SODA 1996

Chapter 21

The Complexity of Flat Origami

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Abstract

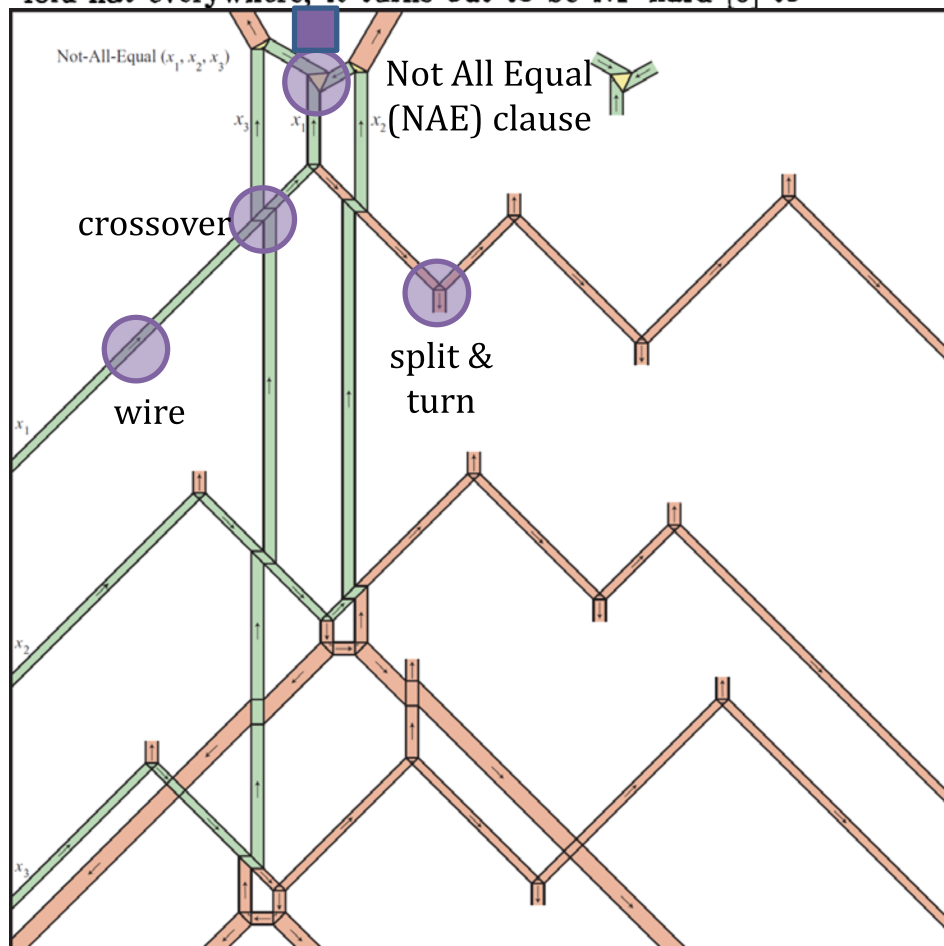
We study a basic problem in mathematical origami: determine if a given crease pattern can be folded to a flat origami. We show that assigning mountain and valley folds is NP-hard. We also show that determining a suitable overlap order for flaps is NP-hard, even assuming a valid mountain and valley assignment.

1 Introduction

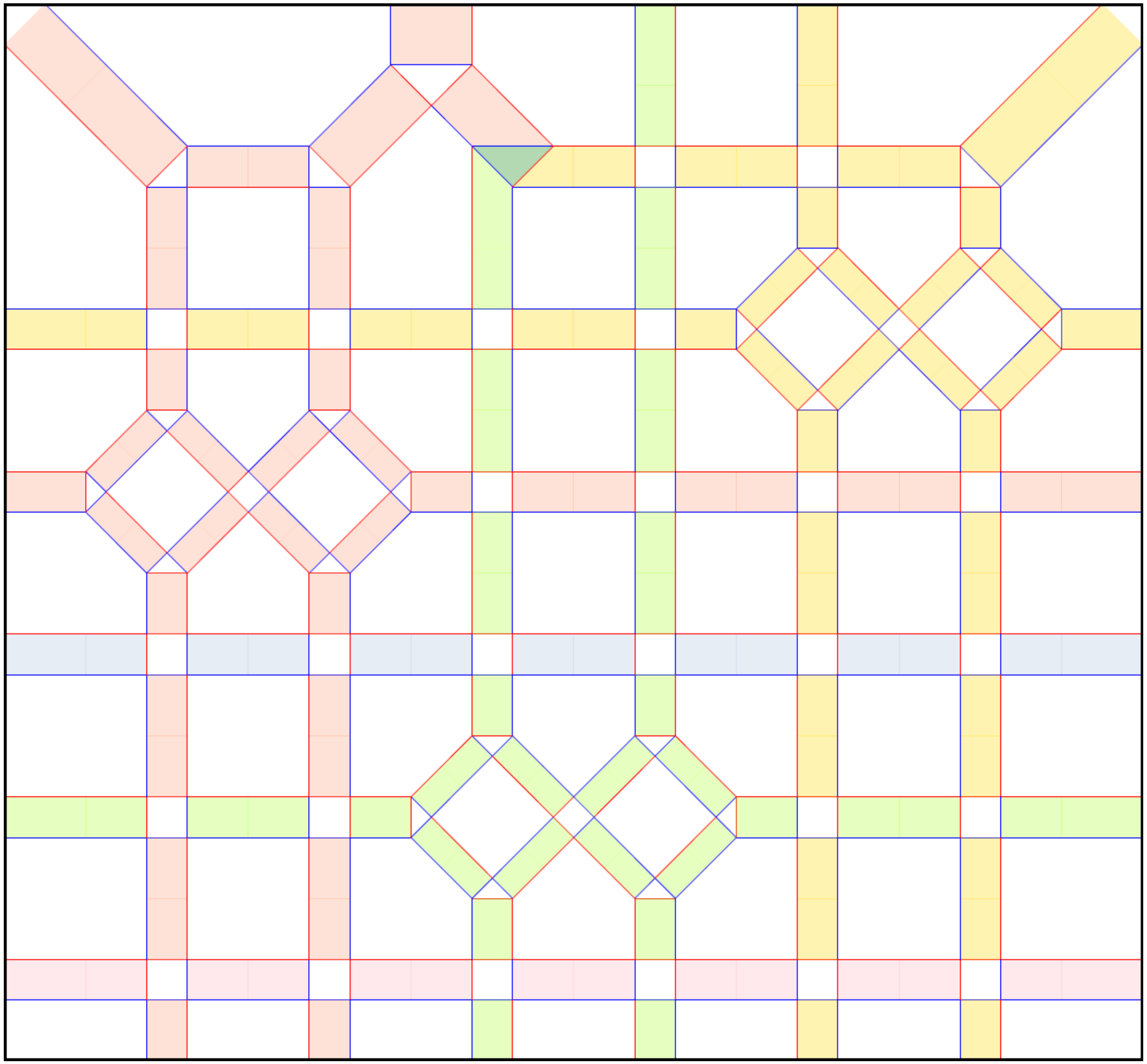
Origami, the centuries-old art of folding paper into sculpture, is currently enjoying a renaissance. Contemporary origami artists invent new models of great beauty and intricacy. To achieve these stunning results, artists such as Engel, Fuse, Lang, and Maekawa have taken a geometric approach to origami design. One useful technique, incorporated into Lang's *TreeMaker* program, uses the centers of non-overlapping disks to determine the tips of "flaps" [10, 11]. Another technique [1, 12] builds complicated crease patterns out of repeating blocks called "molecules".

Alongside this "technical" approach to origami design, some mathematicians have started to study origami. Huffman [5] gives relations between face angles in polyhedral models, using an approach related to network flow. A number of authors [4, 6, 7, 9] have discovered angle conditions for an origami to fold flat in the neighborhood of a single vertex. In this paper, we extend this study to "global flat foldability". Although

"mountain" and "valley" orientations and determines an overlap order for flaps. If we require that the origami fold flat everywhere, it turns out to be NP-hard [3] to



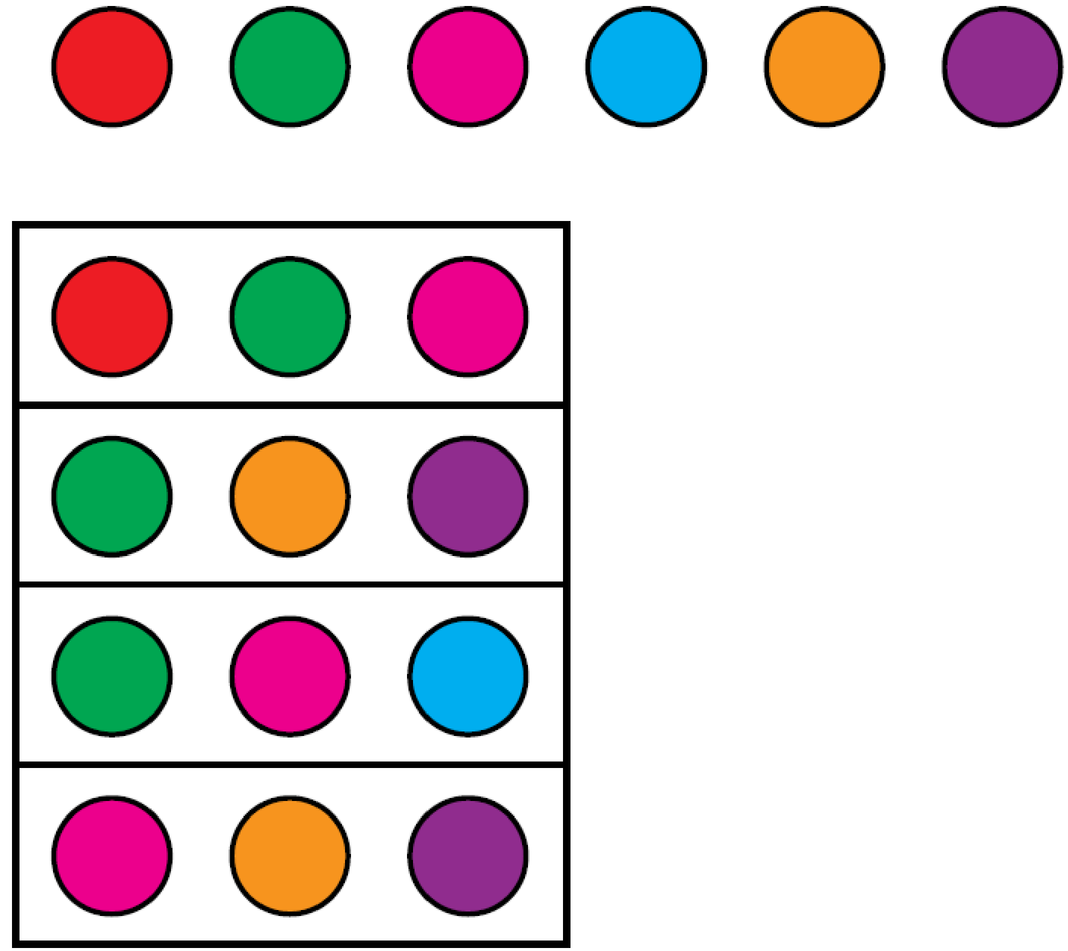
Flat Folding of Box-Pleated Crease Pattern is NP-complete



