

**Does the first triangulation  
work for arbitrary  $n$ ?**

**Do triangulations which use a  
mix of the two fall in between  
the two in terms of what  
angles and  $n$  are possible?**

**Have you tried the same thing  
with a large  $k$ -gon?**

**Can you explain what  $C^1$   
and  $C^2$  are?**

**Could you go over the  
definition/meaning of  
semi-creases?**

**Why does that  $n(p)$  is  
perpendicular to the boundary  
edge imply that  $n'(p)$  is?**

**You're proving things are impossible, even though we have paper examples of them existing! So my question is, what about the mathematical model is more restrictive than the real world? What choice do we make in modeling the paper that allows us to prove something is impossible in the model which is possible in real life?**

# Algorithmic Folding Complexity\*

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**Abstract.** How do we most quickly fold a paper strip (modeled as a line) to obtain a desired mountain-valley pattern of equidistant creases (viewed as a binary string)? Define the *folding complexity* of a mountain-valley string as the minimum number of simple folds required to construct it. We first show that the folding complexity of a length- $n$  *uniform* string (all mountains or all valleys), and hence of a length- $n$  *pleat* (alternating mountain/valley), is  $O(\lg^2 n)$ . We also show that a lower bound of the complexity of the problems is  $\Omega(\lg^2 n / \lg \lg n)$ . Next we show that almost all mountain-valley patterns require  $\Omega(n / \lg n)$  folds, which means that the uniform and pleat foldings are relatively easy problems. We also give a general algorithm for folding an





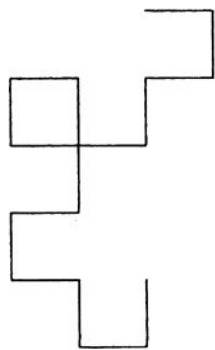
render by  
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1990



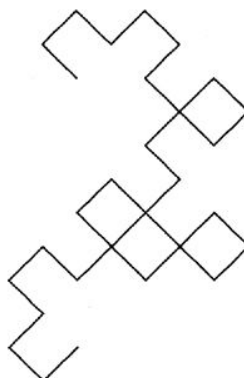
### FIRST ITERATION



*"At the earliest drawings of the fractal curve, few clues to the underlying mathematical structure will be seen."*

