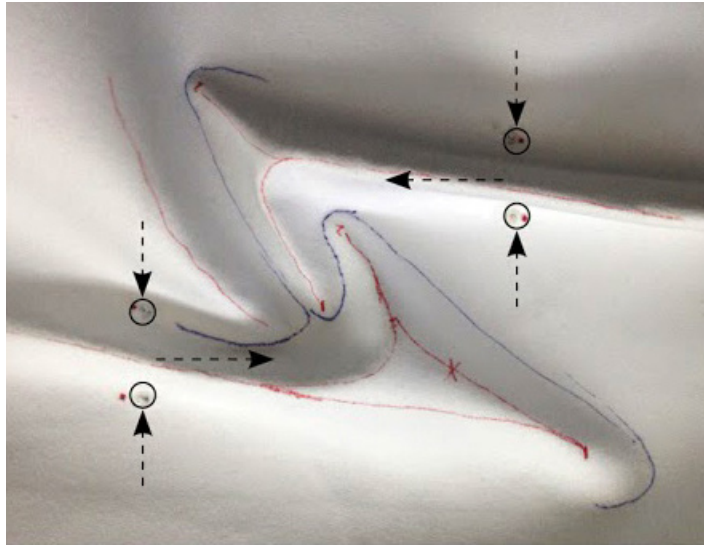

Iterations of a design made with intersecting curves (Ashley Hickman)

Sculpting and Post-rationalizing Surfaces

Fig 2.4.1 Images from The Ron Resch Paper and Stick Film (1968, Ron Resch)

Sculpting, Notation of pinching, Crease pattern
Digitized crease pattern, Paper model (UnJae Pyon, Lauren Greer)