- Funny comments
- Positive comments
  "Such a powerful theorem, so quickly"

- Folding practice: numbers 6, 8, 4, 9!
  - cf. Jason Ku's universal alphabet & Jeannine Mosely's 4-fold alphabet
  - **PROJECT**: design 4-fold digits

- History: why "silhouette" & "gift wrapping"?

- **Practical?** strips occasional in origami
  - pretty good n x n checkerboard design (see L4)

- **Pseudopolynomial upper/lower bound?**
  \[ \ell \text{ polynomial in } n = \# \text{ vertices} + \text{edges} + \text{faces} \]
  \[ \leq 5(nr)^c \] & geometric ratio r
  - here, \( r = \frac{\text{max. diameter of face}}{\text{min. altitude of triangle}} \)
  & want to bound # folds & aspect ratio
  - upper bound claimed, but not explicit
    - \( O(nr) \)? \( O(n+r) \)?
    - presumably a lower bound e.g. \( \Omega(n+r) \)
0. **Seam placement**
   - Convex seam patterns all possible:
     - Visit seam polygons in a tour
     - Transition increases/decreases width of strip via log ratio width gadgets & offsets strip to "cover" next polygon
   - Some nonconvex possible:
     - OPEN: which?

0. **Hide gadget via simple folds?** (some layers)
   - Silhouette easy: valley fold, not mountain
   - OPEN: 2-color pattern?
     - Idea: bicolor turn gadget/excess
     - TRY TO SOLVE
   - OPEN: convex seam placement

**Simple Folds:** [Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena, 2000/2004]

0. **Motivation:** Bending rigid material

0. **Definition:** Single line segment
   - $\pm 180^\circ$ rotation
   - No collision during motion
Example:

\[ \begin{array}{c}
v \quad \text{crimp} \\
\downarrow \quad \text{V}
\end{array} \quad \text{is mingling (but not "forever")} \]

not left mingling

\[ \begin{array}{c}
v \quad \text{mingling} \\
\downarrow \quad \text{V}
\end{array} \quad \text{not right mingling} \]

\[ \Rightarrow \begin{array}{c}
v \quad \text{V} \\
\downarrow \quad \text{V}
\end{array} \quad \text{is not mingling} \]

& not flat foldable

Another:

\[ \begin{array}{c}
v \quad \text{crimp} \\
\downarrow \quad \text{V}
\end{array} \quad \text{is mingling} \]

& flat foldable

\[ \Rightarrow \begin{array}{c}
v \quad \text{V} \\
\downarrow \quad \text{V}
\end{array} \quad \text{is mingling} \]

& flat foldable

\[ \Rightarrow \begin{array}{c}
v \quad \text{V} \\
\downarrow \quad \text{V}
\end{array} \quad \text{done} \]
Algorithm: (NEW) (covered in C3)
- search (left to right) for segment that's crimpable or end foldable
- if none found: STOP ~ not flat foldable
- else: do fold $x \rightarrow y \rightarrow z$
  merge segments $x, y, z \rightarrow x - y + z$
  go back one segment (left of $x$)
  continue search

Correctness:
- doing fold changes foldability only of adjacent segments
  $\Rightarrow$ enough to back up 1 step

Running time: $O(n)$
- # right steps $= n + $ # left steps
  $= $ # folds done
  $\leq 2n$
  (amortization: charge left steps to fold just done)
0 Every mountain-valley pattern can be made flat foldable by adding creases:
   - between consecutive \( MM \)'s add \( V \)
   & between consecutive \(VV\)'s add \( M \)
   \( \Rightarrow \) alternating \( M/V \)
   \( \Rightarrow \) flat foldable
   (globally smallest segment is crimpable)

0 \( d \)-dimensional paper
   \( \Rightarrow \) \( (d-1) \)-dimensional creases
   \& \( (d+1) \)-dimensional ambient space
   \( - d \)-dimensional = flat folding