IAN MALCOLM

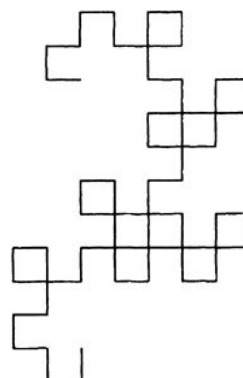
### SECOND ITERATION



*"With subsequent drawings of the fractal curve, sudden changes may appear."*

IAN MALCOLM

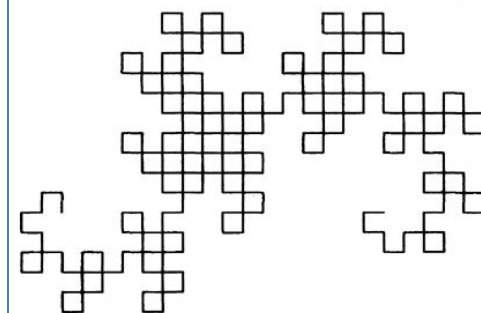
### THIRD ITERATION



*"Details emerge more clearly as the fractal curve is re-drawn."*

IAN MALCOLM

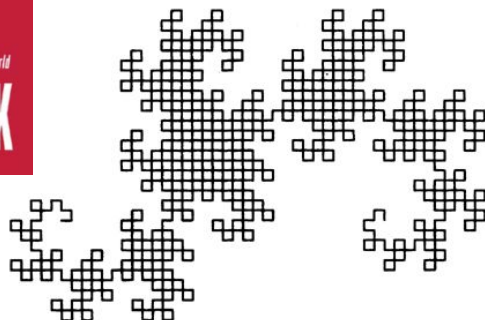
### FOURTH ITERATION



*"Inevitably, underlying instabilities begin to appear."*

IAN MALCOLM

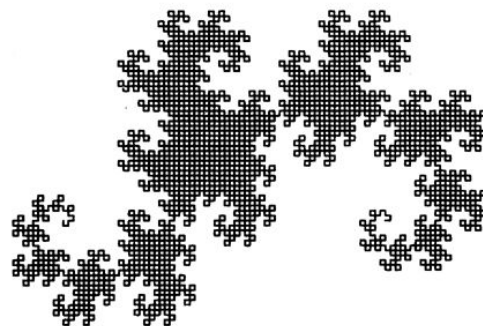
### FIFTH ITERATION



*"Flaws in the system will now become severe."*

IAN MALCOLM

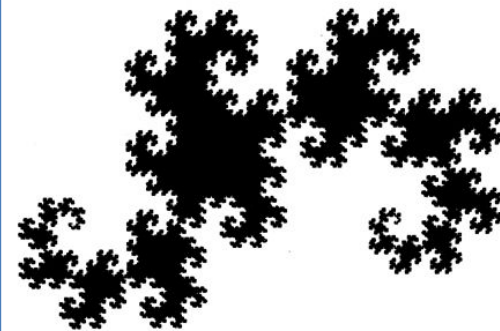
### SIXTH ITERATION



*"System recovery may prove impossible."*

IAN MALCOLM

### SEVENTH ITERATION



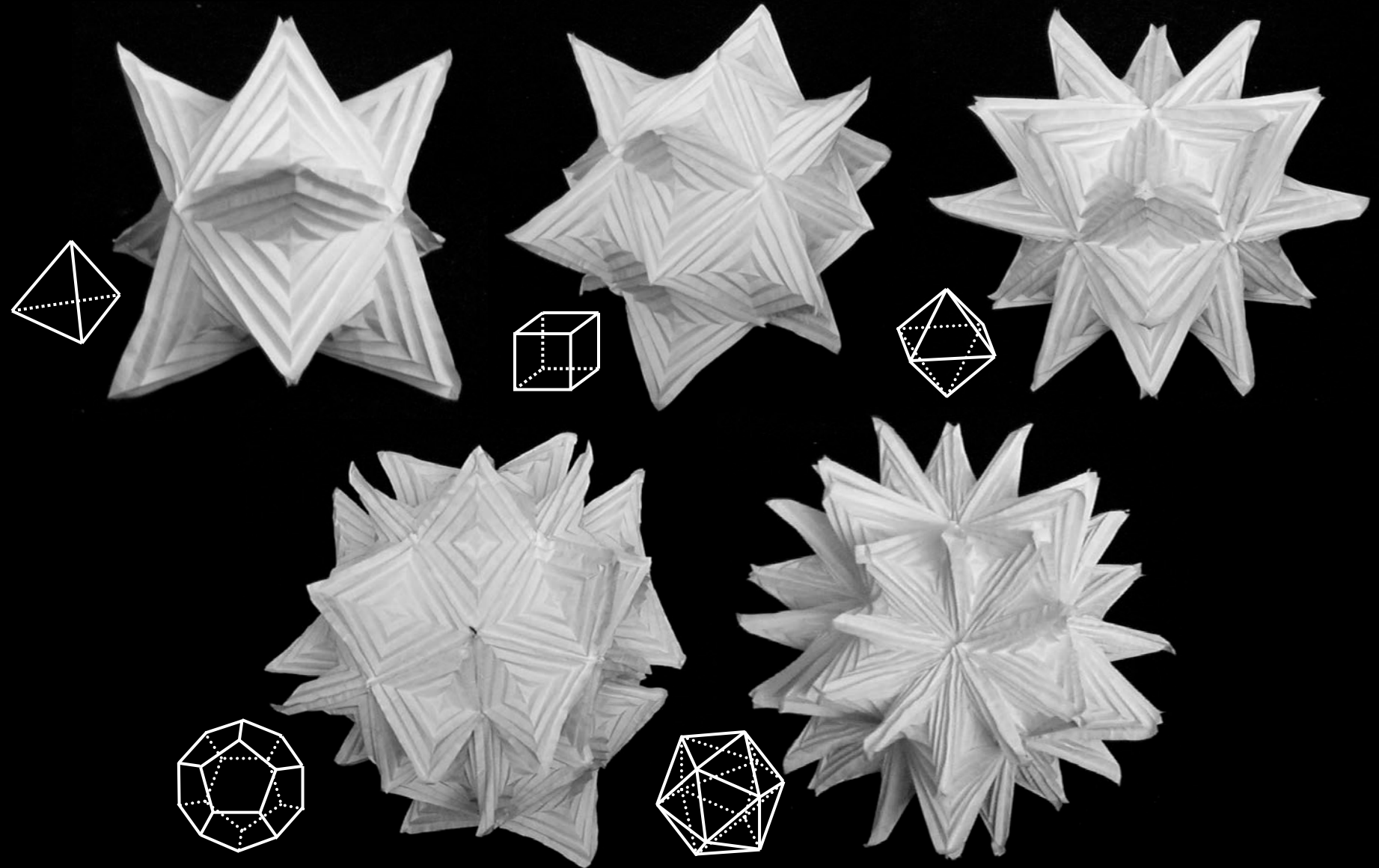
*"Increasingly, the mathematics will demand the courage to face its implications."*

