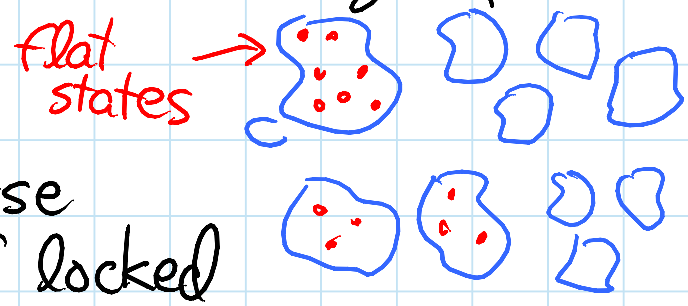
6.885	Lecture 17	Nov. 7, 2007

Fixed-angle linkages: (entire lecture)

Flat-state connectivity [Aloupis et al. 2002 & 2002]

- connected if there's a motion between any two non-self-intersecting flat configurations
  - weaker form of connected config. space
  - ⇒ flat states are "canonical" for C



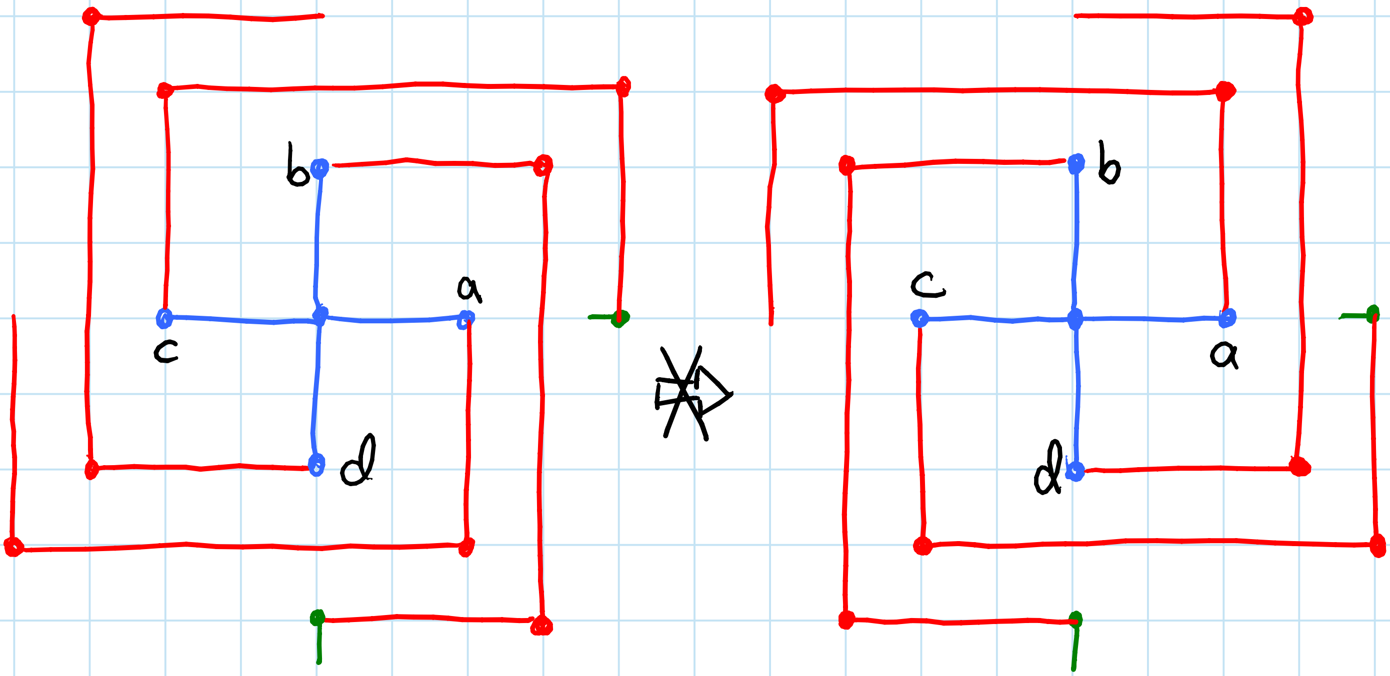
- disconnected otherwise
  - stronger notion of locked