[Akitaya, Cheung, Demaine, Horiyama, Hull, Ku, Tachi, Uehara 2015]

Not-All-Equal 3SAT

- Given **variables** & **triples** of variables
- Assign binary values to variables so that every triple has **both** values
- **NP-hard**



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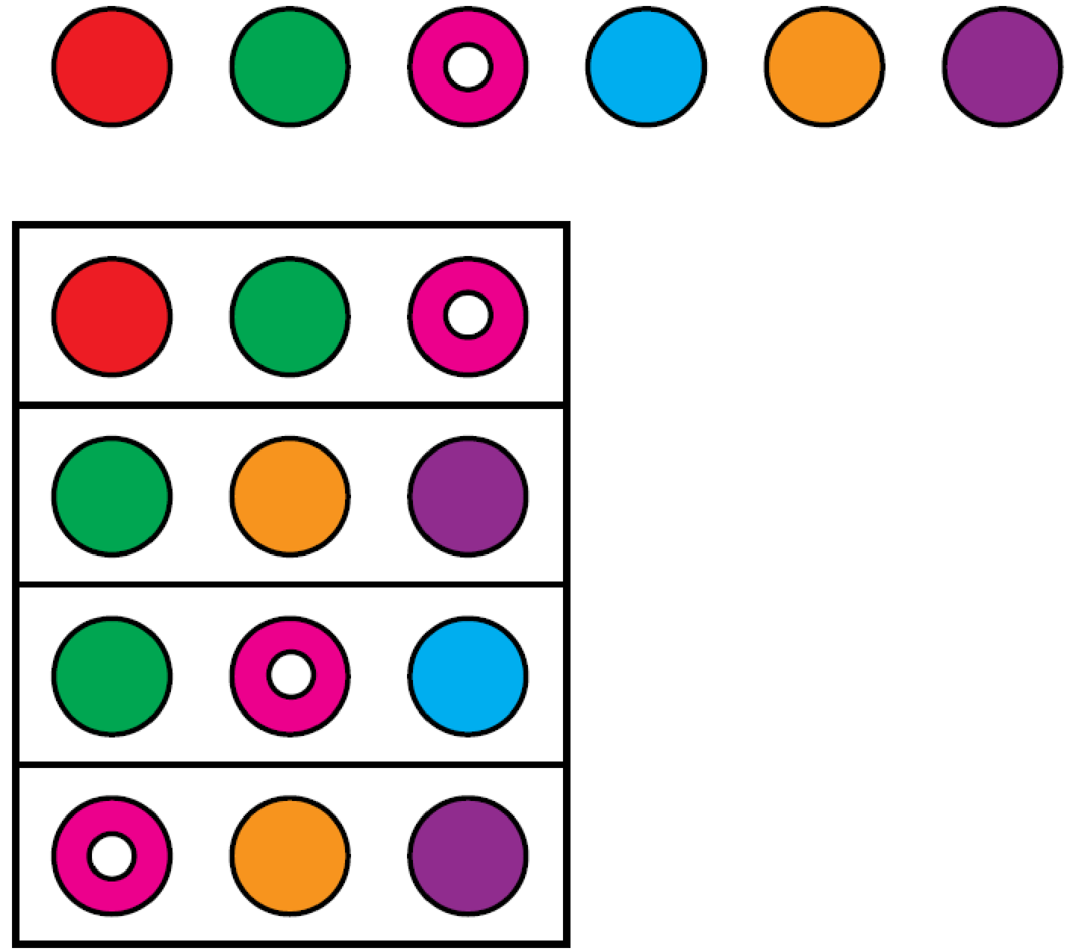

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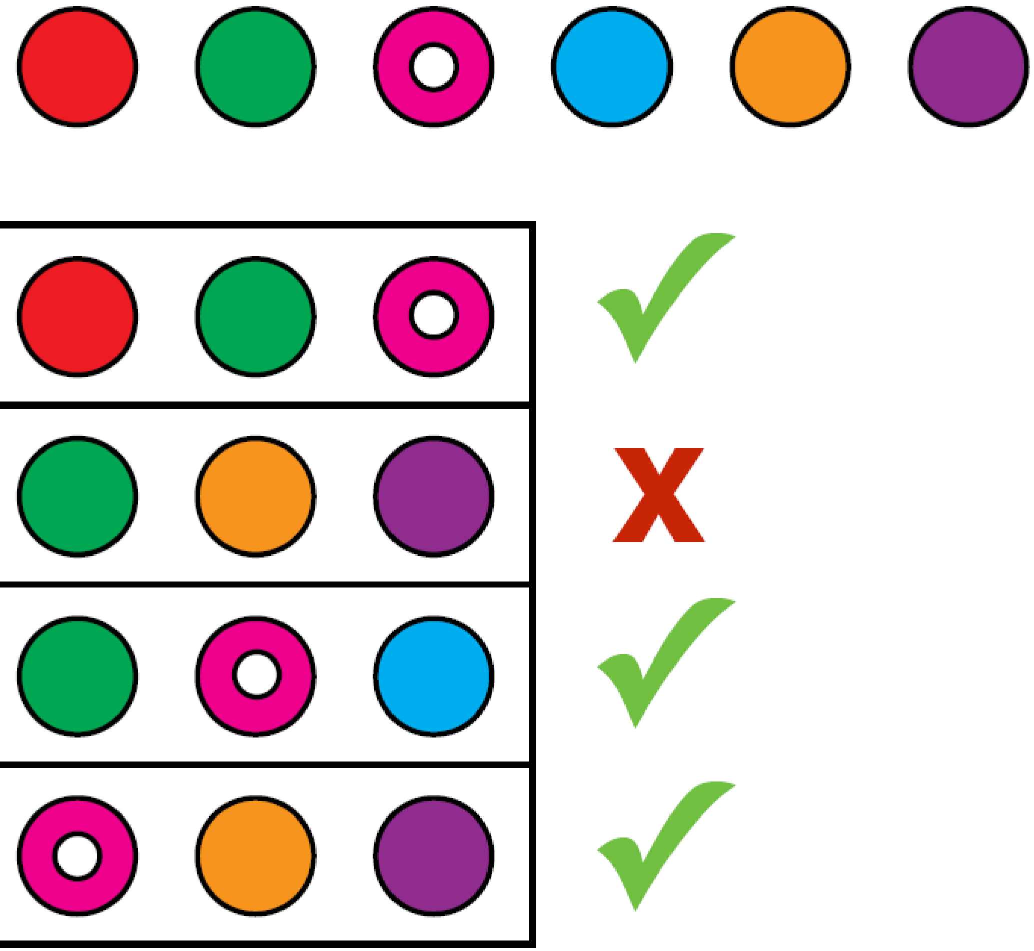