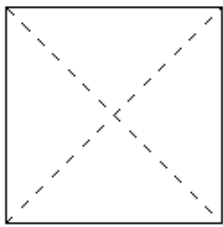
IAN MALCOLM



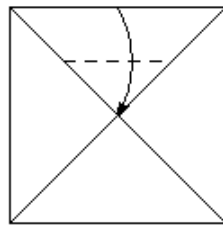
# Hyparhedra: Platonic Solids

[Demaine, Demaine, Lubiw 1999]

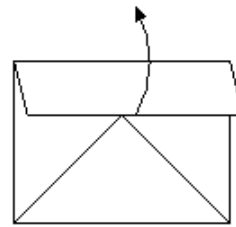




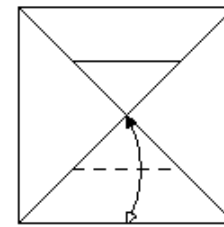
Crease the diagonals



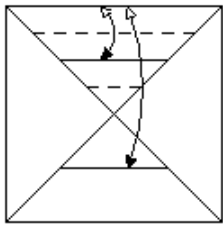
Fold the top edge to the center point, creasing only between the diagonals



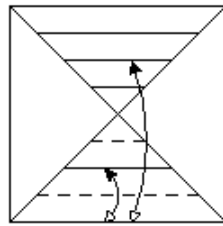
Unfold



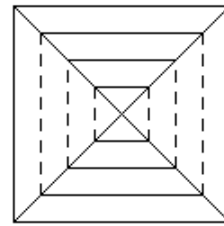
Repeat on the bottom (fold and unfold)



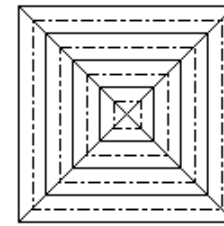
Fold and unfold on 1/4 and 3/4 marks



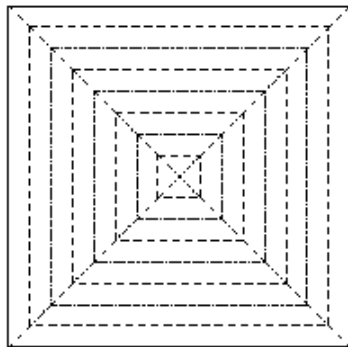
Repeat on the bottom



Repeat on left and right sides



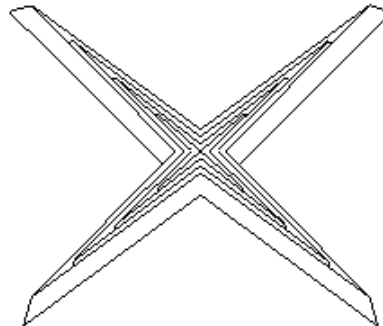
Turn over, and crease in between the squares in the opposite direction



