Why does having a hole in your 2D sheet of a paper pose a problem to your rolling method? I can imagine a piece of paper having not a hole, but a little transparent patch of cellophane. Of course we would be able to roll the paper in the same way into a little triangle and then unfold. What difference does the hole make?
Have there been any investigations into linkages that allow for points that can “slide” along edges?
I made a linkage from paper strips to demonstrate how a parallelogram can flip into a contraparallelogram. But, I didn’t understand why this flip page breaks the gadgets.
Could you go over the fix for the contraparallelogram gadget again?
[Jim Loy, 2000]
“Faster”
Arthur Ganson
In the case where $d = 3$ and the output follows a surface ..., do we have two degrees of freedom in the linkage (to account for the two-dimensional surface)?
Has anyone looked at a Kempe construction for curves not represented by polynomials? In particular, for piecewise-defined curves ... say ... cubic splines?

The intersection gadget is clear, but what’s the union gadget?
I would definitely be interested in learning more about these axioms. It seems so mind-boggling (at least, from someone used to compass-and-straightedge constructions) that one axiom [...] would make the difference between solving fourth-degree and any-degree polynomials.
straight edge & compass
Huzita axioms

A1.

A2.

A3.

A4.

A5.

A6.

Hatori axiom:
Solution (0.0000,0.3333): err = 0.0000 (rank 3)
Fold the right edge to the left edge, making line A.
The intersection of the bottom edge with line A is point P.
Form a crease connecting the top left corner with point P, making line B.
Bring the bottom left corner to line B, making line C.
The intersection of the left edge with line C is point Q.

Solution (0.0000,0.3318): err = 0.0015 (rank 3)
Fold the right edge to the upward diagonal, making line A.
Fold the top edge to line A, making line B.
The intersection of the left edge with line B is point P.
Angle Quintisection

Designed by Robert J. Lang
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Angle quintisection is division of an arbitrary angle into fifths. This requires solution of an irreducible quintic equation and thus is not possible with the 7 Huzita-Hatori axioms, each of which defines a single fold by simultaneous alignment of points and lines. By permitting the simultaneous creation of two or more folds that satisfy various combinations of point/line alignments, it is possible to solve higher-order equations, as this example illustrates.

1. Start with a long strip 1 unit high and 5–6 units long. Angle EAB is the angle to be quintisected. Make a vertical crease about 1/3 unit from the right side.

2. Fold line FG down to lie along edge AB.

3. Fold point F over to point A.

4. Fold and unfold.

5. Make a horizontal fold aligned with point C.

6. Fold point C to point A and unfold, making a second longer horizontal crease.

7. Mountain-fold corner D behind.

7. Here’s where it all happens. Fold edge AE down along crease AJ. At the same time, fold the left flap up so that point F touches crease HI at the same point that edge AE does and point C touches crease AJ. You will have to adjust both folds to make all the alignments happen at once.

8. Here’s what it looks like folded. Yours may not look exactly like this, depending on the angle you used and the length of your strip. Unfold to step 7.

9. Bisect angle EAJ.

10. Fold crease AK down to AM and unfold.

11. Bisect angle LAM.

12. Angle EAM is now divided into fifths.
$k$-gon

$k$-hat

[Demaine, Demaine, Lubiw 1999]