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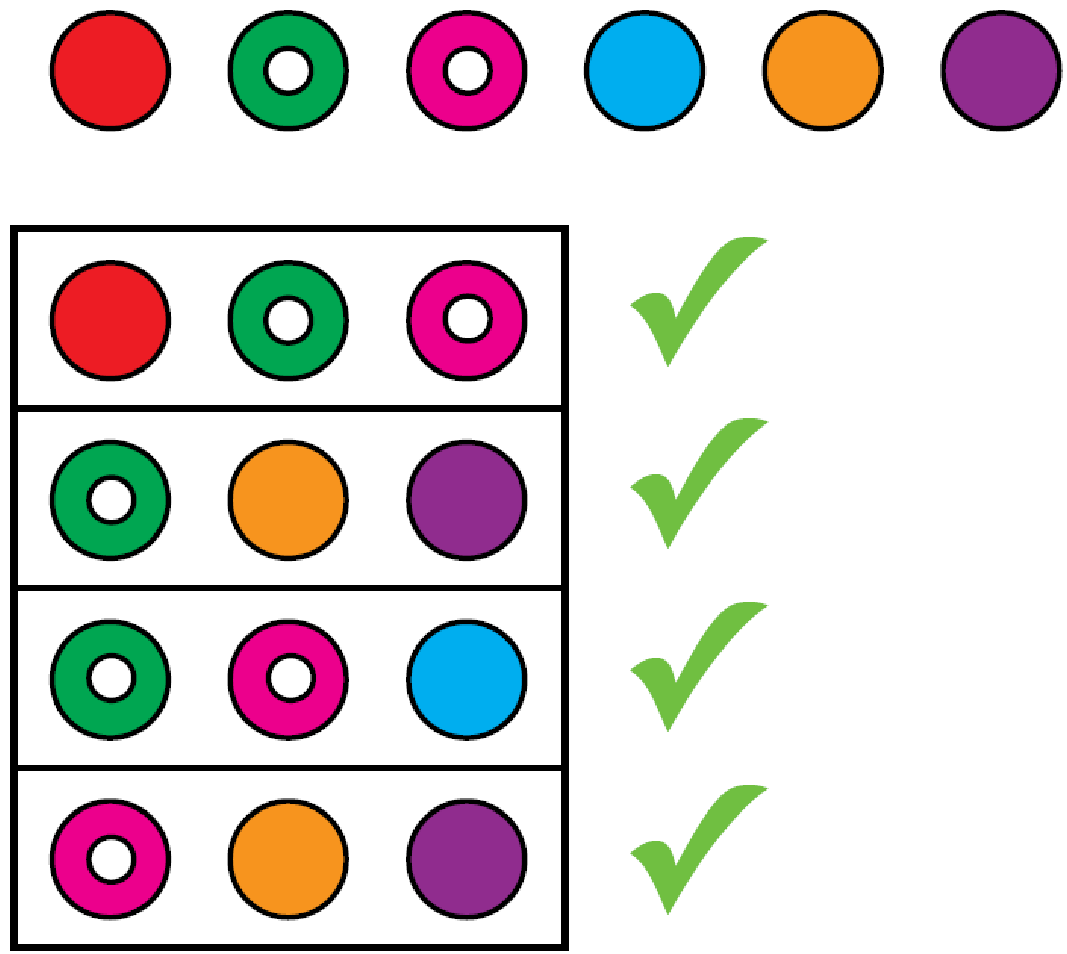

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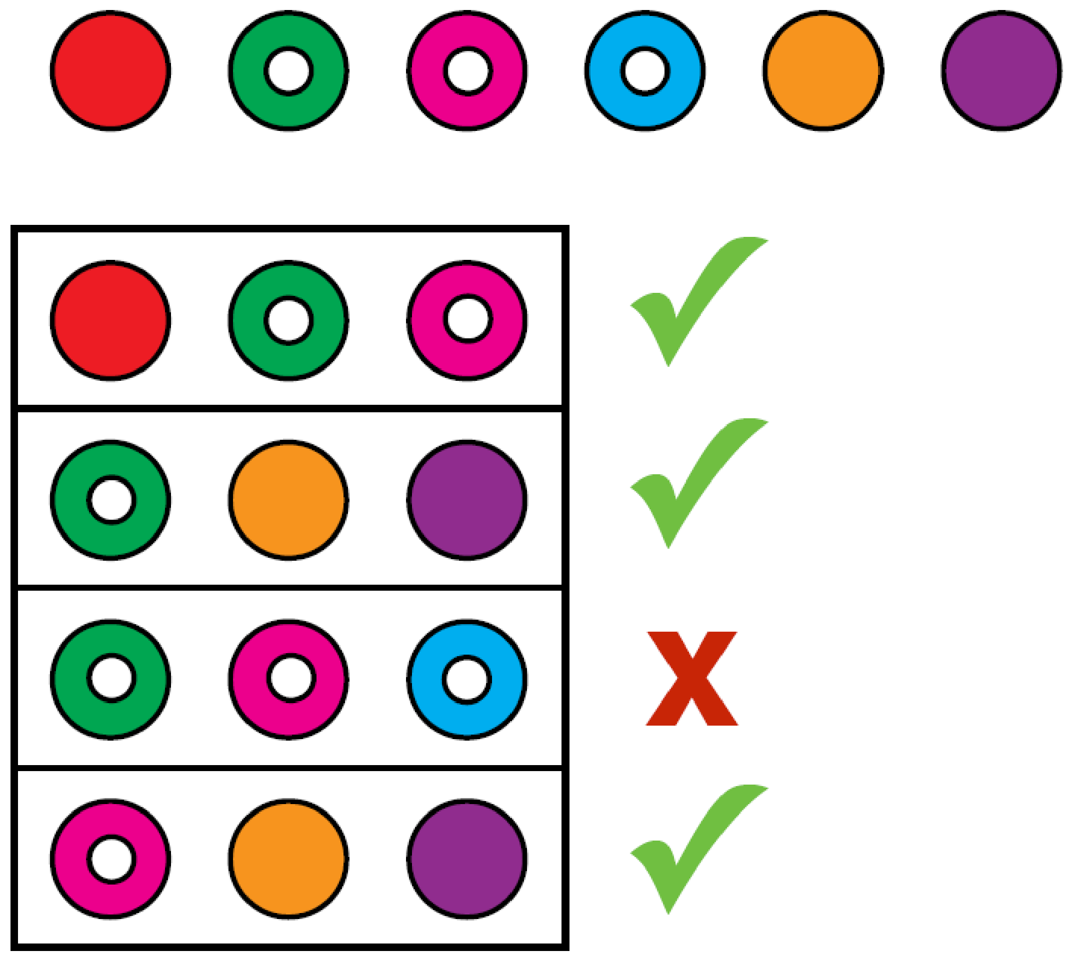
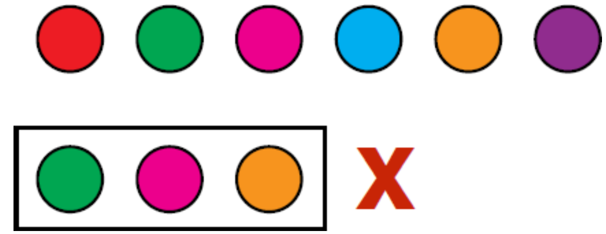
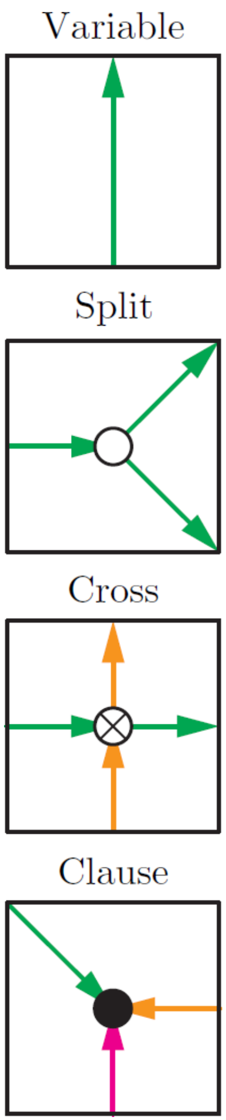
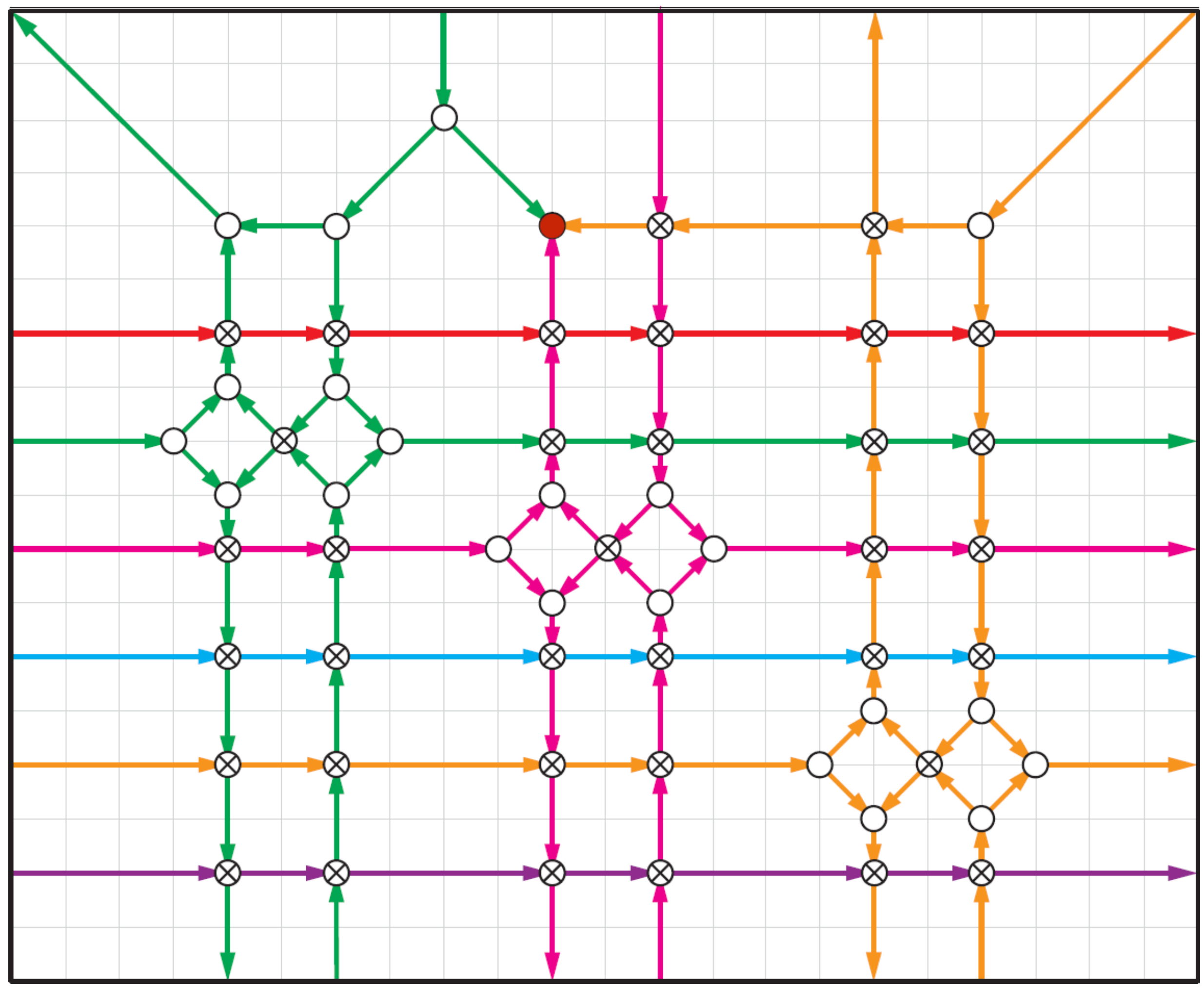