Final crease pattern

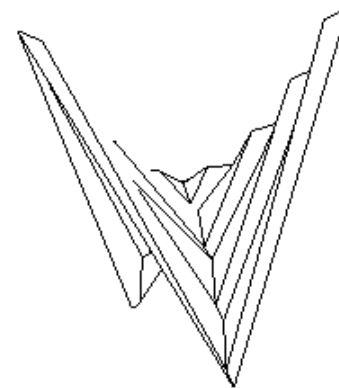
--- Valley fold

---- Mountain fold



Folding the crease pattern completely forms an "X" shape

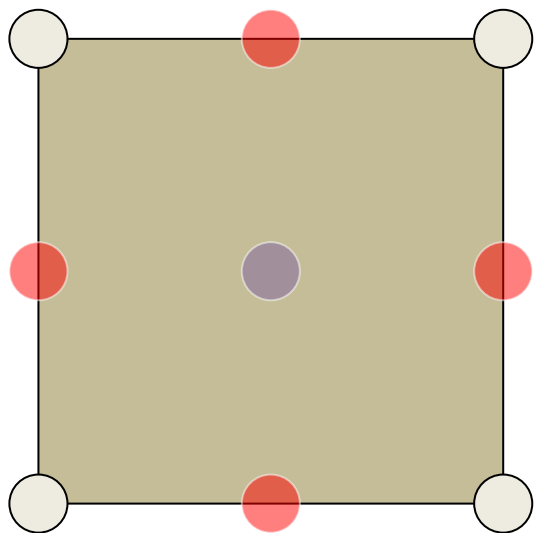
Partially opening it forms a hyper



