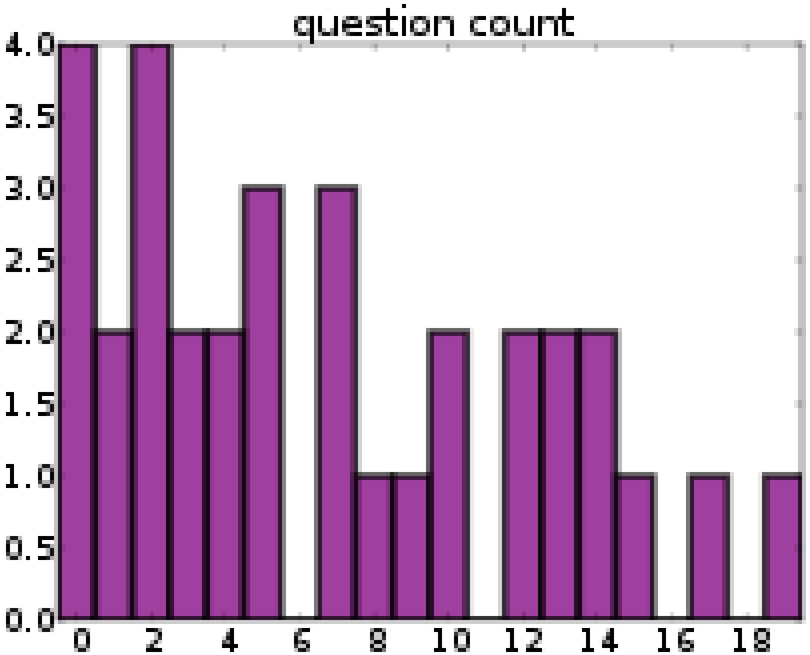
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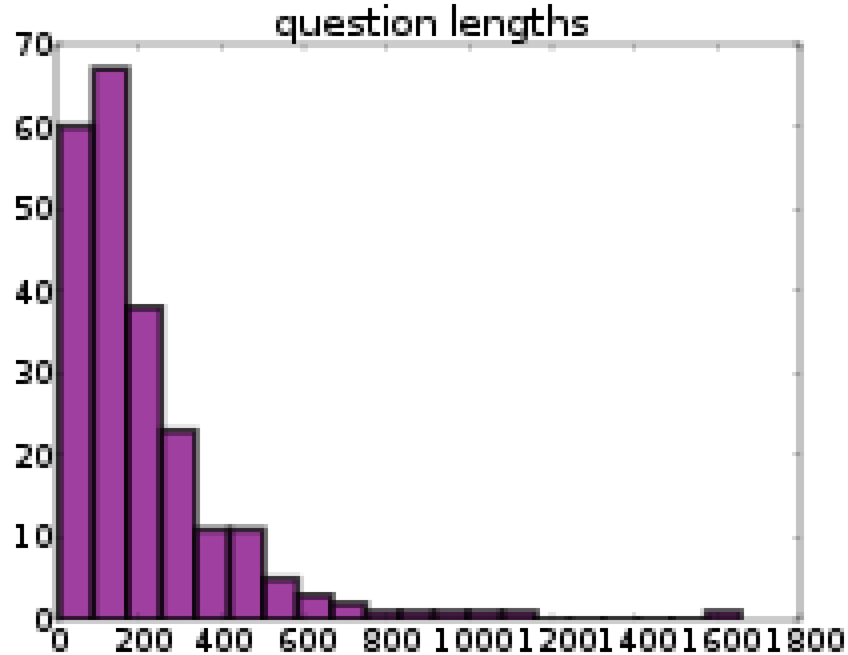
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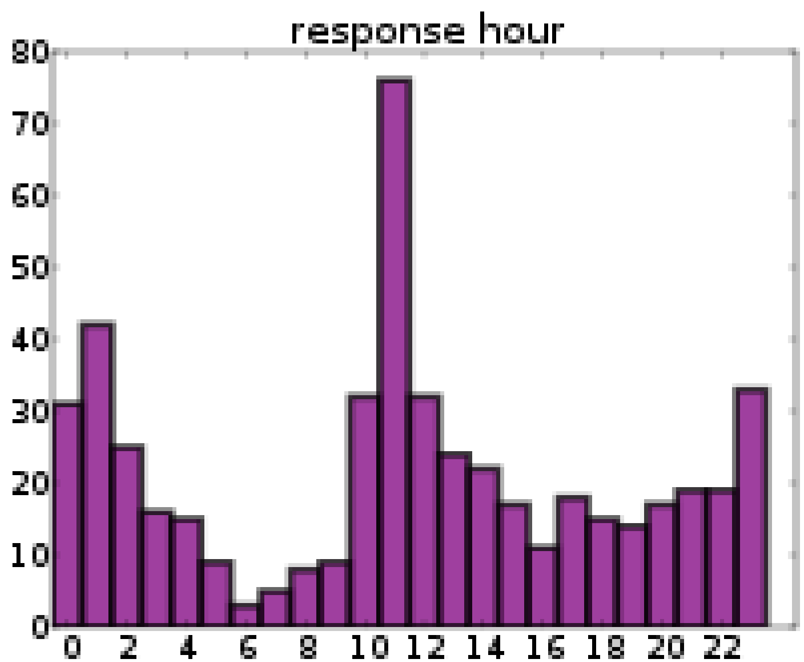
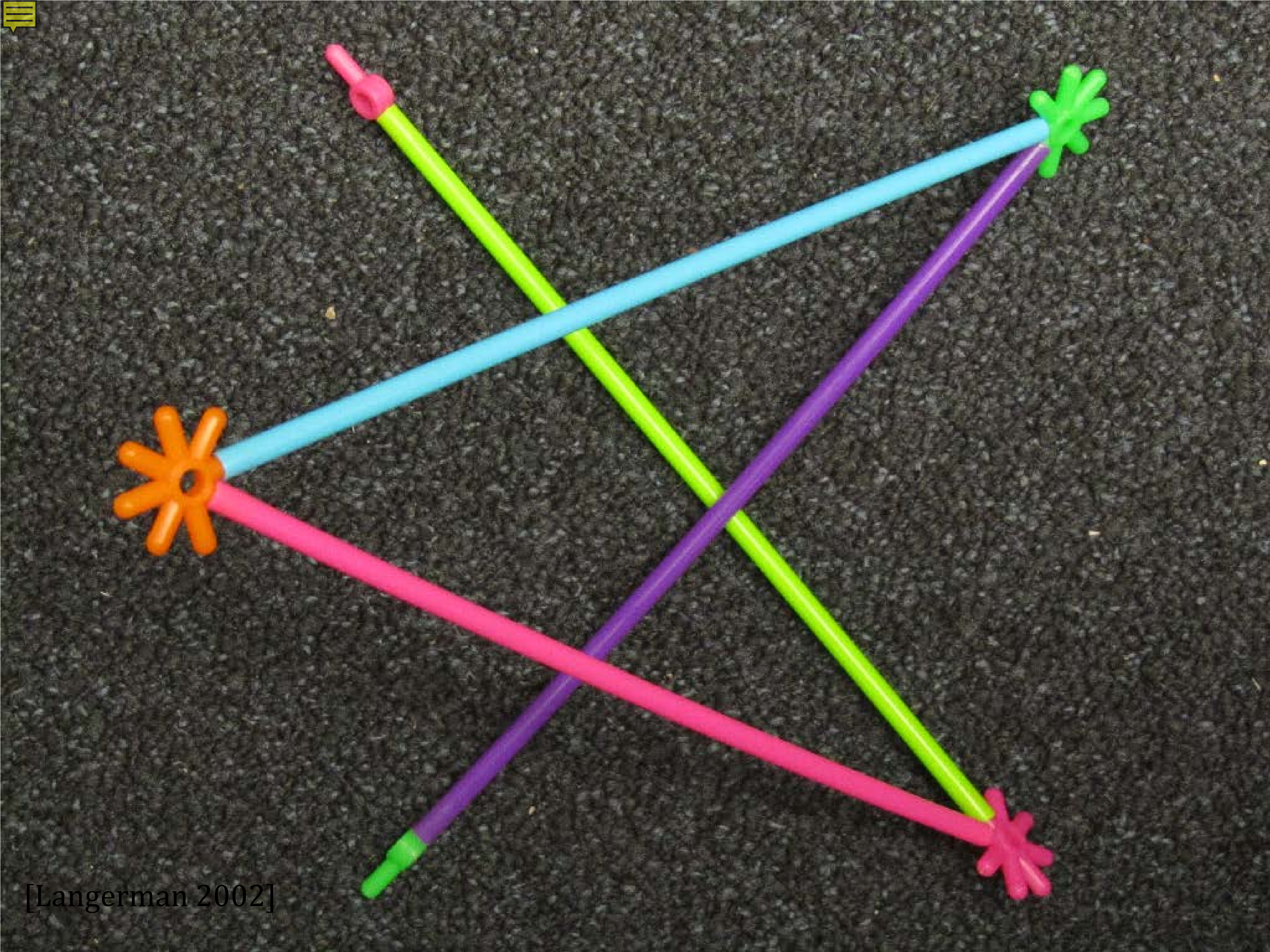
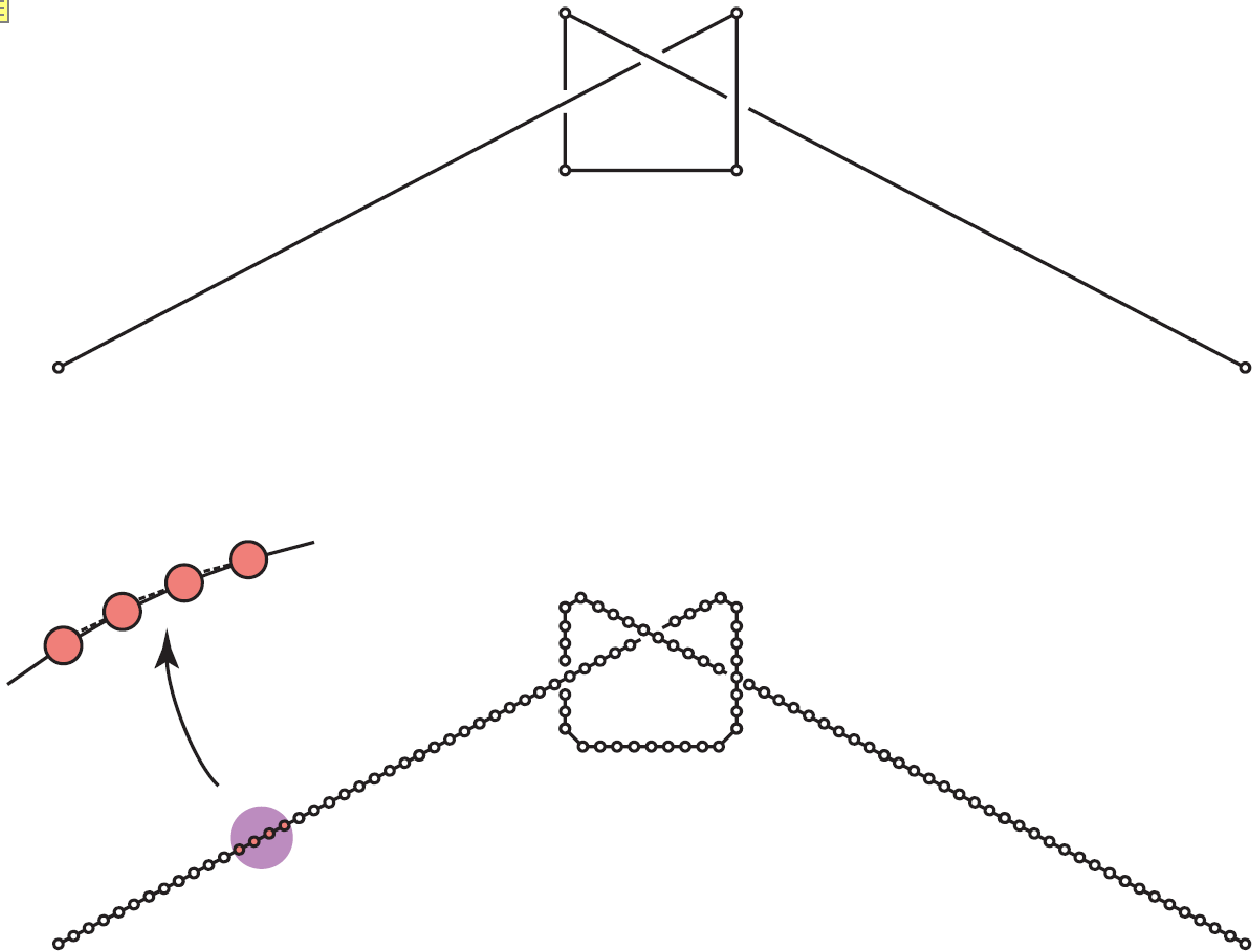