Graph Orientation







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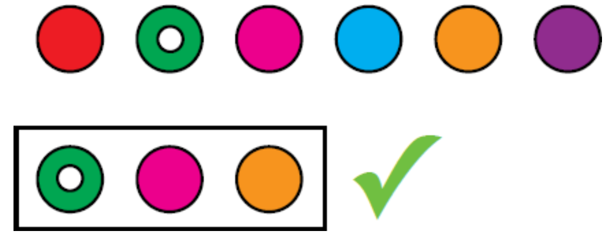

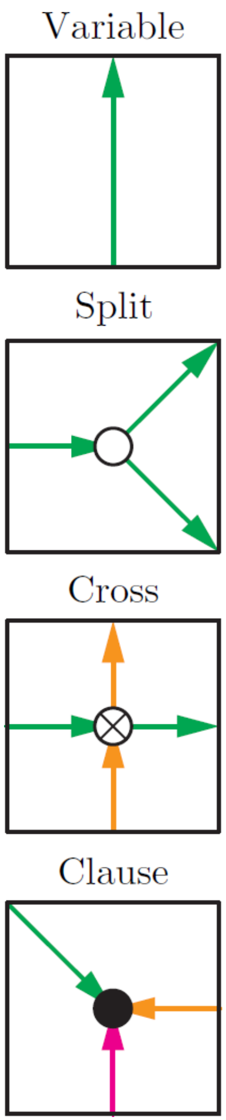

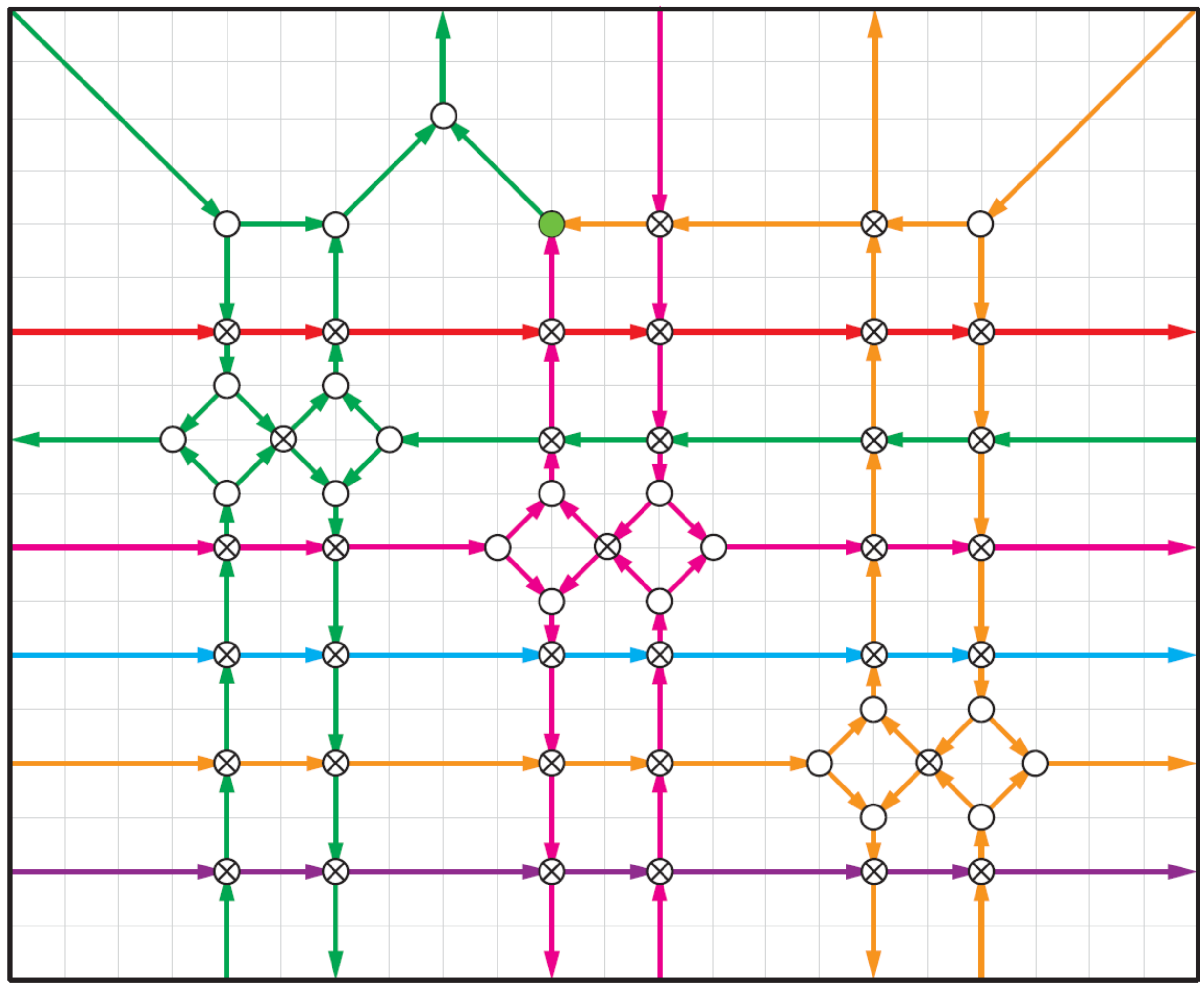

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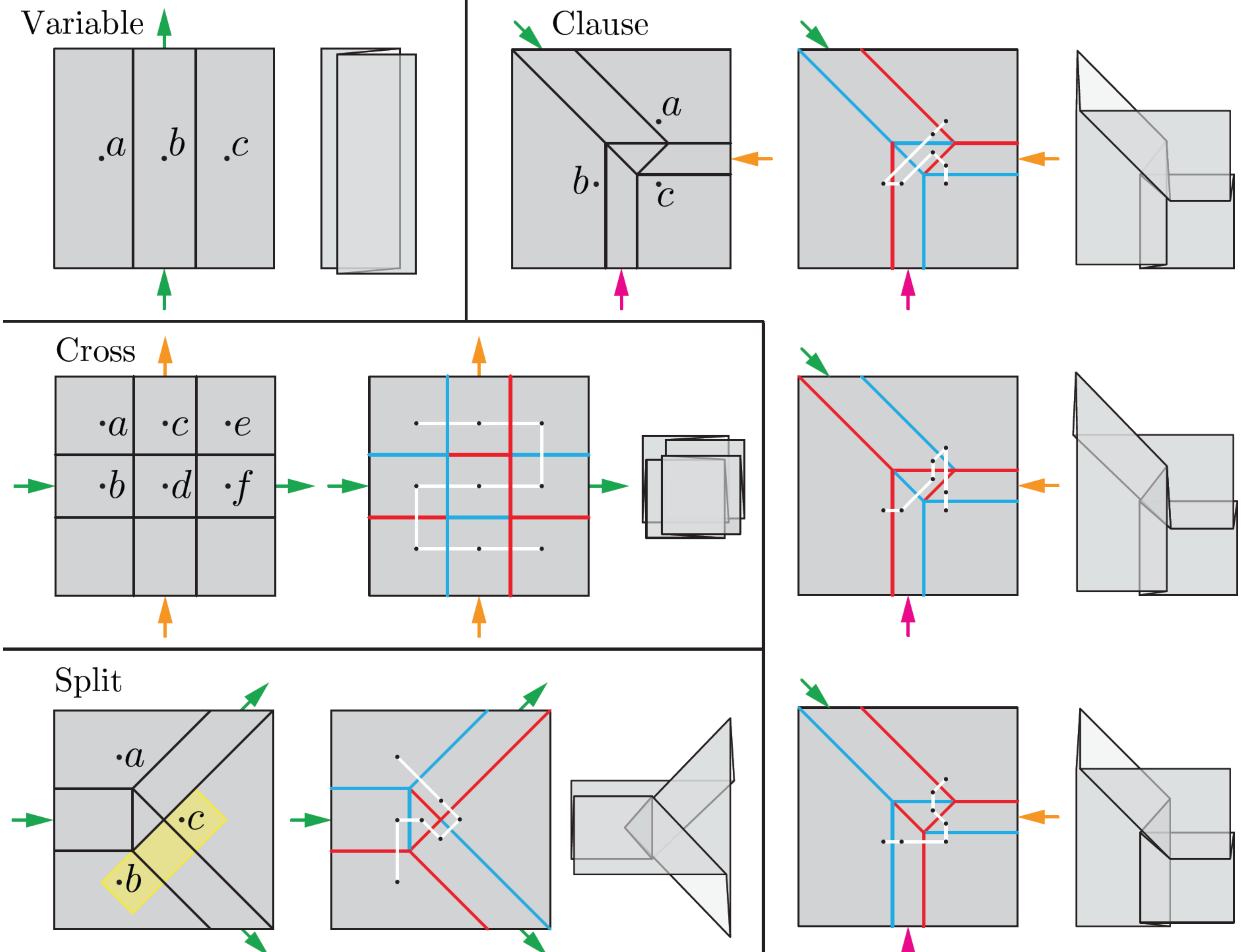