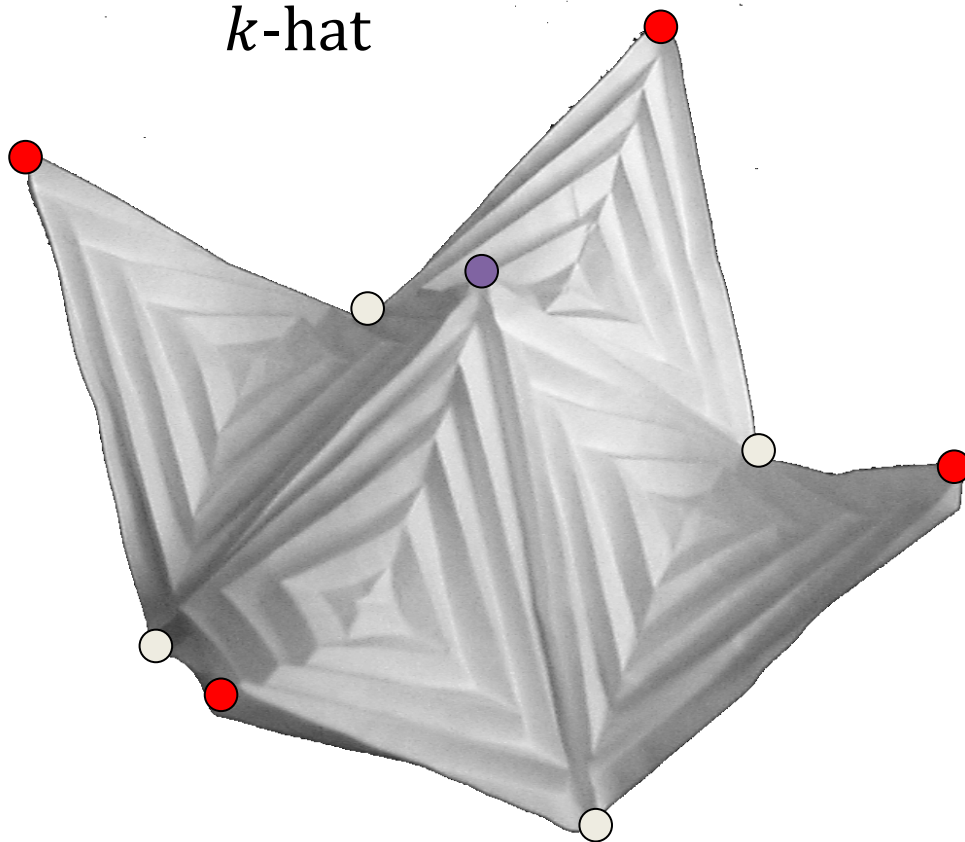
Demaine,  
Demaine,  
Lubiw  
1999



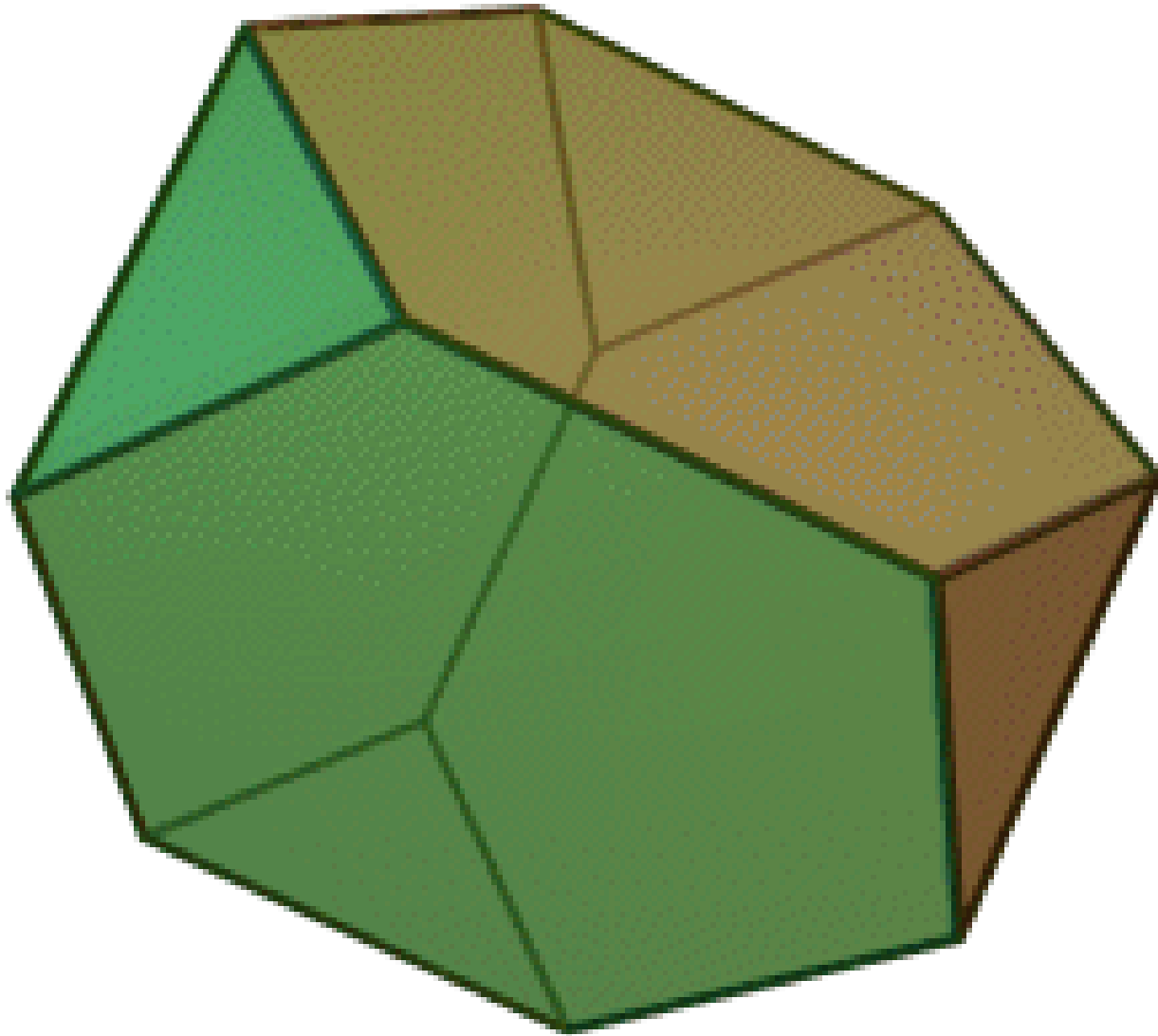
$k$ -gon



$k$ -hat



[Demaine, Demaine, Lubiw 1999]



<http://en.wikipedia.org/wiki/File:Truncatedtetrahedron.gif> by Cyp