response hour
80
70
60
50
40
30
20
10
0
0 2 4 6 8 10 12 14 16 18 20 22

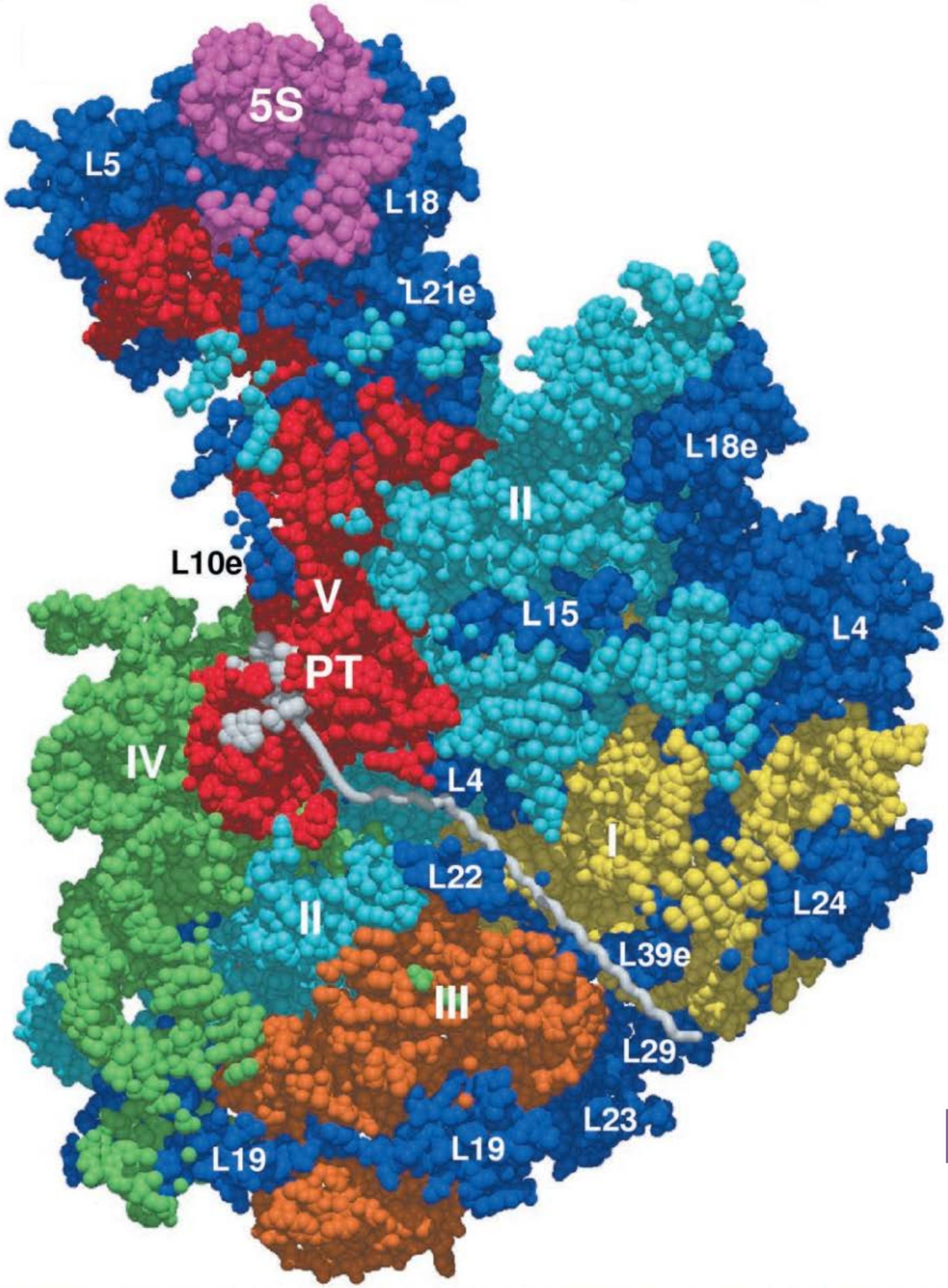
We've done several combinations of the constraints of equilateralness, equiangularity, and obtusehood for 3-D chains we want to know whether can be locked. What about the others?



[Langerman 2002]



The geometric (cone) model for a ribosome seems too simple. Is it actually based on some verified model from biology?



[Nissen, Hansen, Ban, Moore, Steitz 2000]

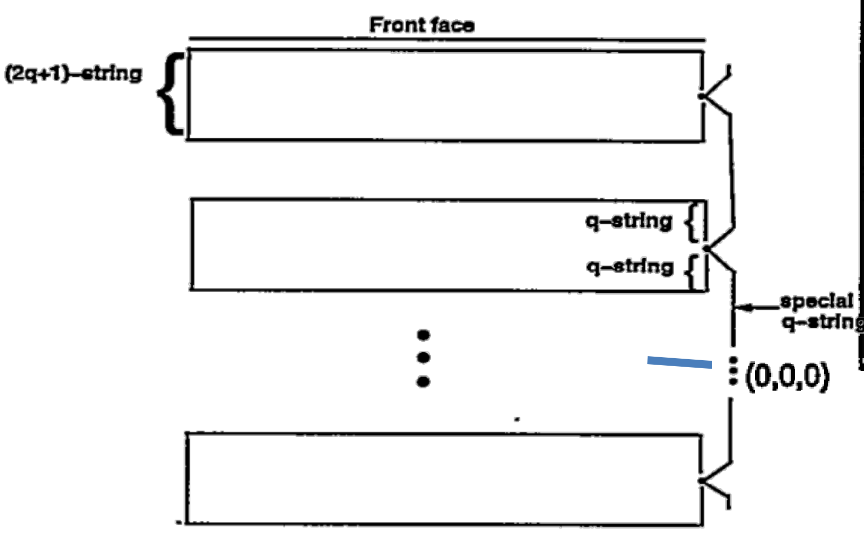
[L21] In proving the NP-hardness of the 2D HP-model folding problem, what are the NP-hard problems used in various reductions?

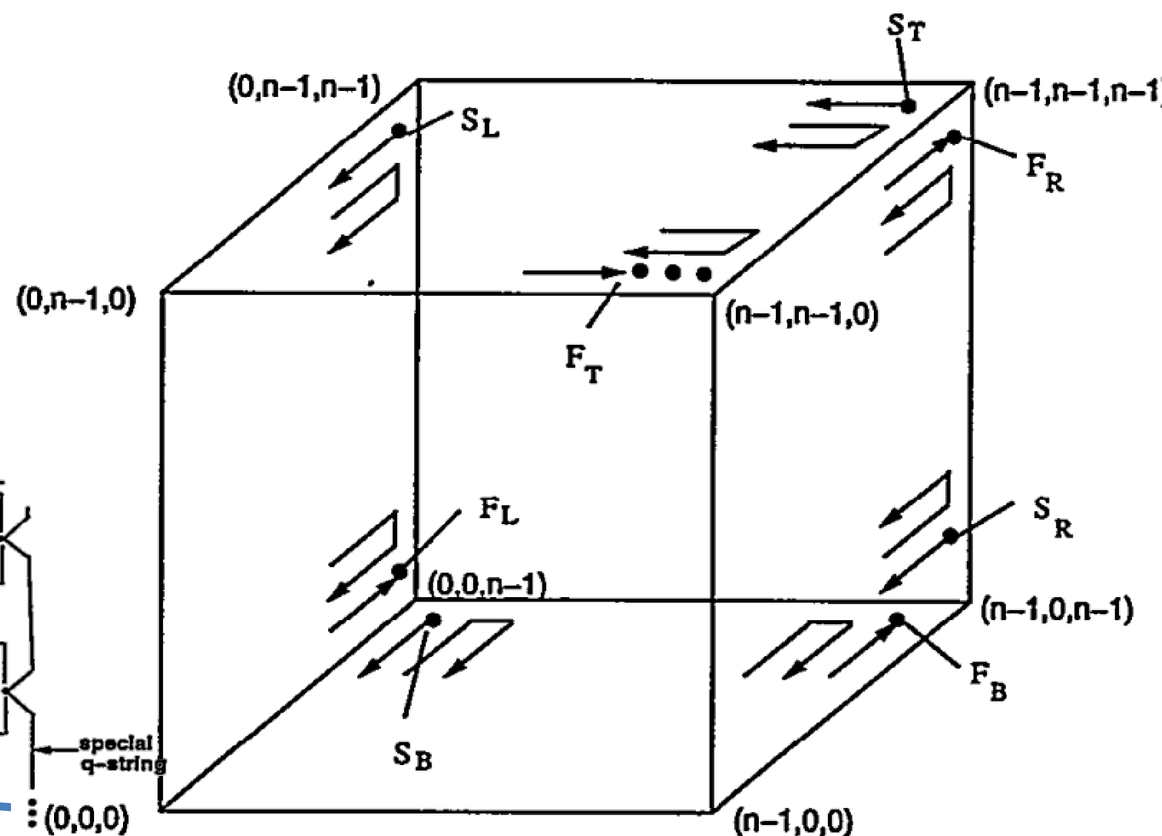
Protein Folding in the Hydrophobic-Hydrophilic (HP) Model is NP-Complete

