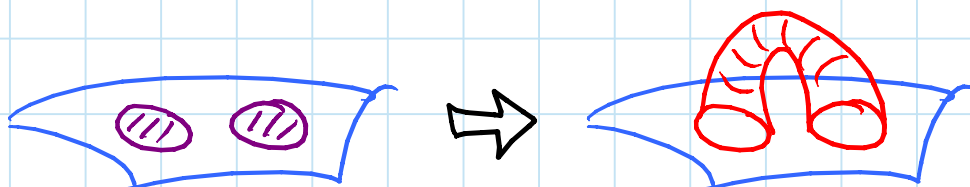


- What's a handle?
 - transformation on surface with 2 disks:

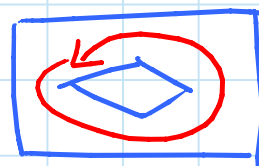


- every orientable manifold without boundary
= sphere + some handles

- Holes in unfoldings:

- Gauss-Bonnet Theorem:

turn angle along closed curve
+ curvature enclosed by curve
= 360°



($\Rightarrow$ total curvature of polyhedron = 720°)

- loop around hole has turn angle 360°

$\Rightarrow$ enclosed curvature = 0

- convex polyhedron $\Rightarrow$ no vertices

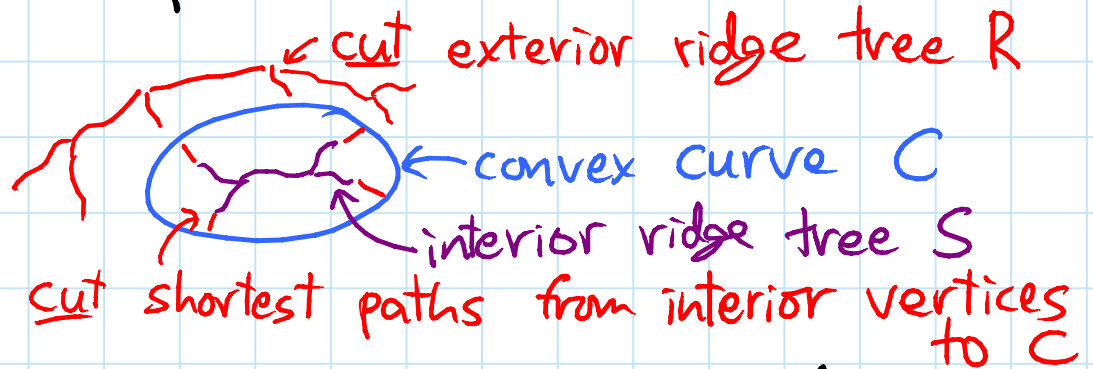
$\Rightarrow$ no need for cuts (suture up slits)

- Leaves of ridge tree:

- indeed, have unique shortest path to x

– limit of nonunique points though

- o Star unfolding: [Demaine & Lubiw 2011]
 star unfold inside & source unfold outside
 convex curve C on convex polyhedron
 $\Rightarrow$ no overlap!



- generalization of source unfolding
 from a point or from a geodesic

OPEN: no overlap if source/star roles reversed? (i.e., C is a reflex curve)

- o Zipper unfolding: [Demaine, Demaine, Lubiw, Shallit, Shallit 2010/2011]

- goal: cutting is a path
- edge zipper unfolding possible for Platonic & Archimedean
 - OPEN: Johnson solids
- but not for e.g. rhombic dodecahedron

OPEN: does every convex polyhedron have a general zipper unfolding?

○ Ununfoldable polyhedra:

- 2 "witch's hat" constructions

[Bern, Demaine, Eppstein, Kuo, Mantler, Snoeyink 1999]

- triangular faces: $4 \cdot 9 = 36$ faces

- convex faces: $4 \cdot 6 = 24$ faces

OPEN: hat $\times$ which polyhedra = unfoldable?

- pointy cube: $6 + 8 \cdot 3 = 30$ faces

[Tarasov 1999]

- starshaped pointy dodecahedron:

[Grünbaum 2001] $12 + 20 \cdot 3 = 72$ faces

- smallest: $4 + 3 \cdot 3 = \underline{13}$ faces

[Grünbaum 2002]



OPEN: 12 faces impossible "ununfoldable"

Strongly NP-complete to decide edge unfoldability of topologically convex orthogonal polyhedra

[Abel & Demaine - CCCG 2011]

- reduction from packing $\square$ s into a $\square$

Heuristics: Pepakura

o Band unfolding:

- bad cut can overlap

[Demaine, Demaine, Lubiw 1999]

- always a good cut avoiding overlap

[Aloupis, Demaine, Langerman, Morin,

O'Rourke, Streinu, Toussaint - CGTA 2008;

Aloupis - PhD 2005]

- also blooms by squishing

OPEN: attach top/bottom face, even for prismoid!
easy? ↩

◦ Continuous blooming: [Demaine, Demaine, Hart, Iacono, Langerman, O'Rourke 2009/2011]

- every unfolding can be refined & bloomed
 - cut along spanning tree of dual
 - ⇒ Hamiltonian dual: cut to make path
 - unroll one face at time
 - at all times:
 - subset of unfolding in xy plane
 - + subset of polyhedron in $z \geq 0$
 - ⇒ no intersection (allowing layering at xy)
- avoid 2D layering via "two step":
 - ① unfold f_i to almost coplanar with f_{i-1}
 - ② finish unfolding f_{i-1}
 - ③ repeat
- avoid 1D layering via "waltz"
 - ① unfold f_i to almost coplanar with f_{i-1}
 - ② unfold f_{i+1} slightly
 - ③ finish unfolding f_{i-1}
 - ④ repeat
- source unfolding unfolds by postorder traversal
 - imagine unrolling each shortest path separately
 - imagine unfolding as growing the polyhedron
 - claim: as long as the shortest path remains on surface, it remains shortest
 - tree just interleaves these path unrolls



PROJECT: implement these algorithms

OPEN: continuous blooming of

- star unfolding?
- sun unfolding?
- all edge unfoldings?
- all unfoldings?
- nonconvex polyhedra?