Hinged dissection software: just specific examples

**PROJECT:** hinged dissection animator
- implement slender adornments (refinement + expansive motion)
- implement general algorithm?
- implement polyform algorithm

**PROJECT:** design elegant hinged dissections

Polyform = \( n \) copies of one shape glued together along corresponding edges

**Inductive construction:**
- base case: hinge-dissect 1 copy such that every edge has incident hinge
- step: take spanning tree of copies remove leaf copy induct on \( n-1 \) remaining copies rotate base case to meet them reconnect \( \Rightarrow \) get same hinging

\( \Rightarrow \) folded states (use slender for motion)

Also: poly\( \Delta \) \( \Rightarrow \) poly\( \Box \), etc.
3D [Demaine, Demaine, Lindy, Souvaine 2005]

Physical:
- in liquid
- DNA
- Macrobot/Decibot
- related: reconfigurable robots

0 Rectangle $\to$ rectangle [Montucla 1778]
- superposing strips method

- same method for Dudeney's $\triangle \to \square$
- more stable table [Frederickson 2008]

**PROJECT:** build reconfigurable furniture

0 # pieces doubles? at least, in worst case
Pseudopolynomial: say integer

if polygon vertices lie on common grid,
# pieces = poly(n,r)
⇒ # grid positions = \( \frac{\text{size}}{\text{cell size}} \)

- idea: ensure constant-depth recursion

*1* triangulate polygons with grid vertices
⇒ matching Δ areas of \( \frac{1}{2} \) [Pick's Theorem]

*2* chainify \( \text{△} \Rightarrow \text{△} \)
⇒ vertices on \( \frac{1}{3} \) grid

*3* fix which vertices connect which Δs
by only modifying parent in subtree move

*4* \( \text{Δ} \Rightarrow \text{Δ} \) by overlaying 3 constructions:

A

B

C

Feedback

... using pseudocuts

⇒ simulate cut overlays
3D dissection:
- Volumes must match
- Insufficient by Dehn's solution [1901]
  to Hilbert's Third Problem [1900]
- Dehn invariants must match:
  \[ \sum_{\text{edge } e} l(e) \otimes [\Theta(e) + Q \cdot \pi] \]
  (ignore added rational multiples of \( \pi \Rightarrow \) "irrational part"

Tensor product

- Tensor product space: linear combination
  of pairs \( l \otimes \Theta \) where
  \[ l_1 \otimes \Theta + l_2 \otimes \Theta = (l_1 + l_2) \otimes \Theta \]
  \[ l \otimes \Theta_1 + l \otimes \Theta_2 = l \otimes (\Theta_1 + \Theta_2) \]
  \[ c(l \otimes \Theta) = (cl) \otimes \Theta = l \otimes (c \Theta) \quad \forall c \in \mathbb{Q} \]

- Dehn's Theorem: invariant under dissection
  - e.g.: cut edge \((l_1 + l_2) \otimes \Theta \rightarrow l_1 \otimes \Theta + l_2 \otimes \Theta\)
  slice angle \(l \otimes (\Theta_1 + \Theta_2) \rightarrow l \otimes \Theta_1 + l \otimes \Theta_2\)
  \(\Rightarrow\) no dissection of cube \(\rightarrow\) regular tetrahedron

\[ 12 \left( 1 \otimes 90^\circ \right) \]
\[ = \emptyset \]
\[ 6 \left( 2.04\ldots \otimes 70.5288\ldots \right) \]
\[ \text{arccos} \left( \frac{1}{3} \right) \]
- 3D dissection exists $\iff$ volumes & Dehn Invariants match [Sydler 1965] [Jessen 1968]
- ditto in 4D

**OPEN**: 5D & higher?

**OPEN**: efficient algorithm to check Dehn match
- decidable [Kreinovich–Geomb. 2008]

**OPEN**: algorithm to find dissection
- refinement into hinged dissection still works [Abel et al.]