Bonnie Berger* Tom Leighton†

bin packing

$$P^{cn}(PHHP)^{x/2}$$





On the Complexity of Protein Folding

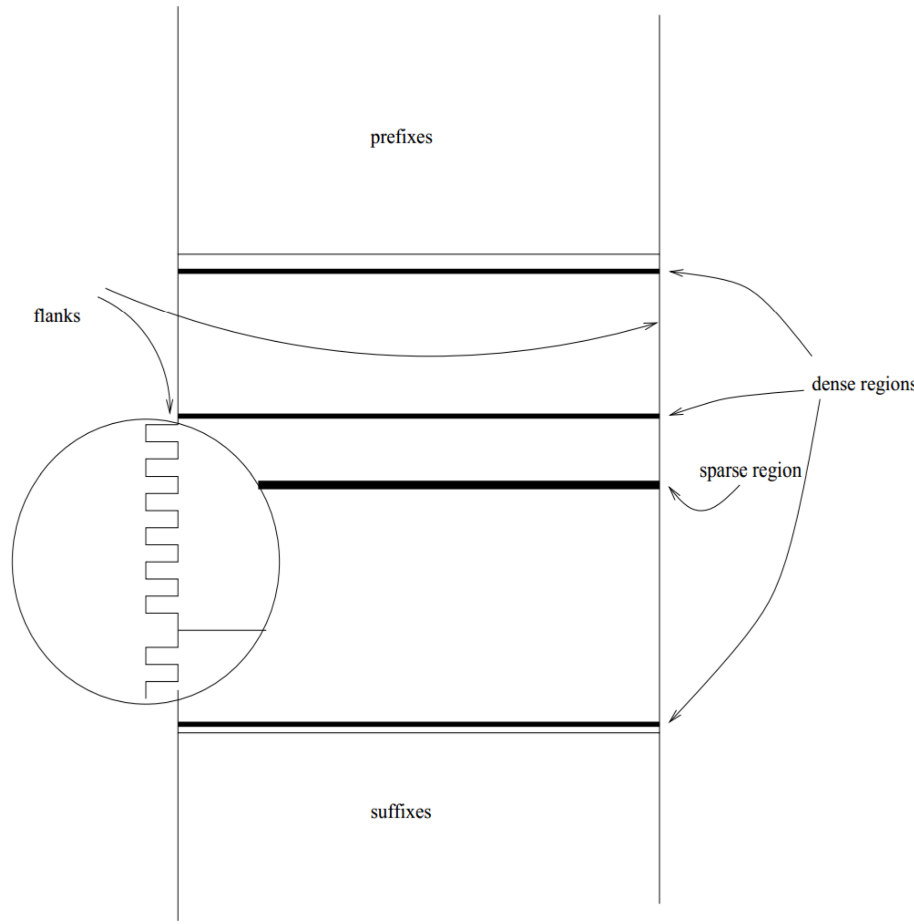
Pierluigi Crescenzi, Deborah Goldman, Christos Papadimitriou
Antonio Piccolboni, Mihalis Yannakakis

Hamiltonicity in max-degree-4 graphs

$$t(i)' = \prod_{l=1}^{8n} \begin{cases} ((10^4 1)^{cq/2} T(v_i)_l)^2 & \text{if } T(v_i)_l = 0 \\ (10^4 1)^{cq/2} T(v_i)_l 0^4 1 (10^4 1)^{cq/2-1} 10^4 T(v_i)_l & \text{if } T(v_i)_l = 1 \end{cases}$$

Trevisan code

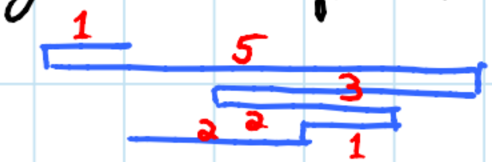
$$s_k = 0^{L^4} d'(10^4 1)^{L/20} d'(10^4 1)^{L/10 - E(cq+1)} m' t(i)' (10^4 1)^{L/5} d' 0^{L^4}$$



Any progress on any of the open problems?

Flattening: weakly NP-hard [Soss & Toussaint 2000]

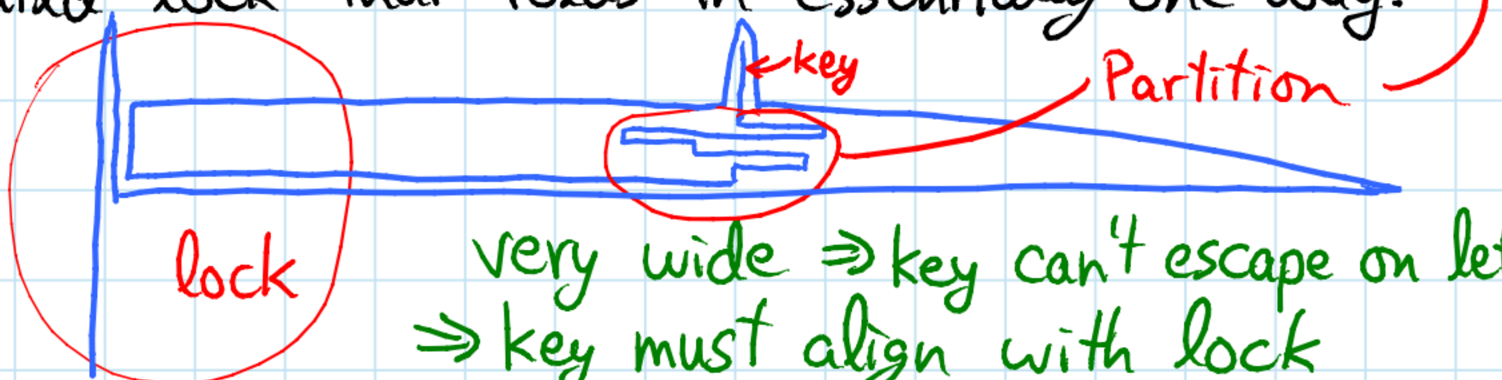
- reduction from Partition: divide n integers into 2 equal sums



- horizontal bars for integers
- vertical bars in between, length $< \frac{1}{n}$

$\Rightarrow$ can flip horizontal bars left & right

- build lock that folds in essentially one way:



very wide $\Rightarrow$ key can't escape on left

$\Rightarrow$ key must align with lock

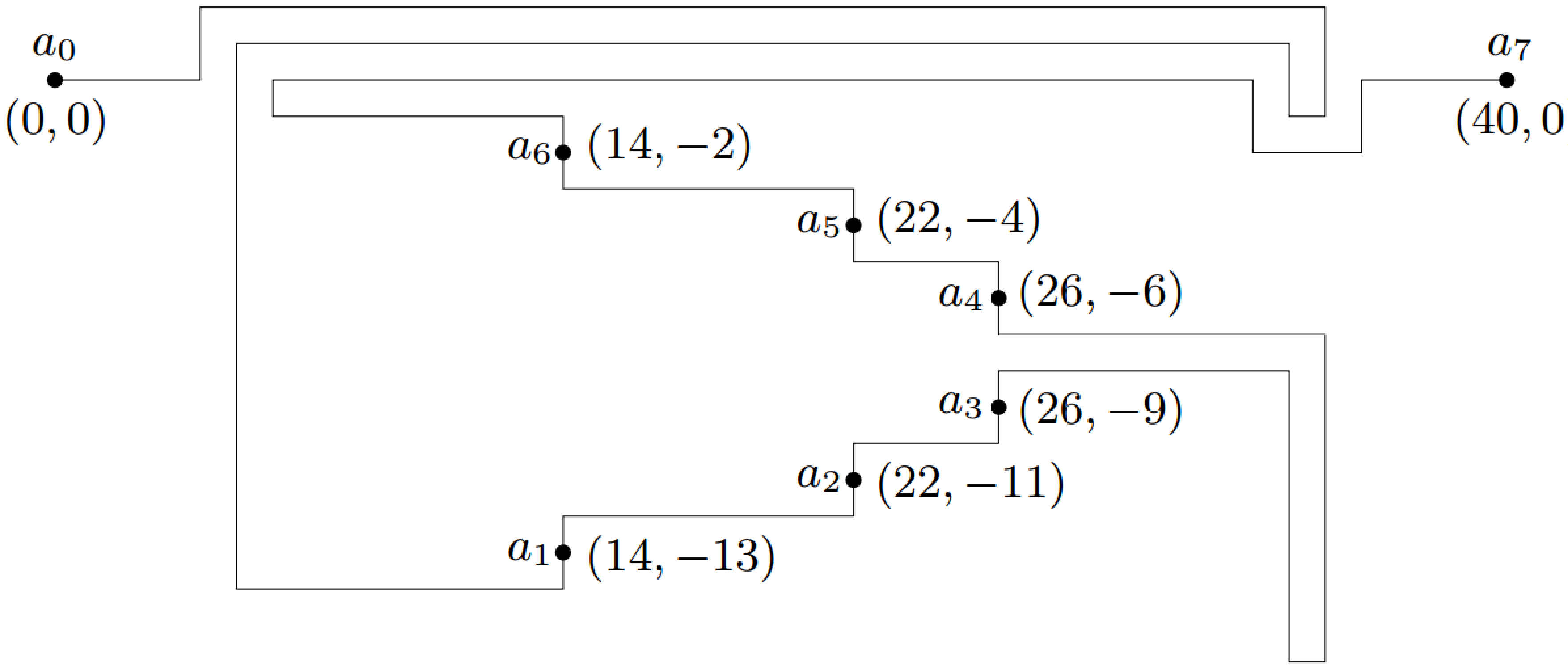
OPEN: pseudopolynomial-time algorithm?

Flattening Fixed-Angle Chains Is Strongly NP-Hard

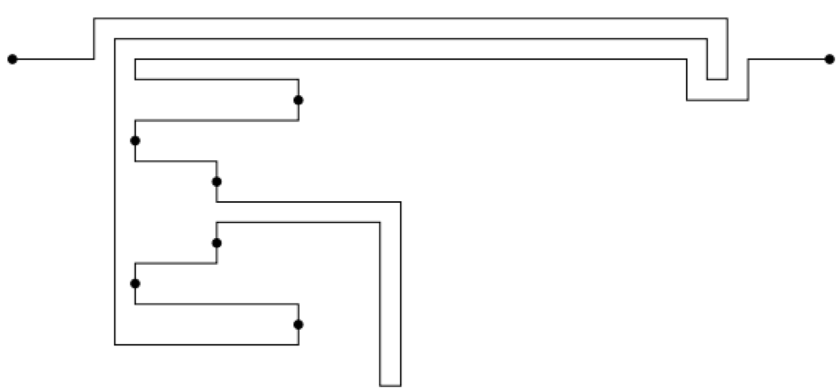
Erik D. Demaine* and Sarah Eisenstat*

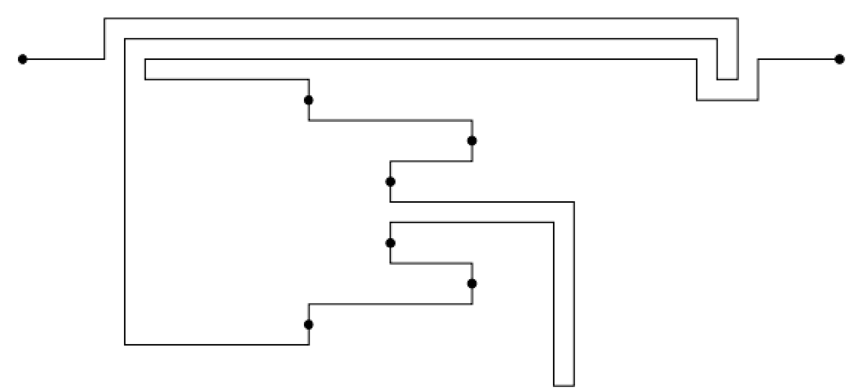
MIT Computer Science and Artificial Intelligence Laboratory,
32 Vassar St., Cambridge, MA 02139, USA
{edemaine,seisenst}@mit.edu