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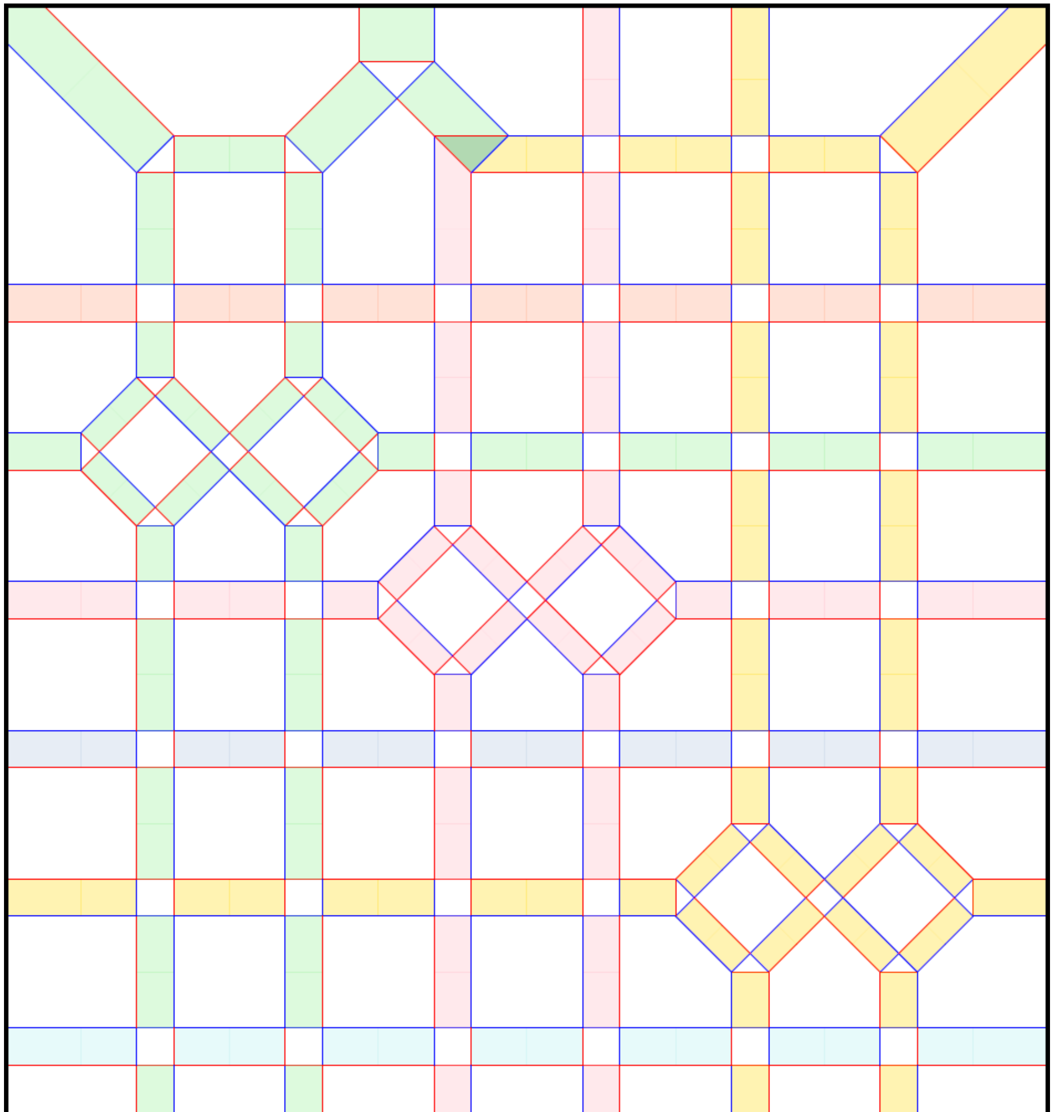

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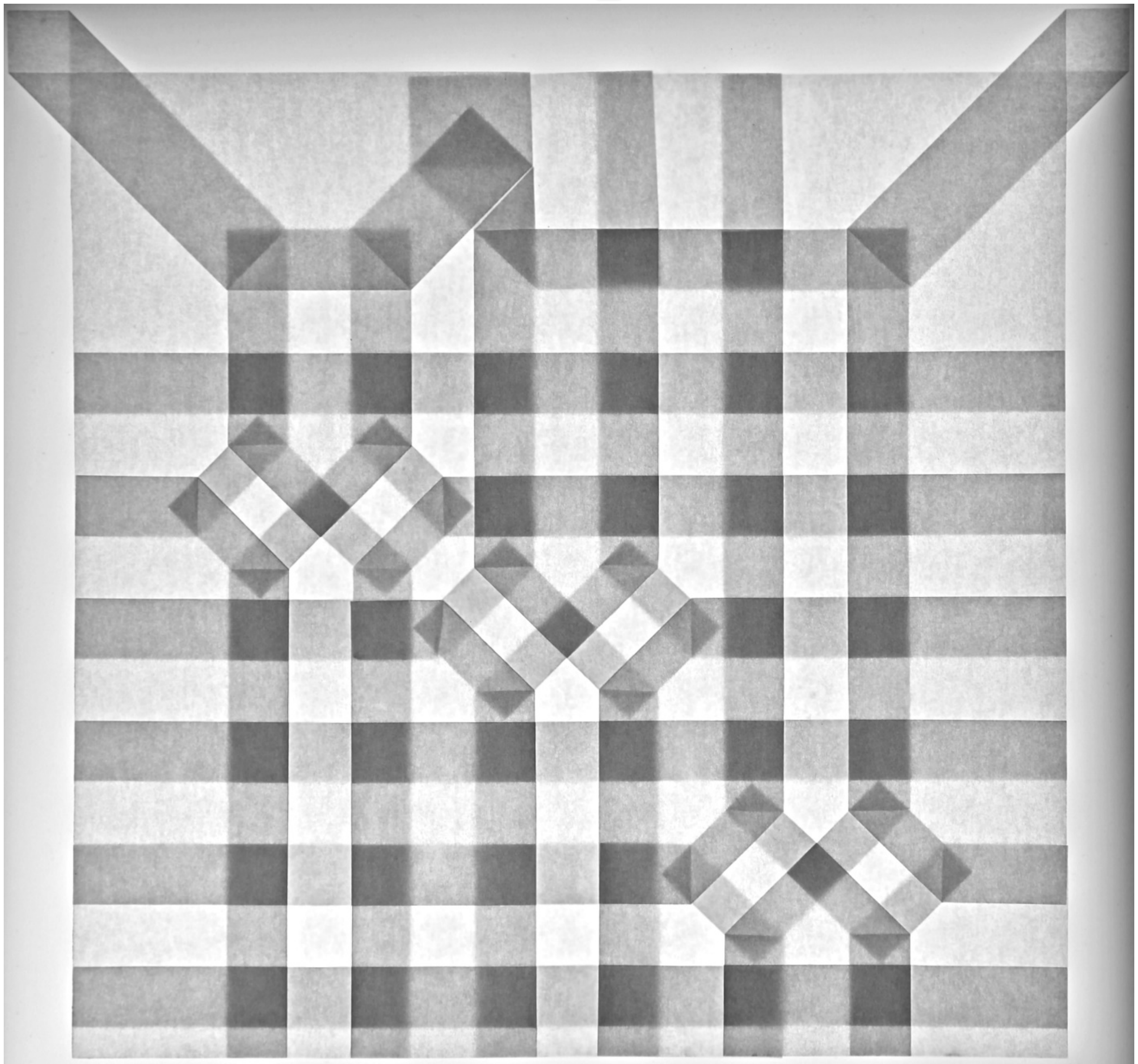

