6.885	Lecture 22	Dec. 3, 2007

Self-touching Carpenter's Rule Theorem:

[Abbott, Demaine, Gassend 2007]

connected configuration space of open or closed chain linkage, even allowing self-touching configurations:

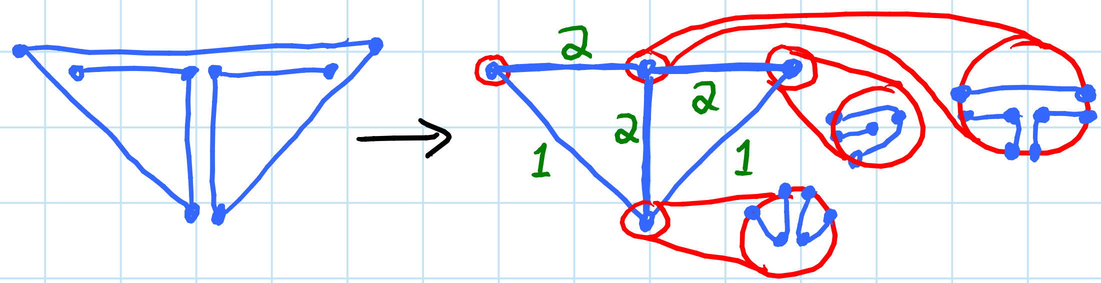
fixing clockwise order

collinear

How to define self-touching configurations?

① combinatorially: where geometry leaves ambiguity, explicitly specify combinatorial order information (as in Lecture 5)

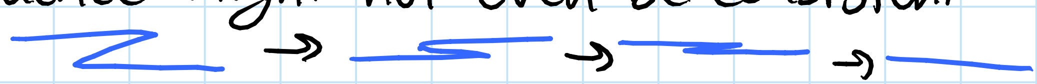
e.g.:



② limits of nontouching configurations:

issues:

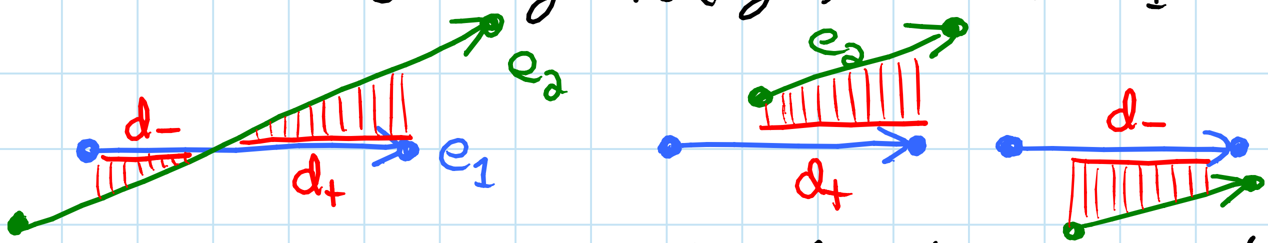
- actual limit loses necessary information
- inconvenient to work with entire sequence
- sequence might not even be "consistent":



- can every self-touching configuration (in sense of ①) be reached by a limit?

Big idea: annotate nontouching configurations with continuous information whose limit gives combinatorial information of touching configuration

- annotate every pair of oriented edges e_1, e_2 with $\mathrm{Ord}(e_1, e_2) = d_+(e_1, e_2) - d_-(e_1, e_2)$
- $d_+(e_1, e_2)$ = length of projection of e_2 on positive (left) side of e_1
- $d_-(e_1, e_2)$ = length of projection of e_2 on negative (right) side of e_1



$\Rightarrow \mathrm{Ord} > \emptyset$ $\quad \Rightarrow \mathrm{Ord} < \emptyset$

- nontouching $\Rightarrow$ all d_+ or all d_-
- Sign in touching limit specifies whether e_2 is on positive (left) or negative (right) side of e_1
- limit of annotated configuration forces consistency: annotation must eventually have same sign as limit
- to get combinatorial self-touching config. as limit of nontouching configurations, use existence of δ-perturbations
  [Ribó Mor — PhD 2006; mentioned in L5]
  - allow edge lengths to perturb arbitrarily slightly
  - can also do easily by adding tiny edges to make turns, which disappear in limit

Proof of self-touching Carpenter's Rule Theorem:

- nontouching configurations are connected
- so are nontouching configurations where we allow edge lengths to perturb by ε
  - straighten/convexify perturbed version
  - fix edge lengths while staying straight/convex
- configurations are also bounded if we pin a vertex: can't get farther than perimeter
- we are annotating these configurations and taking all possible limits (closure)
- closure of connected bounded (compact) semi-algebraic set is connected
- intersection as $\varepsilon \to \emptyset$ preserves connectivity because configuration spaces nest $\square$

From folding to reconfigurable robotics, programmable matter, nanomanufacturing, self-(re)assembly:

(1) hinged dissection: rigid pieces hinged together

- one hinging can fold into all structures made of n squares [Demaine, Demaine, Eppstein, Frederickson, Friedman 1999/2005]
  or
  of n cubes [Demaine, Demaine, Lindy, Souvaine 2005]
  - in 2D, slender adornments apply [L20]
  - $\boxed{\text{OPEN}}$: can 3D be done by continuous motion without self-intersection?
- now can fold into any finite set of desired 2D/3D shapes [Abbott, Abel, Charlton, Demaine, Demaine 2007]

- physically controllable at millimeter scale in 2D [Mao, Thalladi, Wolfe, Whitesides, Whitesides 2002]
- 3D cube hinging in sculpture [Demaine, Demaine, Palmer 2004/2006]

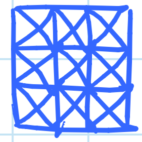
- contrast to previous reconfigurable robots which dynamically attach/detach units
  ~ complicated & fault-prone [Rus et al.]

- programming shape via local instructions/energies [Cheung, Demaine, Griffith, Jacobson 2007]

② DNA origami [Rothemund 2006]

- one medium strand of random DNA
- lots of little "glue" DNA strands
  to zip up parts of medium strand
- works surprisingly well, though size limited
- 3D in process

③ robotic paper [Demaine, Fearing, Rus, Wood 2007]

- goal: build regular grid crease pattern 
- power & computation embedded throughout
- each edge can actuate folding to desired
  angle & can also "lock" into shape

Folding puzzles:

- MAD magazine fold-ins [Jaffee 1964–]
- World War II 5-pigs puzzle [1939]
- pattern puzzles: [Grabarchuk 2005]
  make target pattern [Mitchell 1998]
- patterned puzzles: [Russian] [Schröer 1995]
  paper also has [Israel 1989] [Cox 1981]
  initial pattern [Horse & Cow 1941] [McDonalds 1985]
  [Chicken & Egg 2000]
- Burr puzzles [Lang 2005; Ku 2005]
- Rubik's cube [Vomberg]
- MIT CSAIL puzzles [Demaine & Demaine 2002–2007]
  - 2002: fold anywhere
  - 2003: inspired by map folding problem
    wrote software to test all solutions
  - 2004: inspired by checkerboard problem
  - 2005: inspired by origami tessellations
  - 2006: inspired by polyhedron wrapping
  - 2007: inspired by "turning inside-out"

OPEN: which polyhedral surfaces can be turned inside-out with finite number of creases (rigid origami)?

- square tube possible [Stone; Gardner]
- positive curvature impossible [Connelly]