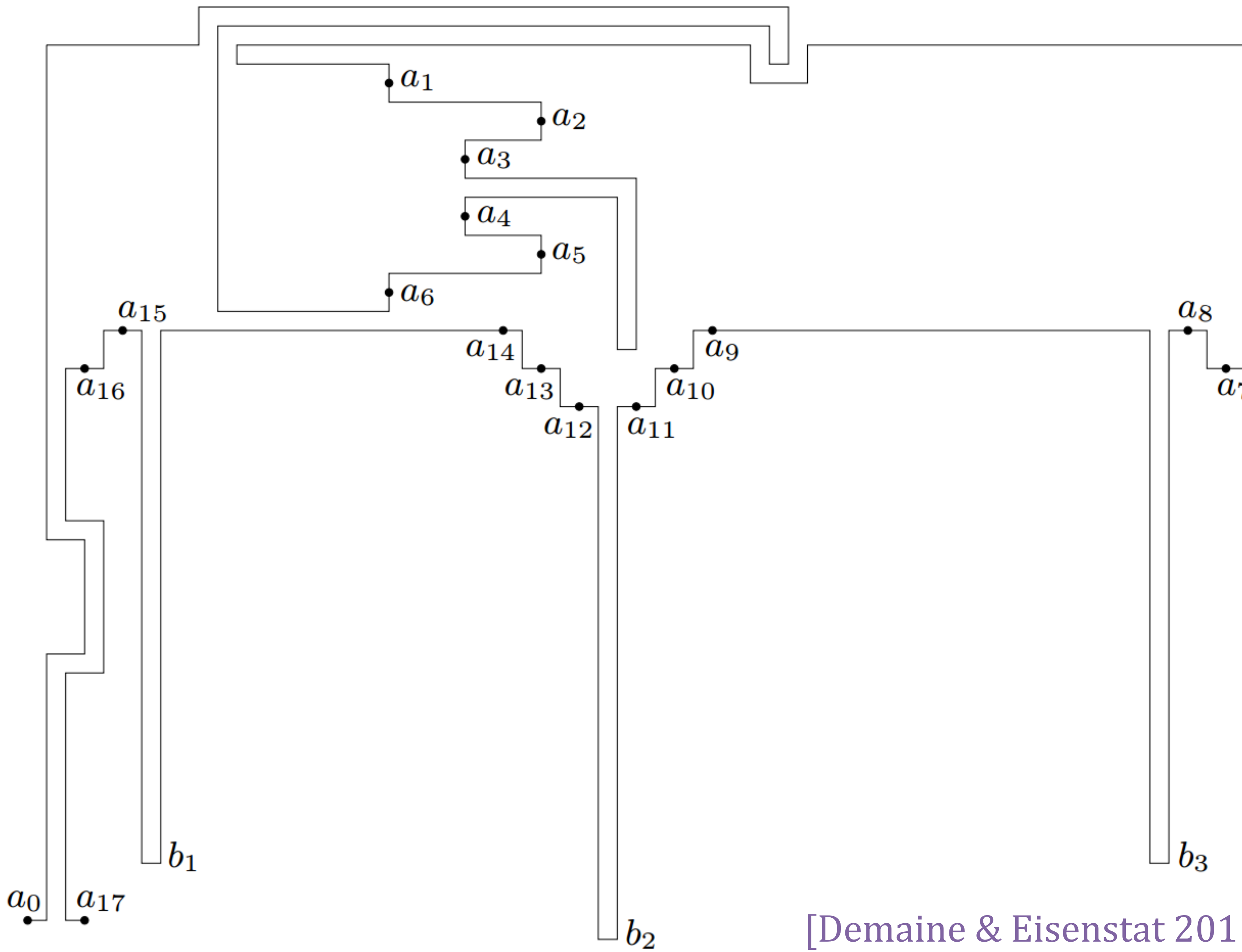
Problem	Linkage	Edge lengths	Angle range
Flattening	fixed-angle chain	equilateral	$[16.26^\circ, 180^\circ]$
Flattening	fixed-angle chain	$\Theta(1)$	$[60 - \varepsilon^\circ, 180^\circ]$
Flattening	fixed-angle caterpillar tree	equilateral	$\{90^\circ, 180^\circ\}$
Min flat span	fixed-angle chain	equilateral	$[16.26^\circ, 180^\circ]$
Max flat span	fixed-angle chain	equilateral	$[16.26^\circ, 180^\circ]$



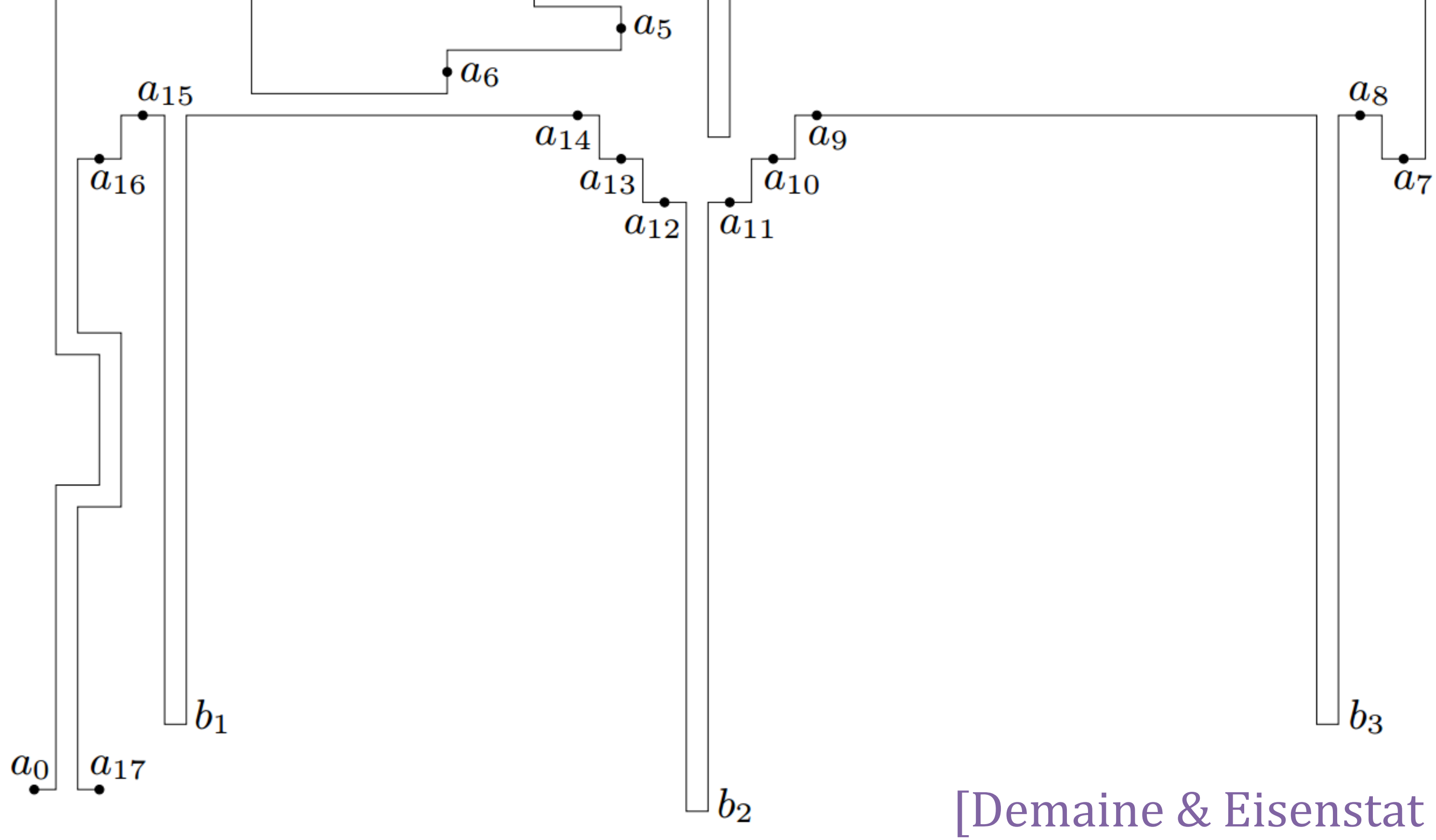
[Demaine & Eisenstat 2011]