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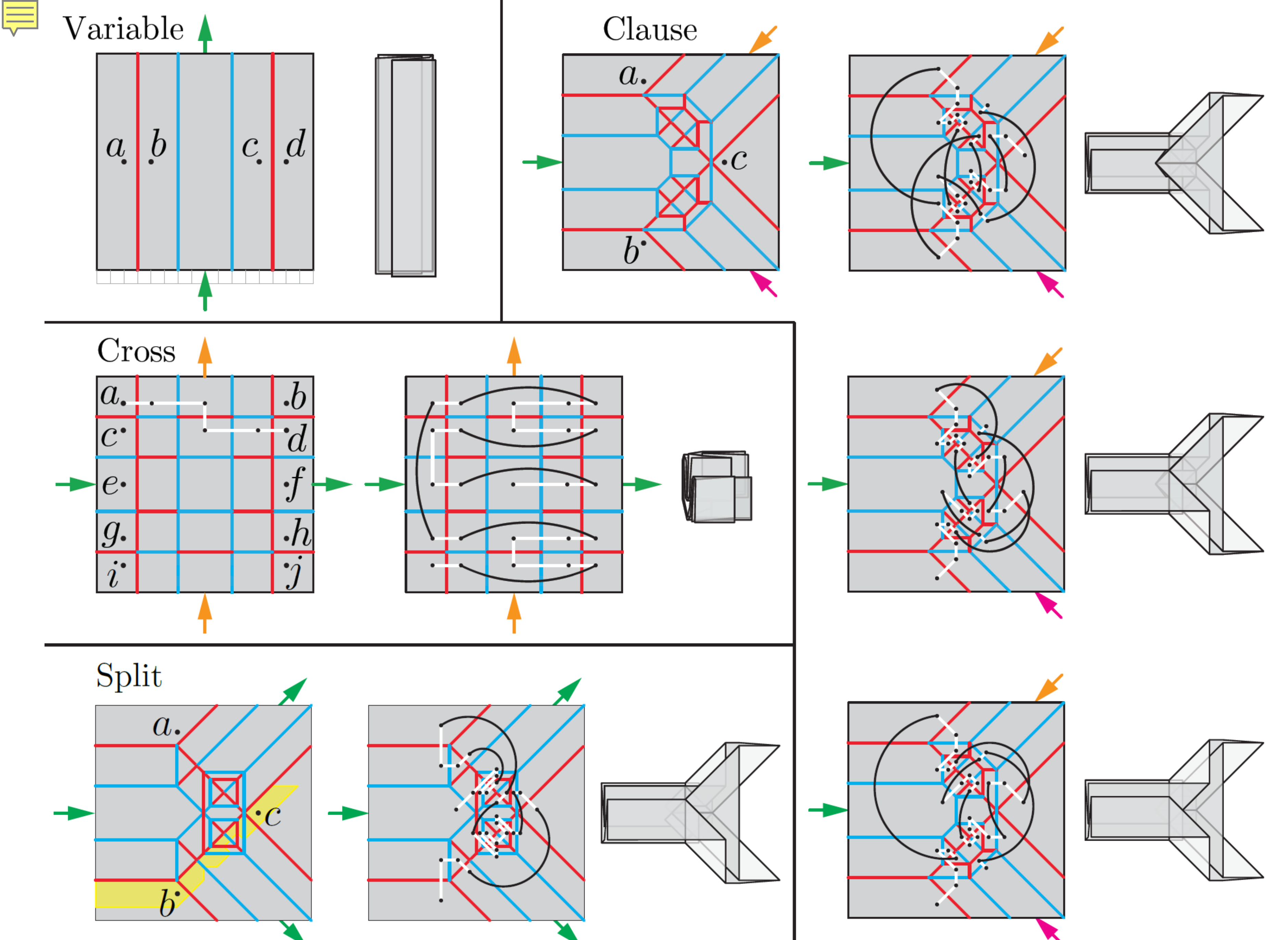

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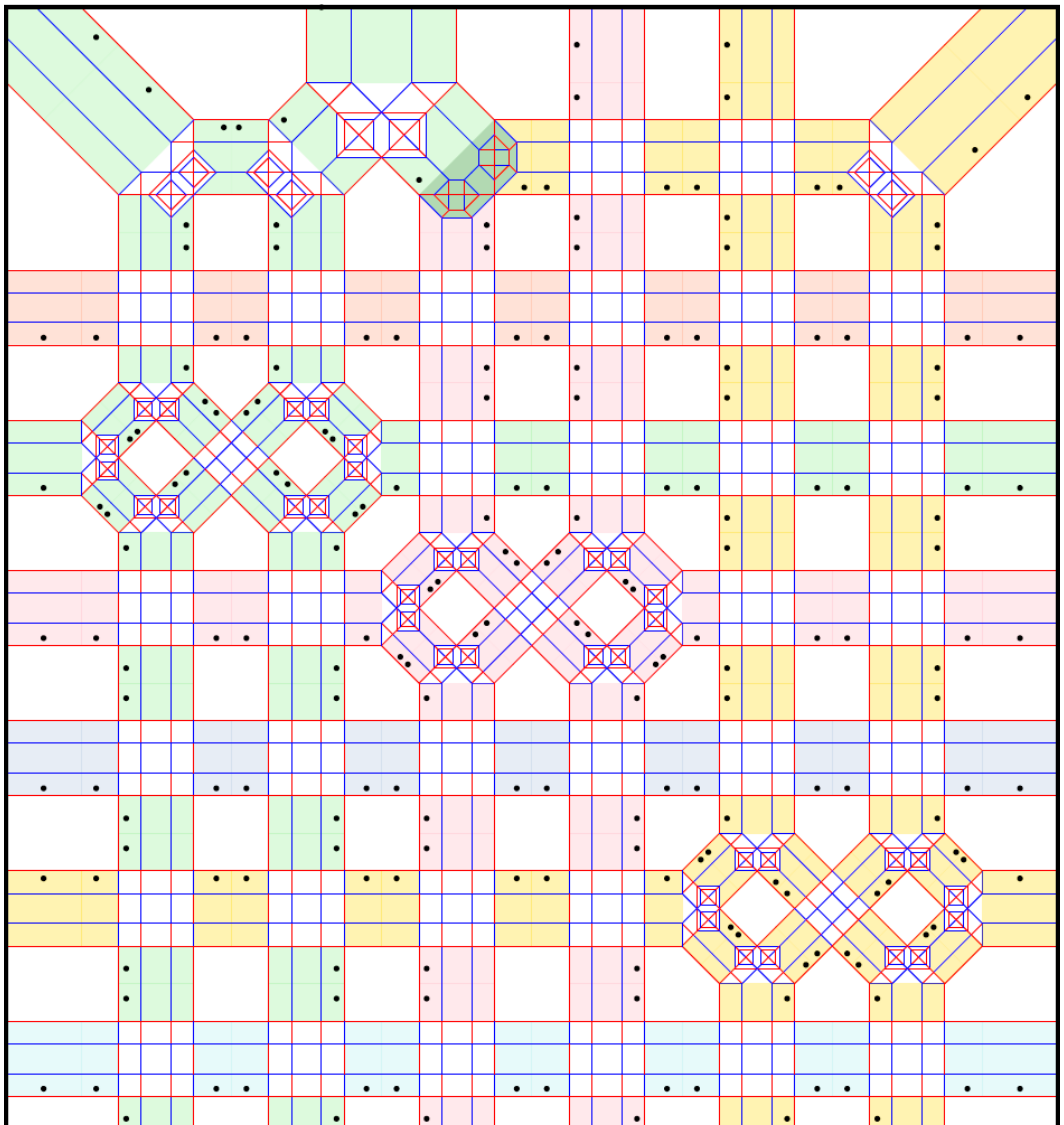
Flat Folding of Box-Pleated Assigned Crease Pattern is NP-complete



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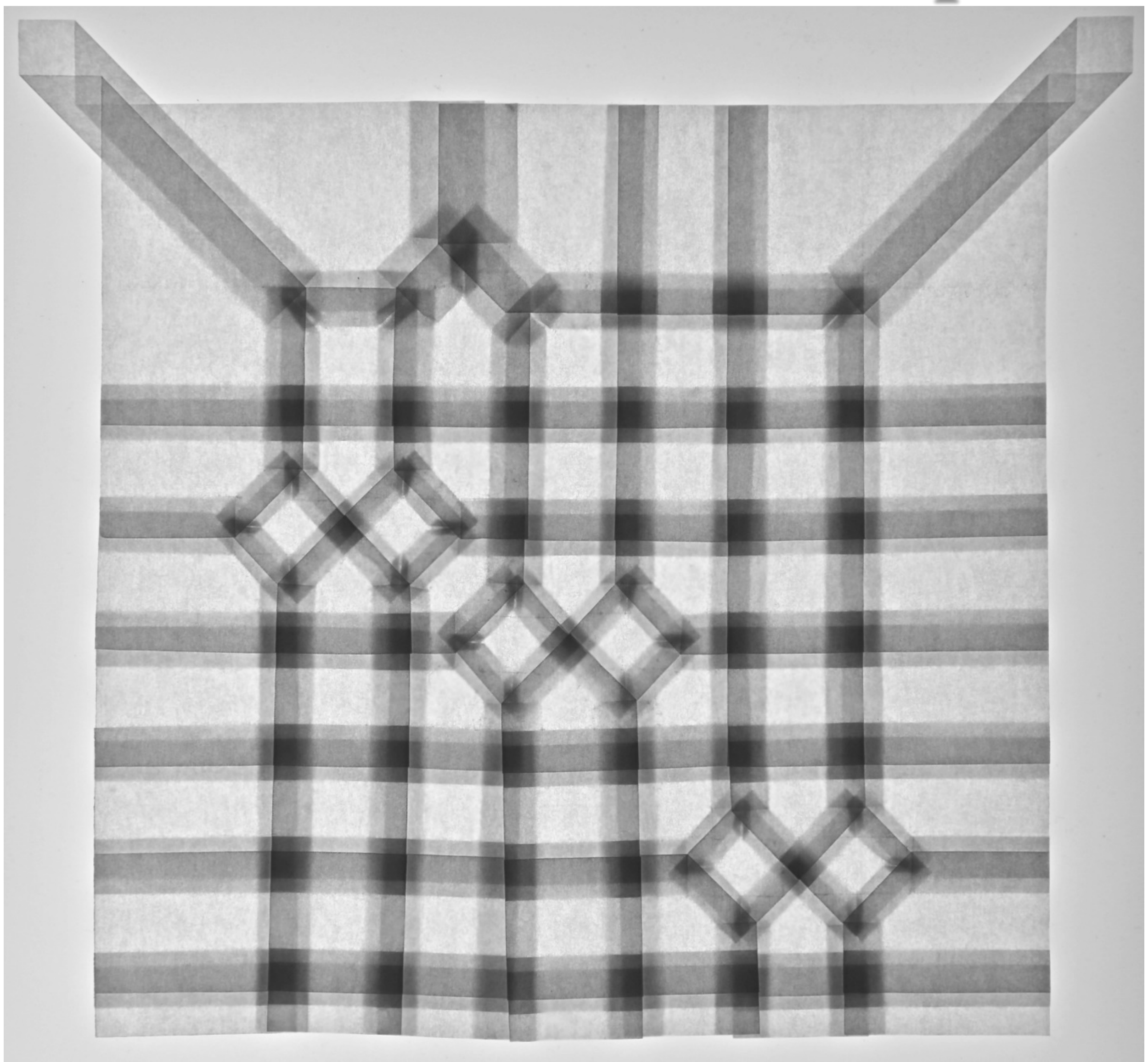