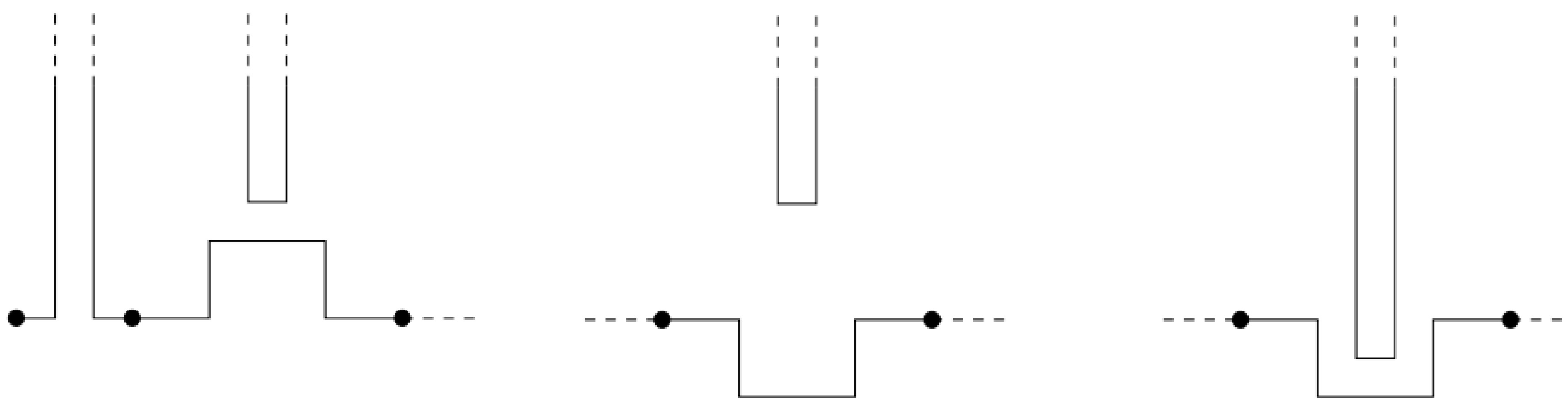


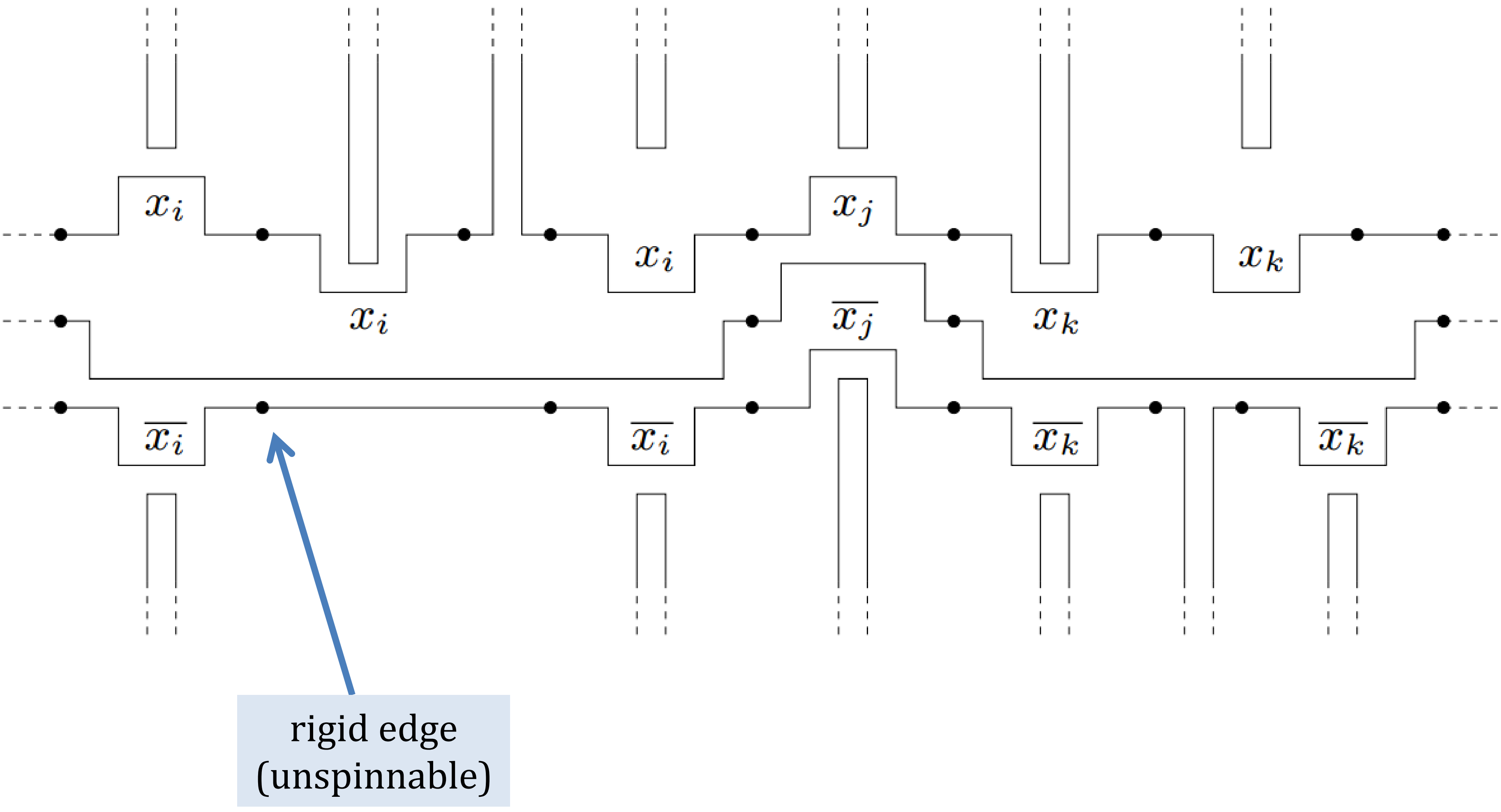


[Demaine & Eisenstat 2011]

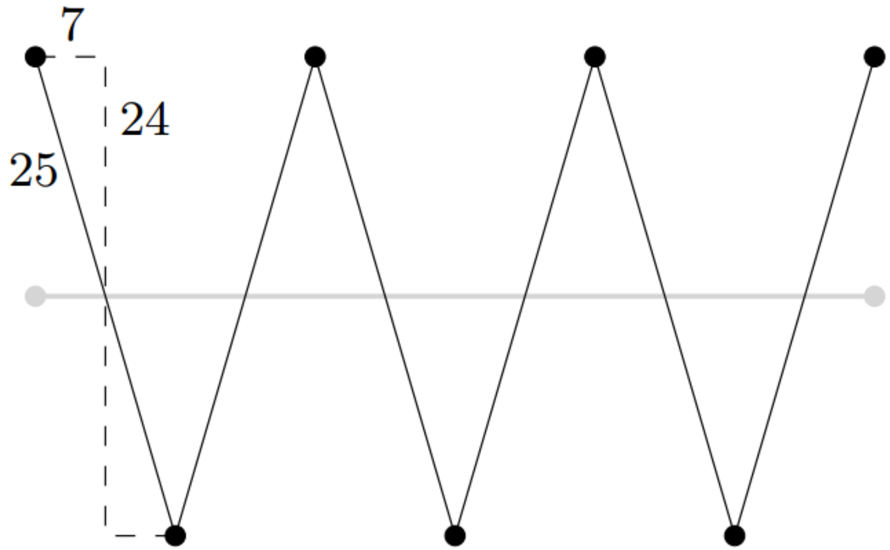


[Demaine & Eisenstat 2011]

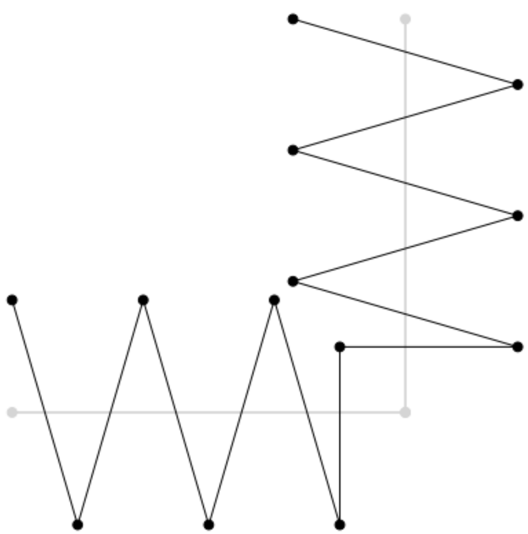




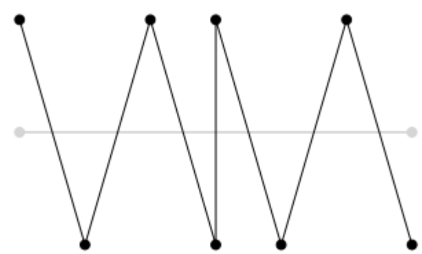
[Demaine & Eisenstat 2011]



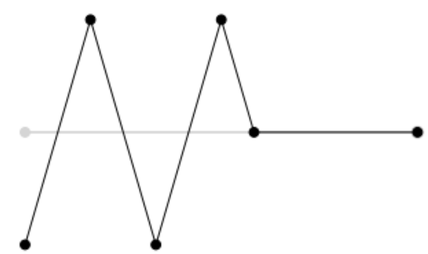
(a) Zig-zag gadget.



(b) Turn gadget.

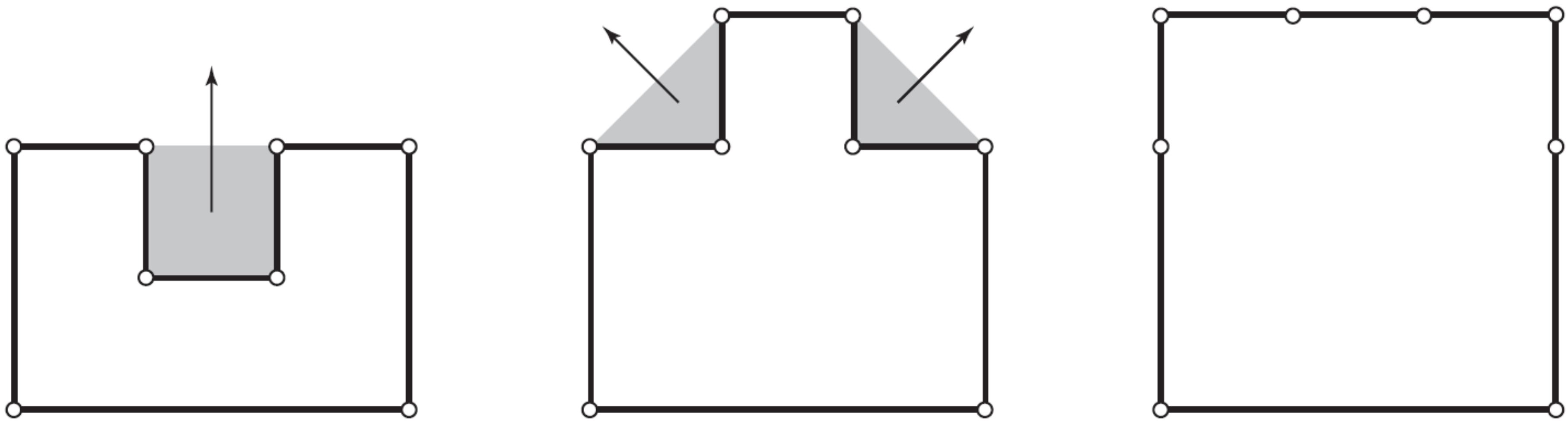


(c) Switch gadget.



(d) Articulation gadget.

[Demaine & Eisenstat 2011]

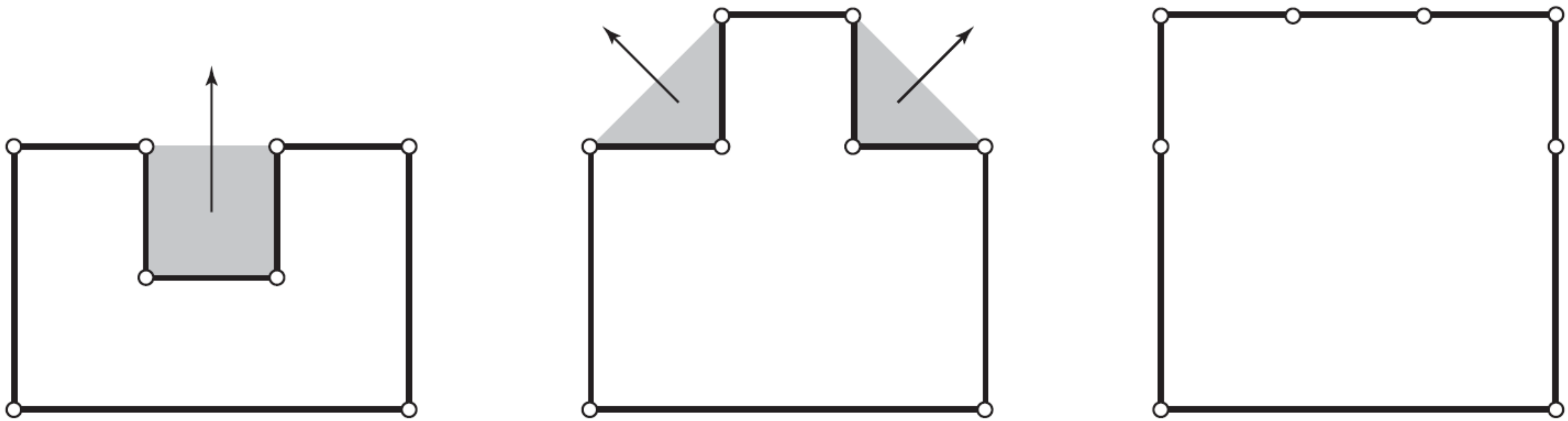


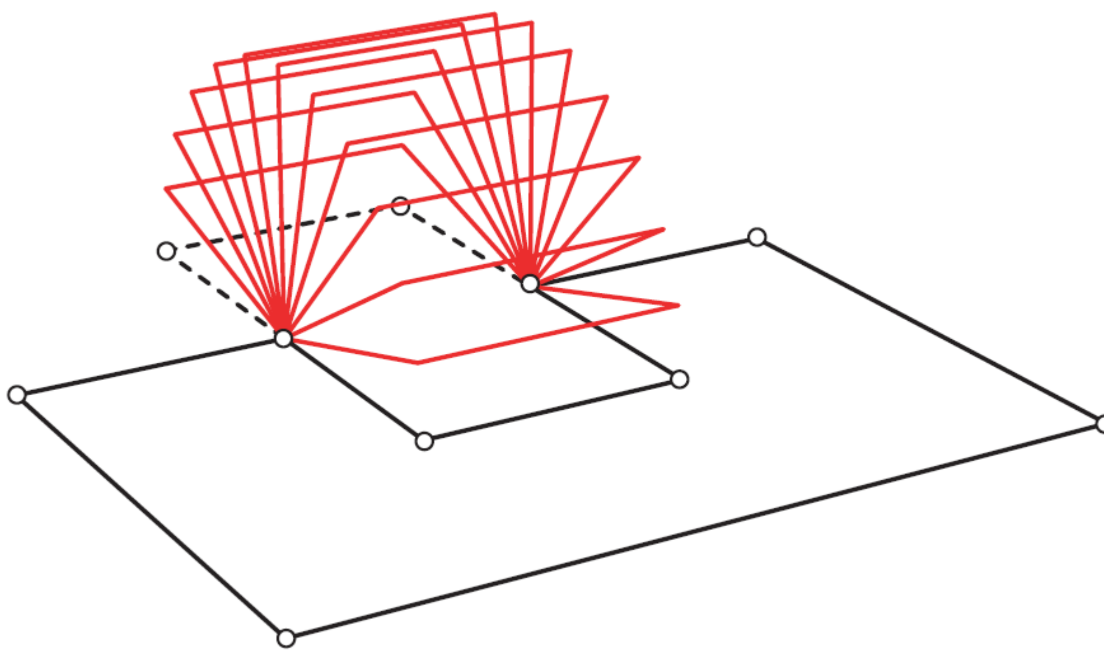
Advanced Problems

3763. *Proposed by Paul Erdös, The University, Manchester, England.*

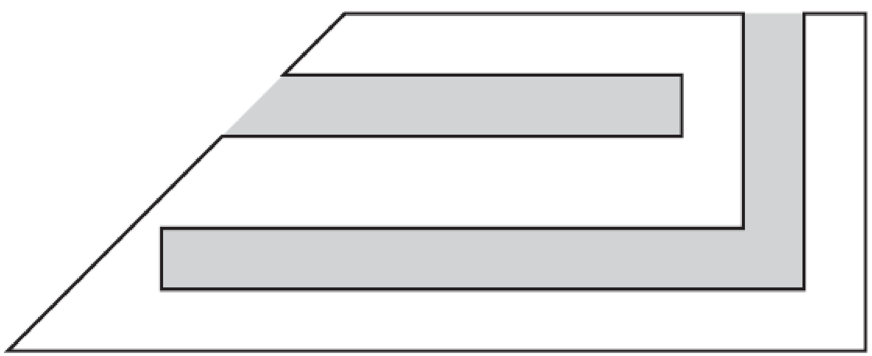
Given any simple polygon P which is not convex, draw the smallest convex polygon P' which contains P. This convex polygon P' will contain the area P and certain additional areas. Reflect each of these additional areas with respect to the corresponding added side, thus obtaining a new polygon P_1. If P_1 is not convex, repeat the process, obtaining a polygon P_2. Prove that after a finite number of such steps a polygon P_n will be obtained which will be convex.

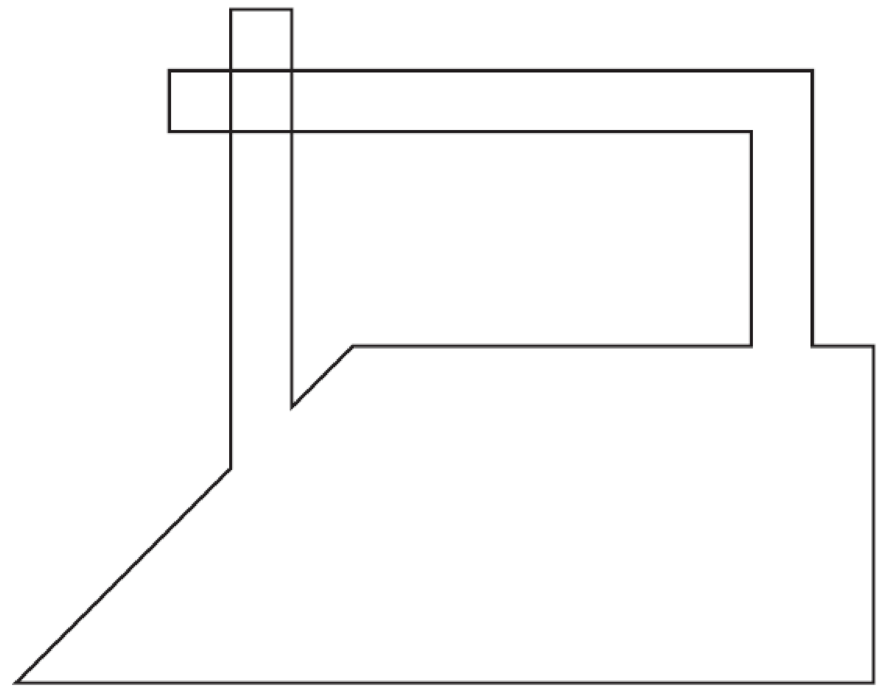
Erdős 1935



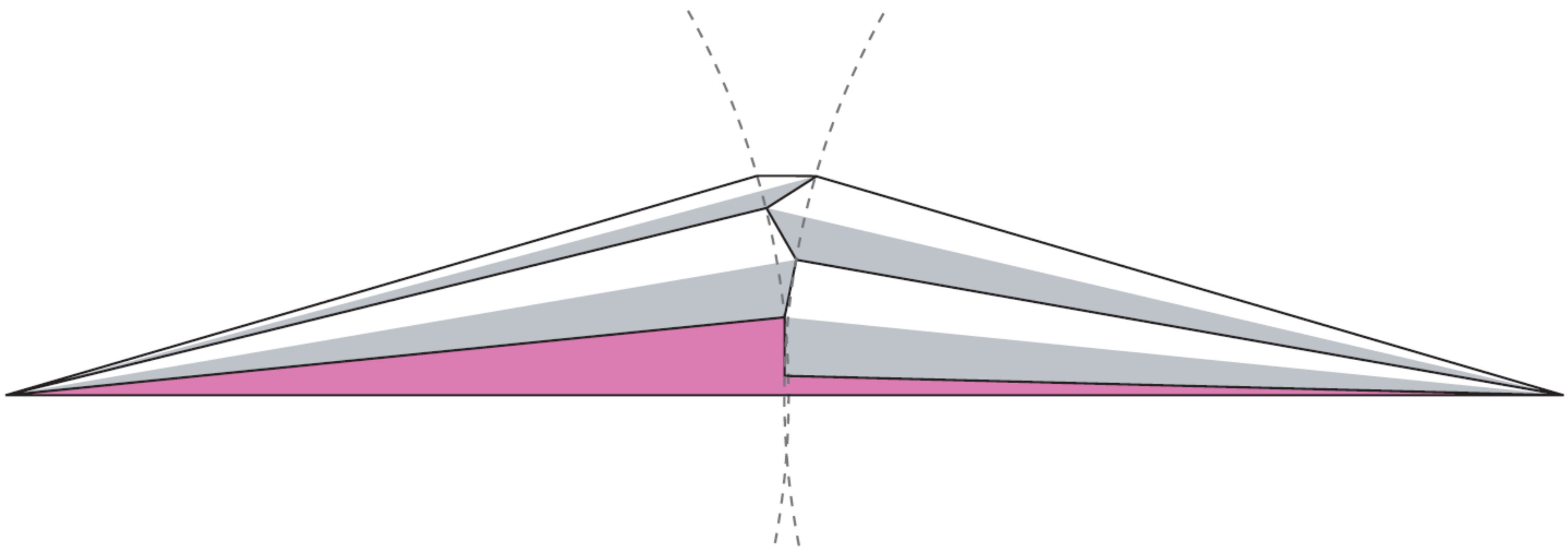


Erdős 1935





de Sz. Nagy 1939



Joss & Shannon 1973

Reference		Genesis
Nagy [**dSN39**]	§3.1	Erdős [**Erd35**]
Reshetnyak [**Res57**]	§3.2	independent[a]
Yusupov [**Yus57**]	§3.3	independent[a]
Bing & Kazarinoff [**KB59, BK61, Kaz61a**]	§3.4	Erdős [**Erd35**], Nagy [**dSN39**], Reshetnyak [**Res57**]
Wegner [**Weg93**]	§3.6	Kaluza [**Kal81**]
Grünbaum [**Grü95**]	§3.7	all of above
Toussaint [**Tou99, Tou05**]	§3.8	all of above

Demaine, Gassend, O'Rourke, Toussaint 2008

Reference		Genesis	Flaws, omissions, comments
Nagy [**dSN39**]	§3.1	Erdős [**Erd35**]	**Flawed:** $C^k \not\subseteq P^{k+1}$.
Reshetnyak [**Res57**]	§3.2	independent[a]	**Correct** though somewhat imprecise.
Yusupov [**Yus57**]	§3.3	independent[a]	**Flawed:** P^* might have pockets, and only some vertices might flatten.
Bing & Kazarinoff [**KB59, BK61, Kaz61a**]	§3.4	Erdős [**Erd35**], Nagy [**dSN39**], Reshetnyak [**Res57**]	**Correct** though somewhat terse. Claims Nagy's proof is incorrect. False conjecture: $2n$ flips suffice.
Wegner [**Weg93**]	§3.6	Kaluza [**Kal81**]	**Flawed:** Area increase can be small.
Grünbaum [**Grü95**]	§3.7	all of above	**Omission:** Why P^* is convex. Based on Nagy's argument. Requires specific flip sequence.
Toussaint [**Tou99, Tou05**]	§3.8	all of above	**Flawed:** P^* might have pockets. Based on Nagy's argument.

Demaine, Gassend, O'Rourke, Toussaint 2008

Доказательство этой теоремы, данное Б. Секефальви-Надем [см. B. Sz.-Nady, Amer. Math. Monthly **46**, 1939, стр. 176—177], неверно.

“The proof of this theorem, given by B. Sz. Nagy, is incorrect”

Reference		**Genesis**	**Flaws, omissions, comments**
Nagy [**dSN39**]	§3.1	Erdős [**Erd35**]	**Flawed:** $C^k \not\subseteq P^{k+1}$.
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Toussaint [**Tou99, Tou05**]	§3.8	all of above	**Flawed:** P^* might have pockets. Based on Nagy's argument.