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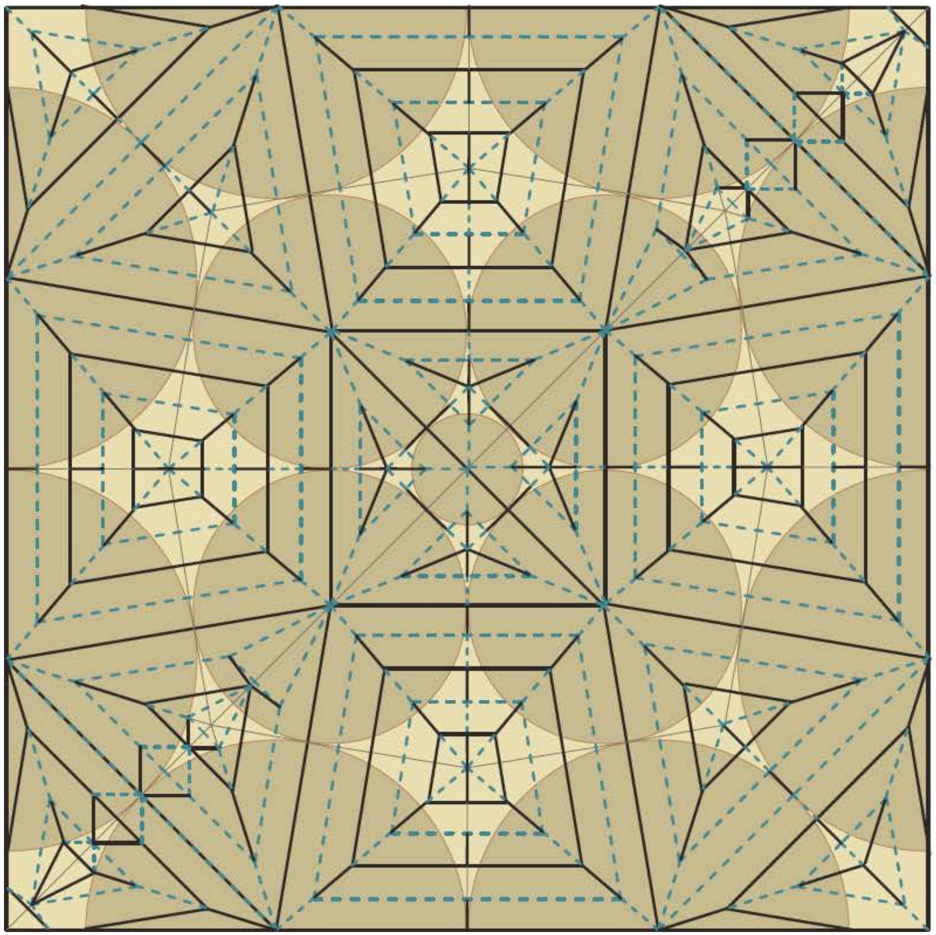
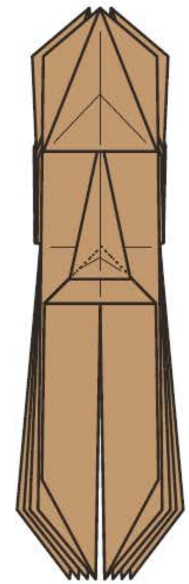
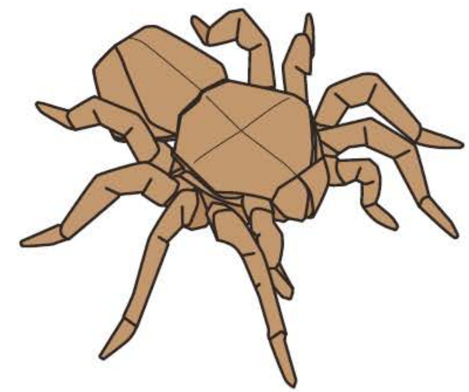
Disk Packing for Origami Design



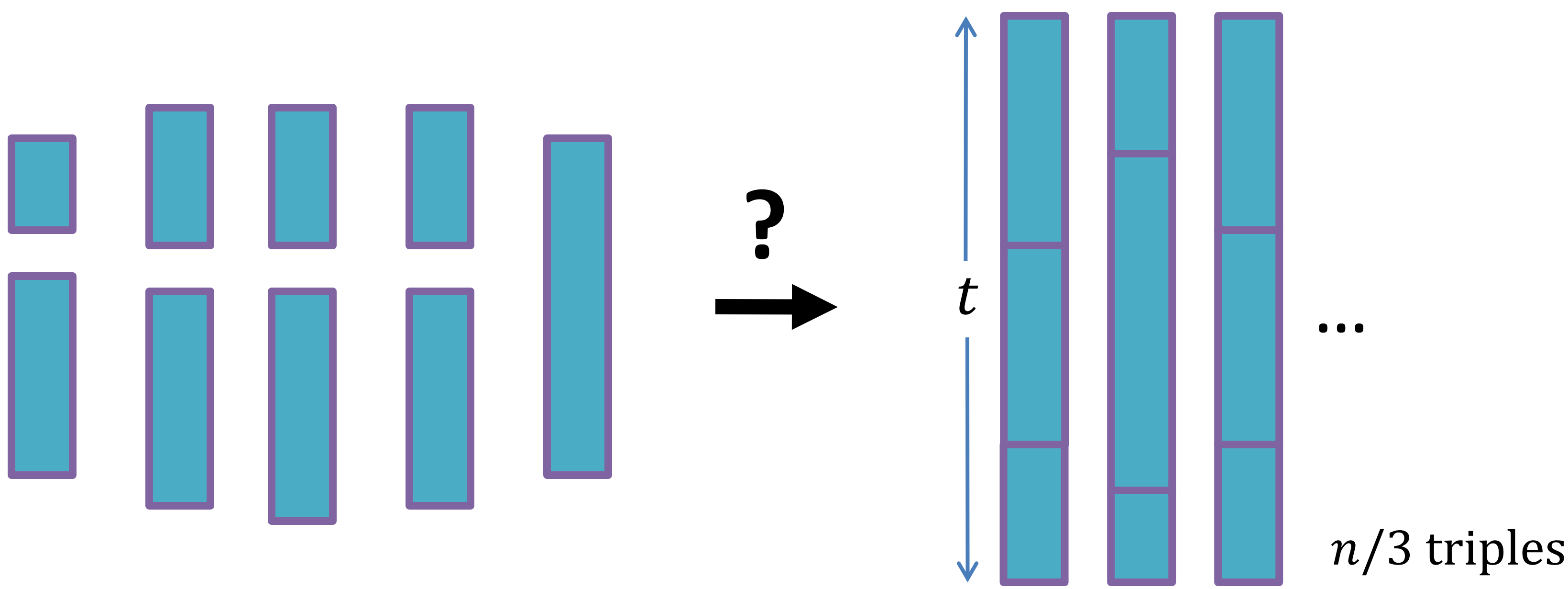




Packing given disks into a given square is **NP-hard**
[Demaine, Fekete, Lang 2010]