“Bing and Kazarinoff remark that Nagy's proof is invalid, but there is no basis for this claim.”

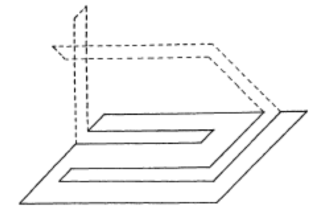
Demaine, Gassend, O'Rourke, Toussaint 2008

3763 [1935, 627]. *Proposed by Paul Erdös, The University, Manchester, England.*

Given any simple polygon P which is not convex, draw the smallest convex polygon P' which contains P. This convex polygon P' will contain the area P and certain additional areas. Reflect each of these additional areas with respect to the corresponding added side, thus obtaining a new polygon P_1. If P_1 is not convex, repeat the process, obtaining a polygon P_2. Prove that after a finite number of such steps a polygon P_n will be obtained which will be convex.

Solution by Béla de Sz. Nagy, Szeged, Hungary.

The process described in the above problem, *i.e.*, the reflection of *all* additional areas, does not always lead from a simple polygon to a simple one, as shown in the following example:



This means that the repeating of this process is not always possible.

In order to avoid this difficulty we modify the process in the following way. Instead of reflecting *all* additional areas mentioned in the problem we reflect *only one* of them, so obtaining obviously always a simple polygon again. We agree to define the process also for convex polygons as the process of leaving them invariant.

Let $A_1^0, A_2^0, \cdots, A_\sigma^0$ be the vertices of the given simple polygon P^0. Applying the process n times leads to a polygon P^n, the points A_ν^0 $(\nu=1, 2, \cdots, \sigma)$ being carried thereby into the points A_ν^n. Let us denote by C^n the least convex polygon containing P^n in its interior. Each polygon in the sequence P^0, C^0, P^1, C^1, P^2, C^2, $\cdots$ contains obviously the foregoing ones in its interior. The lengths of all polygons P^n being plainly the same, there is a circle containing all P^n's in its interior. This implies that the sequence of the points A_ν^n $(n=0, 1, 2, \cdots)$ has at least one point of accumulation.

It follows readily from the nature of the above process that if B is a point on, or inside of, P^m, then dist $(B, A_\nu^n) \leqq$ dist (B, A_ν^{n+1}) for $n \geqq m$. Especially we have: dist $(A_\nu^m, A_\nu^n) \leqq$ dist (A_ν^m, A_ν^{n+1}) for $n \geqq m$. From this it follows that the sequence of the points A_ν^n $(n=0, 1, 2, \cdots)$ may have only a single point of accumulation. It is thus convergent: $A_\nu^n \to A_\nu$ for $n \to \infty$.

The polygon $P = (\overline{A_1A_2}, \overline{A_2A_3}, \cdots, \overline{A_{\sigma-1}A_\sigma}, \overline{A_\sigma A_1})$, being the limit of the sequence P^n, is also the limit of the sequence C^n and is therefore convex.

Denote by $c_\nu(r)$ the interior of the circle of radius r drawn around A_ν as center.

Let A_μ be a convexity-point of P (i.e., such that $A_{\mu-1}, A_\mu, A_{\mu+1}$ do not lie on the same straight line; A_σ being denoted also as A_0, A_1 as $A_{\sigma+1}$). We may find then obviously a straight line L and a positive number ρ such that $c_\mu(\rho)$ lies wholly on one side of L while all $c_\lambda(\rho)$ $(\lambda \neq \mu)$ lie on the other side. For $n \geqq n_0(\mu)$ we shall certainly have: $A_\nu^n \in c_\nu(\rho)$ for $\nu=1, 2, \cdots, \sigma$. L separates thus A_μ^n from the other points A_λ^n $(\lambda \neq \mu)$. Hence A_μ^n is a convexity-point of P^n. It must be therefore invariant: $A_\mu^{n+1} = A_\mu^n$. This implies that for $n \geqq n_0(\mu)$: $A_\mu n_0(\mu) = A_\mu^n$. So is $A_\mu^n = A_\mu$ for $n \geqq n_0(\mu)$.

Let now $A_{\mu_1}, A_{\mu_2}, \cdots, A_{\mu_s}$ be all the convexity-points of P. We have then $A_{\mu_r}^N = A_{\mu_r}$ $(r=1, 2, \cdots, s)$ for $N = \max(n_0(\mu_1), n_0(\mu_2), \cdots, n_0(\mu_s))$.

This involves that $C^N = P$ and therefore also that $P^n = P$ for $n \geqq N$. We thus obtain after a finite number of steps a convex polygon indeed.

de Sz. Nagy 1939

SOLUTIONS

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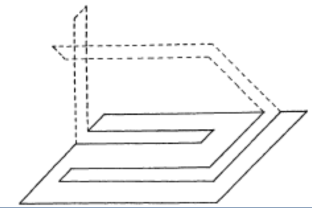

polygon containing P^n in its interior. Each polygon in the sequence P^0, C^0, P^1, C^1, P^2, C^2, $\cdots$ contains obviously the foregoing ones in its interior. The lengths

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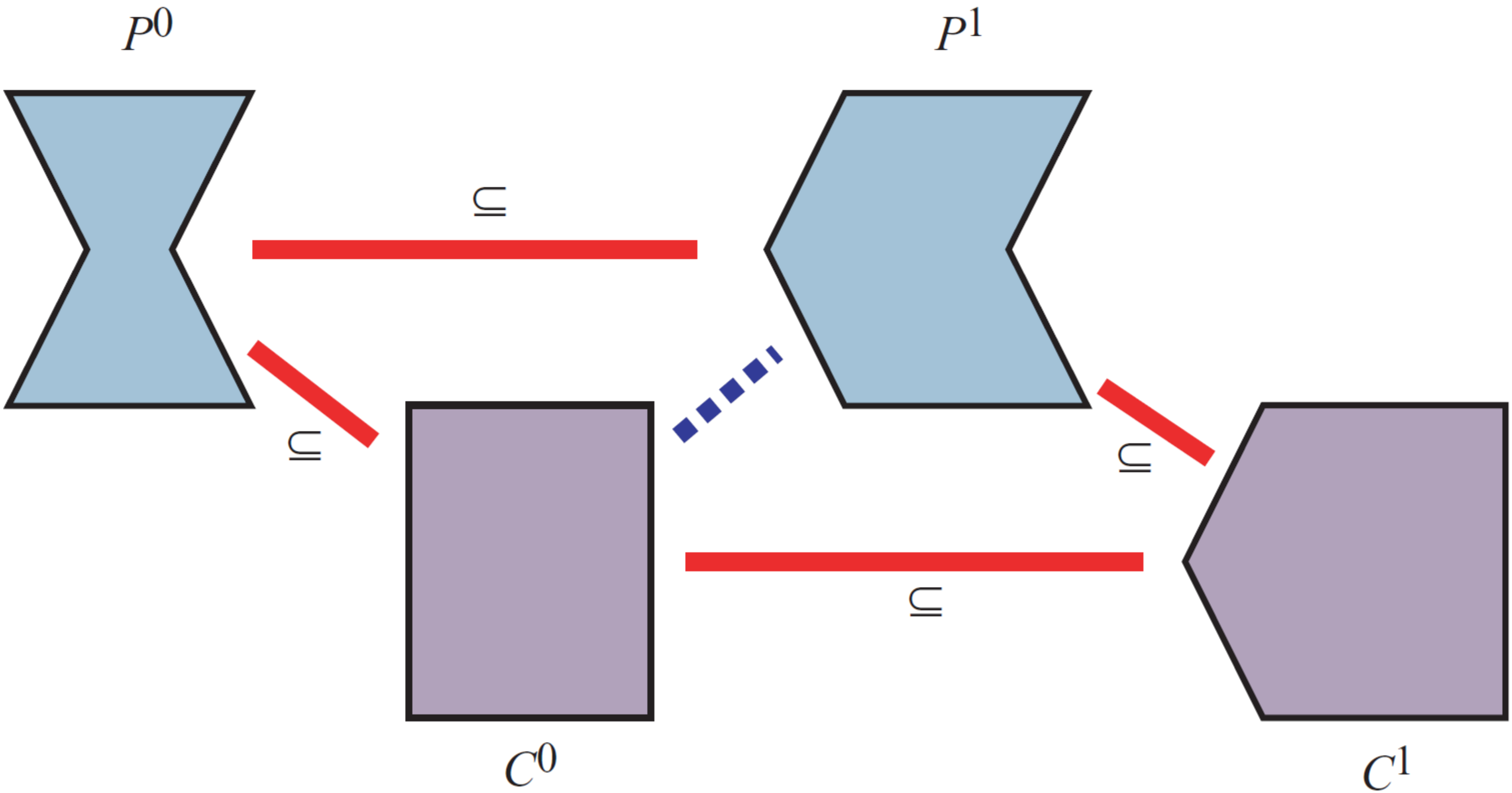
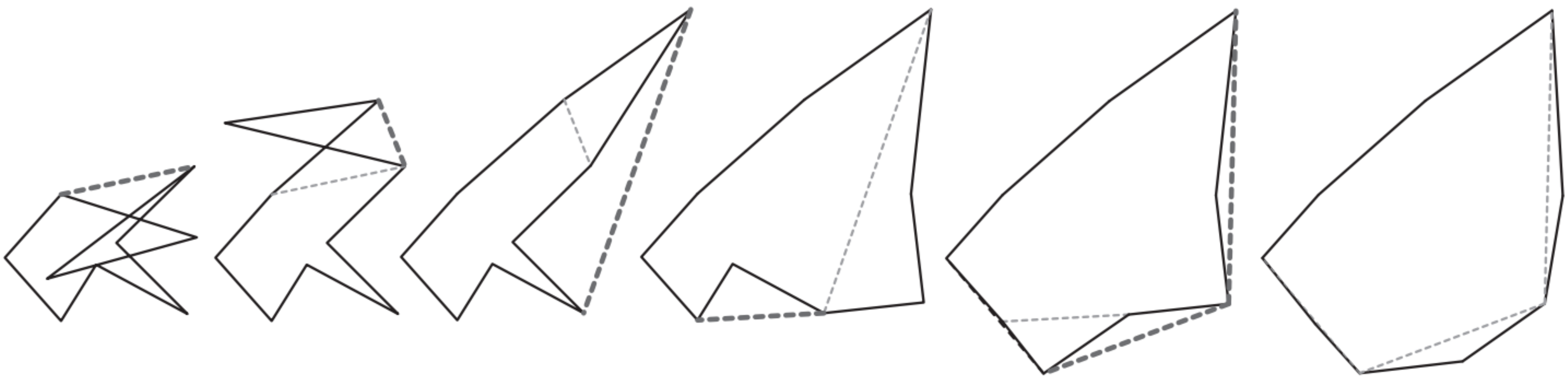
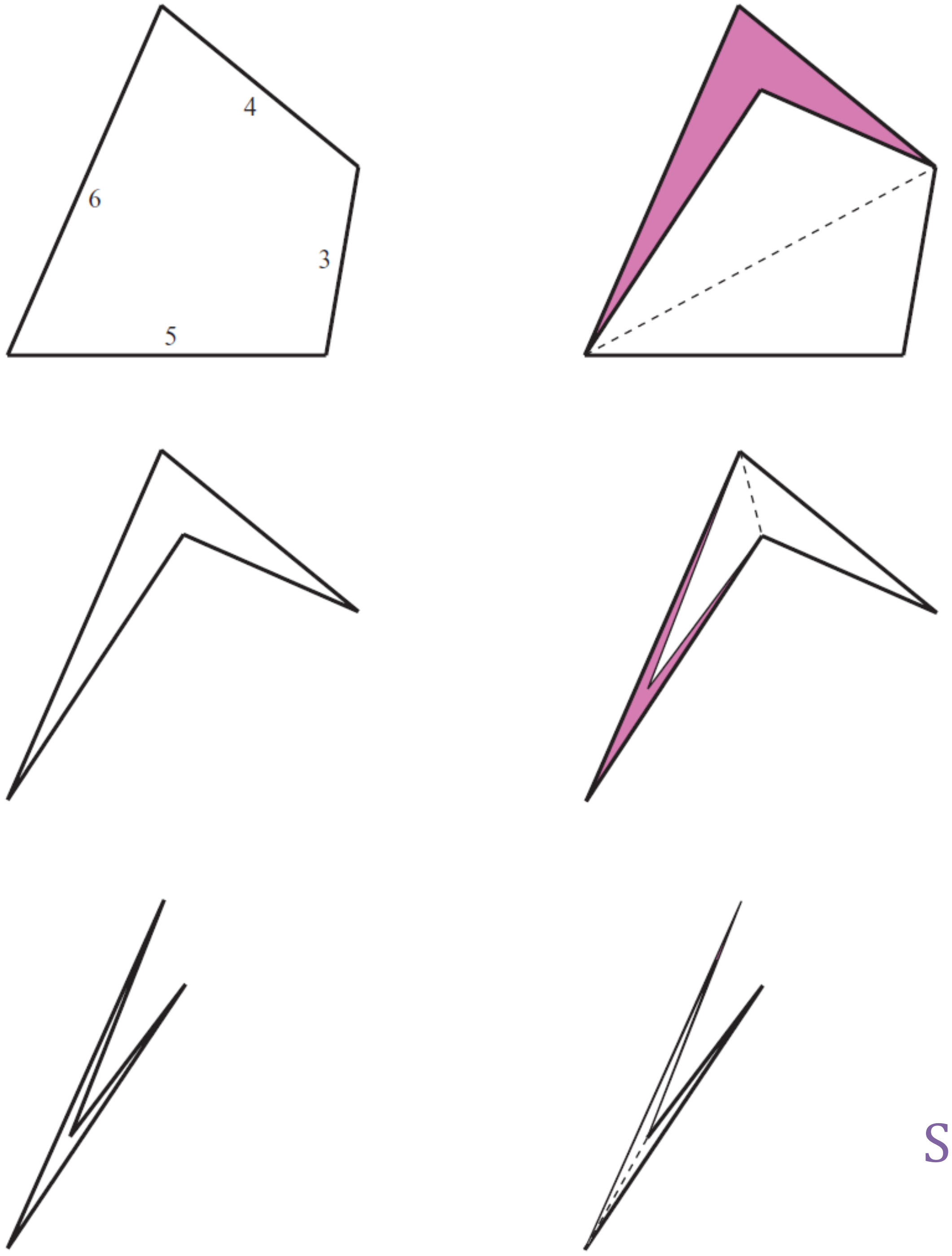
de Sz. Nagy 1939