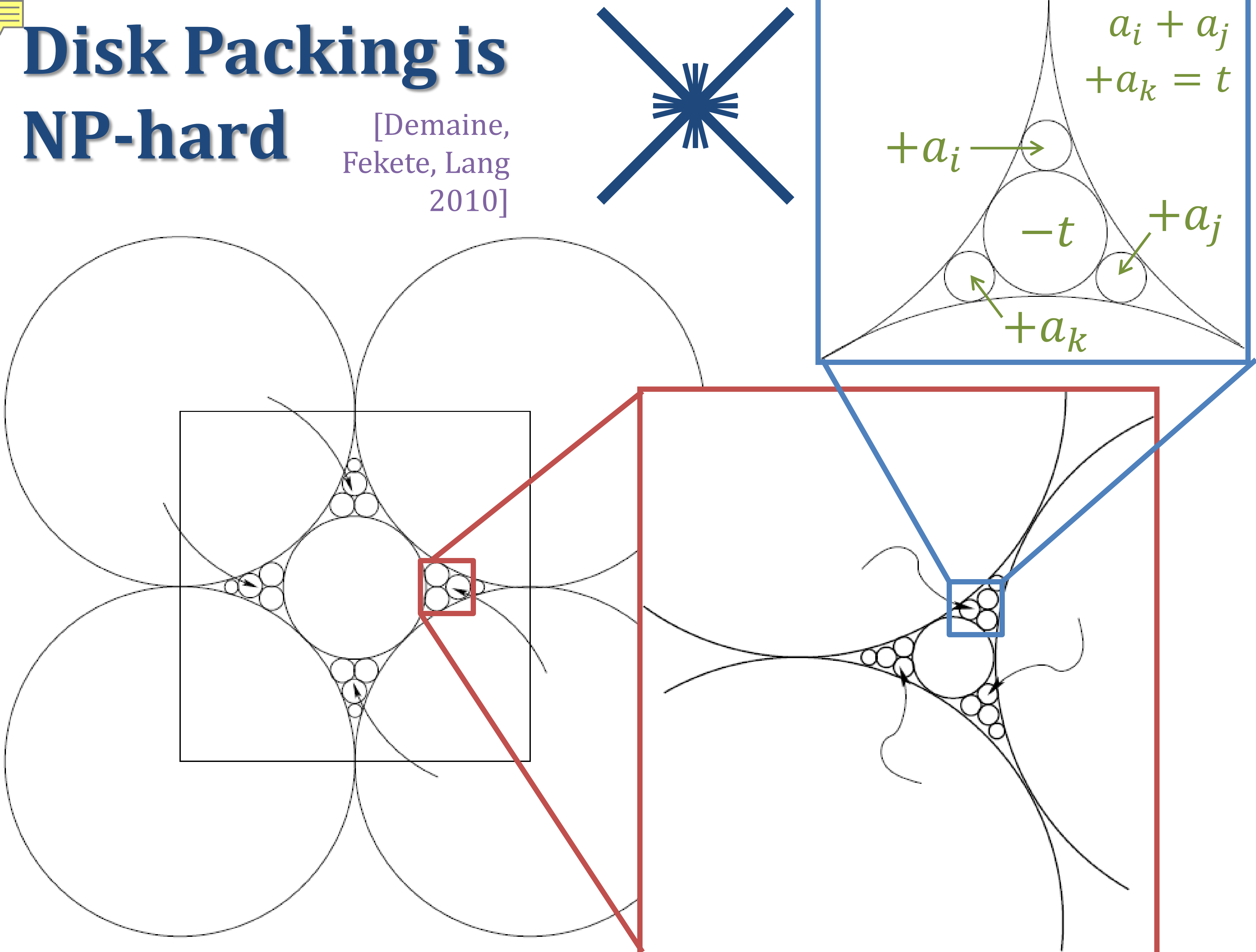
3-Partition

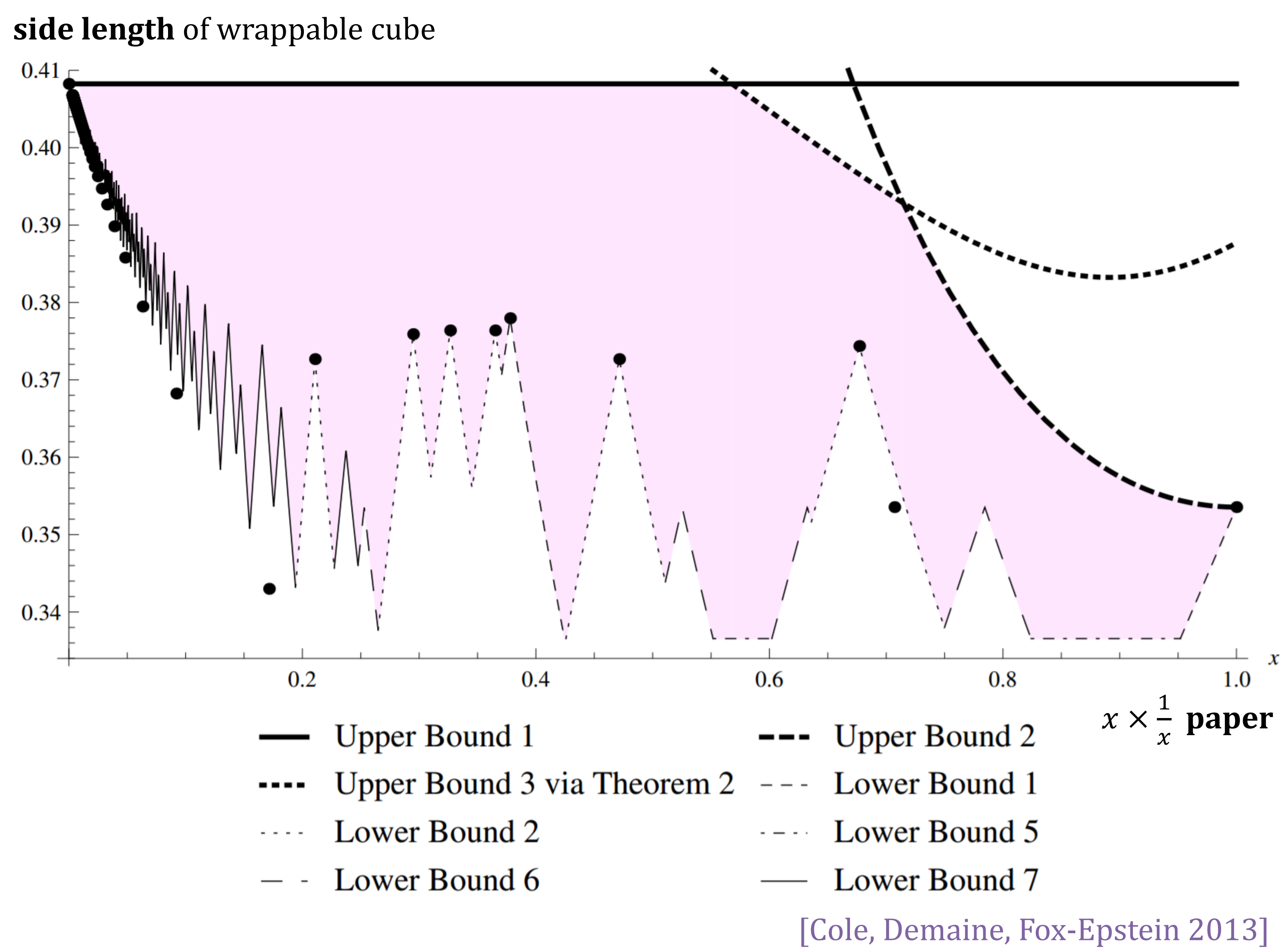
- Given n integers $a_1, a_2, \ldots, a_n$, can you partition into $n/3$ triples with the same sum? $\left(t = \frac{\sum_i a_i}{n/3}\right)$
- This problem is **strongly NP-complete**: NP-complete even if a_i numbers are $n^{O(1)}$

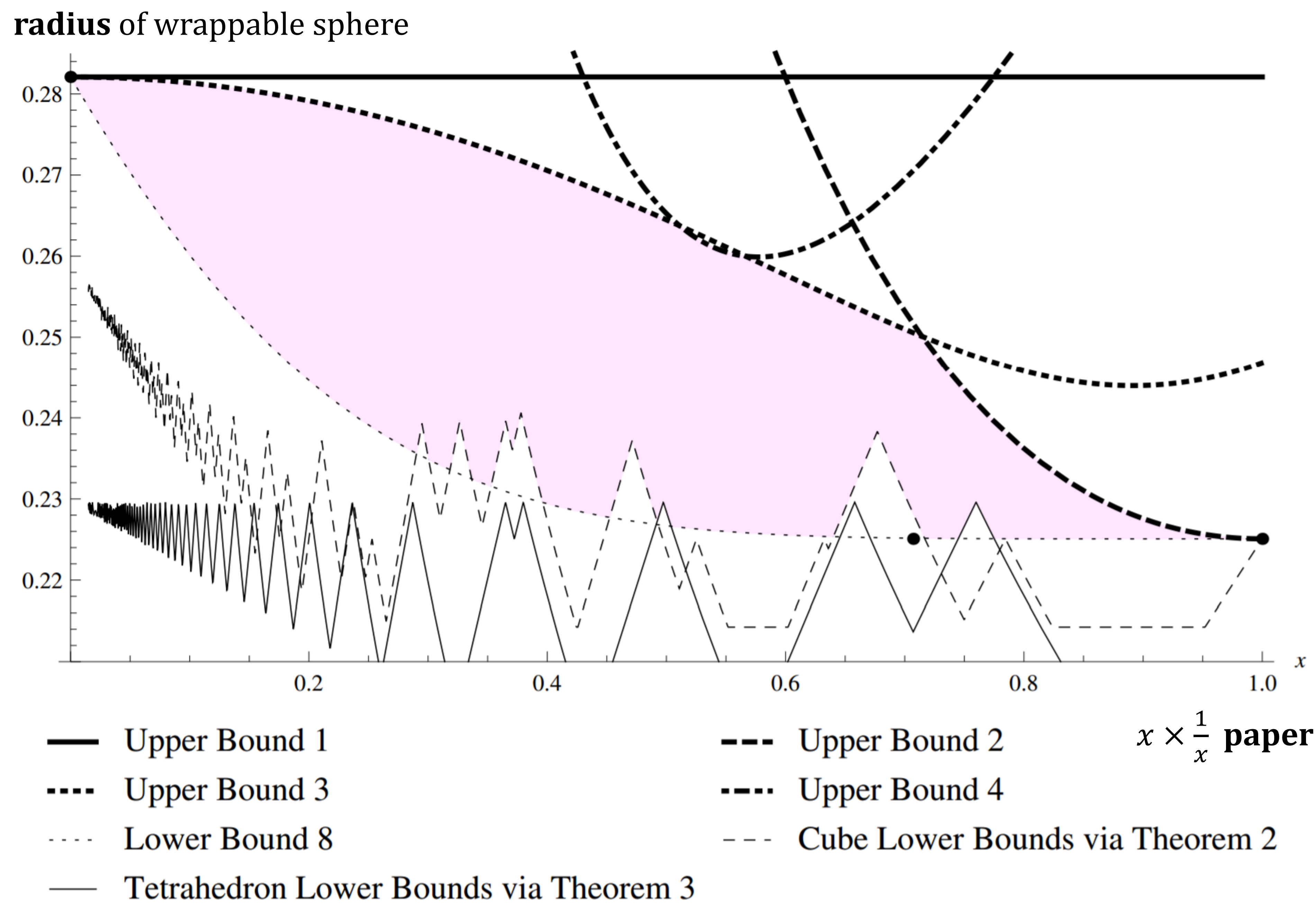


Disk Packing is NP-hard

[Demaine, Fekete, Lang 2010]




side length of wrappable cube
0.41
0.40
0.39
0.38
0.37
0.36
0.35
0.34
0.2
0.4
0.6
0.8
1.0
x
$x \times \frac{1}{x}$ paper
Upper Bound 1
Upper Bound 2
Upper Bound 3 via Theorem 2
Lower Bound 1
Lower Bound 2
Lower Bound 5
Lower Bound 6
Lower Bound 7
[Cole, Demaine, Fox-Epstein 2013]



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