Demaine, Gassend, O'Rourke, Toussaint 2008

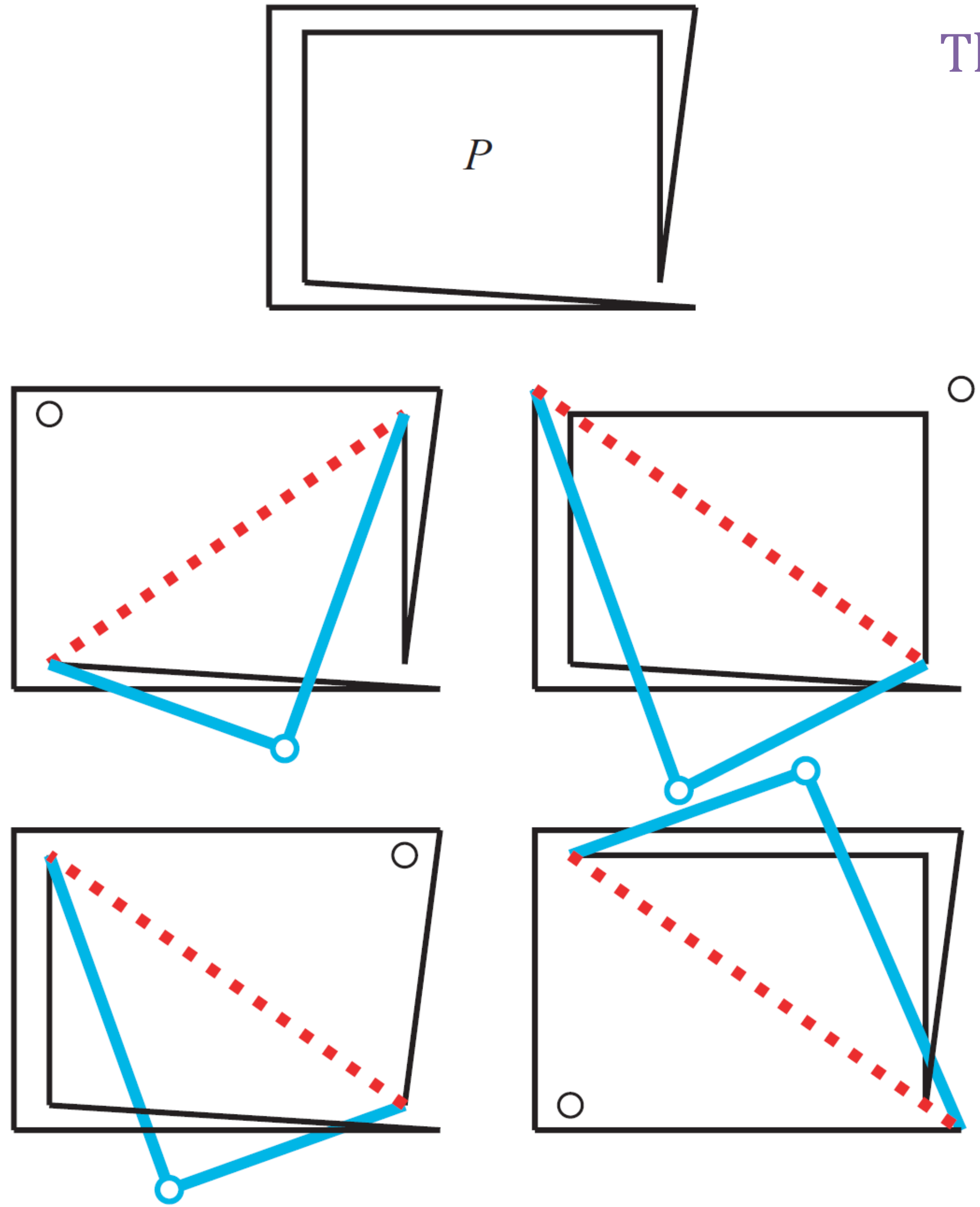


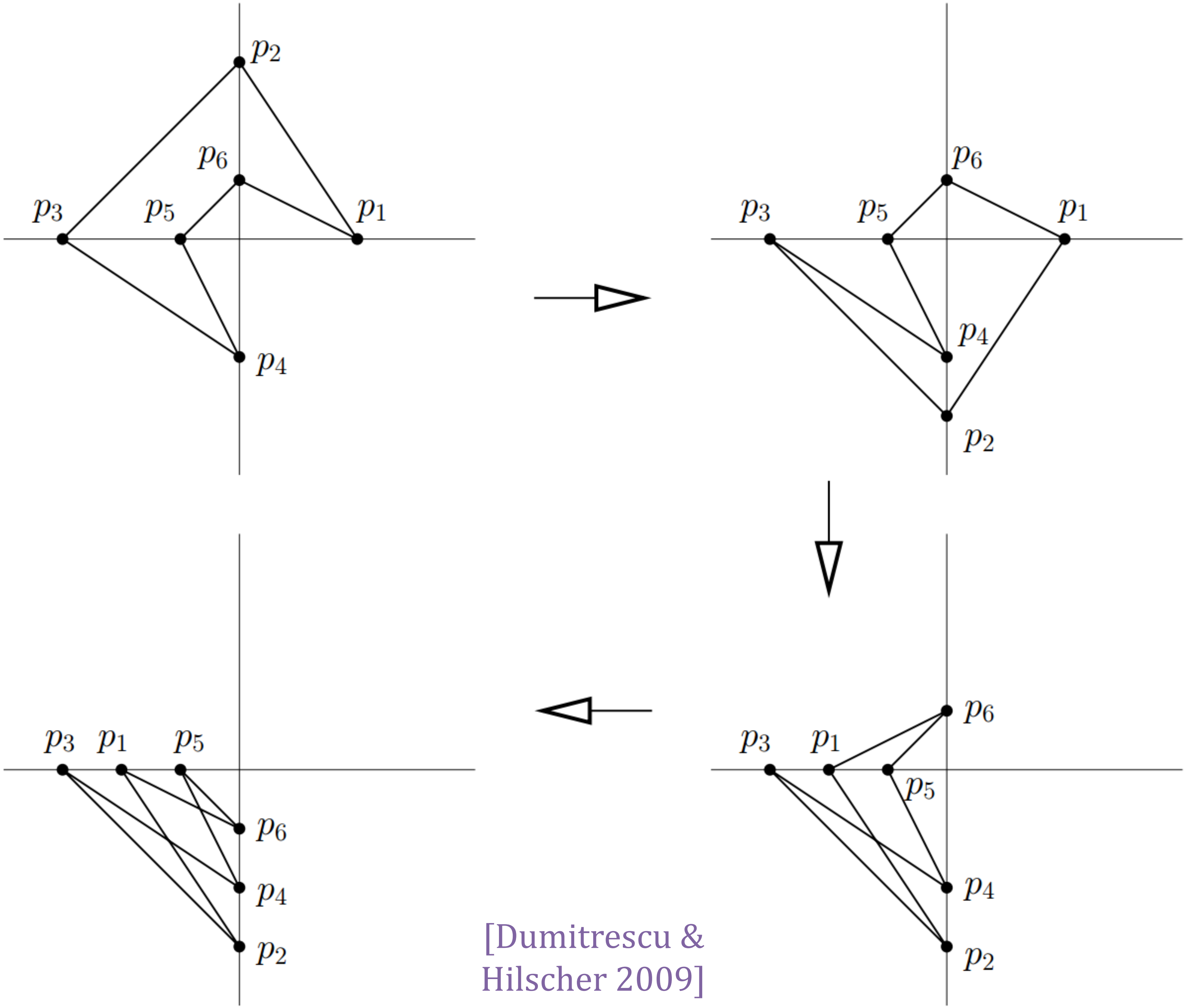
Demaine, Gassend, O'Rourke, Toussaint 2008



Fevens, Hernandez, Mesa, Morin, Soss, Toussaint 2001

Thurston 2001





[Dumitrescu & Hilscher 2009]