- fixed-angle chain might have no flat states (even NP-hard to know which) but proteins do, and seems important

Flat-state connectivity: summary of results

open chain	$\boxed{\text{OPEN}}$
– nonacute angles	connected
– equal acute angles	connected
– angles strictly between 60° & 90° & unit edge lengths	connected
– has a monotone state	connected
– angles strictly between 60° & 150° & unit edge lengths	connected [Aloupis & Meijer 2006]
– using 180° edge spins	disconnected
– orthogonal & using 180° edge spins	connected
set of open chains, pinned at one end	
– orthogonal	connected
– orthogonal & $\underbrace{\text{partially rigid}}_{\text{some edges can't spin}}$	disconnected
closed chain	$\boxed{\text{OPEN}}$
– nonacute	$\boxed{\text{OPEN}}$
– orthogonal	$\boxed{\text{OPEN}}$
– orthogonal & unit edge lengths	connected
tree	$\boxed{\text{OPEN}}$
– orthogonal	$\boxed{\text{OPEN}}$
– orthogonal & partially rigid	disconnected
graph – orthogonal	disconnected

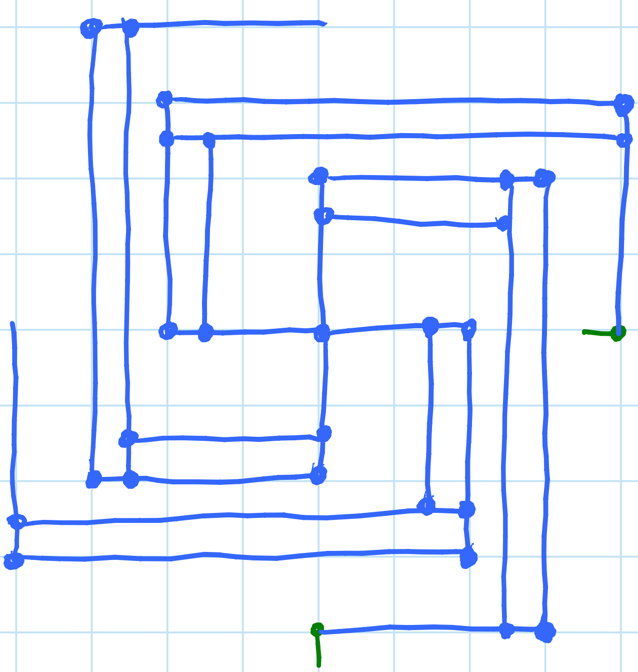
Flat-state disconnected partially rigid tree:

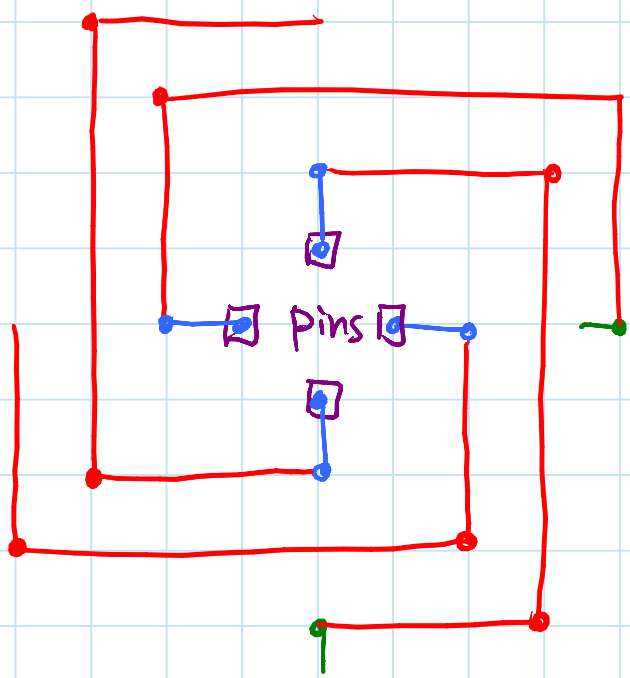


- inner edges flexible; rest rigid
- pins to remove reflectional symmetry

Variations:

① four pinned chains, partially rigid





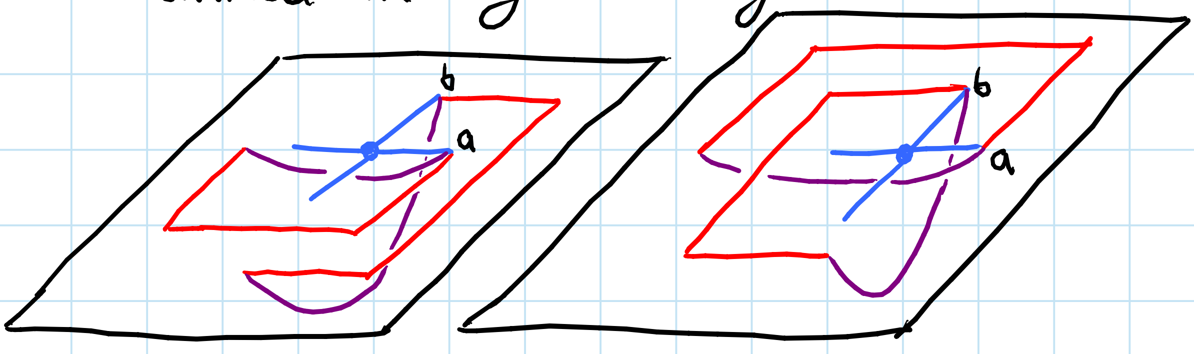
② orthogonal graph

Flat-state disconnected partially rigid tree: (cont'd)

Claim: these two flat states are disconnected

Proof: view plane abcd as stationary
- four branches & two sides of plane

$\Rightarrow \geq 2$ branches must flip through same side
- opposite branches (ac or bd) can't share:
  - geometric argument
  - links parallel to axis of rotation hit exactly
  - can shrink a & b edges for proper collision
- adjacent branches (say, ab) can't share:
  - topological argument
  - connect shallow rope a → end of a branch
  - connect deeper rope b → end of b branch
  - unlinked in left config.
  - linked in right config



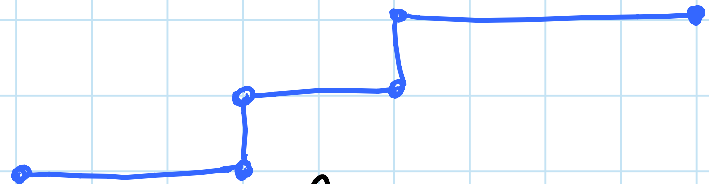
  - ropes stay as-is during motion above plane

  $\Rightarrow$ a & b branches intersect □

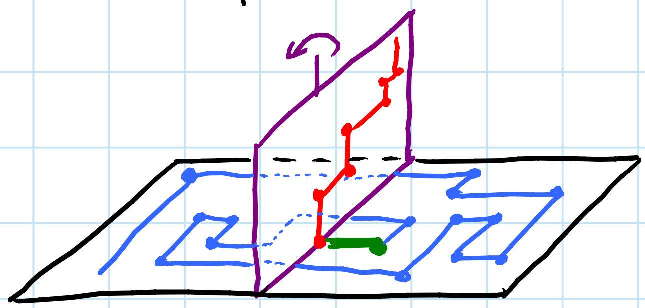
OPEN: flexible tree? orthogonal tree?

Orthogonal open chains are flat-state connected:

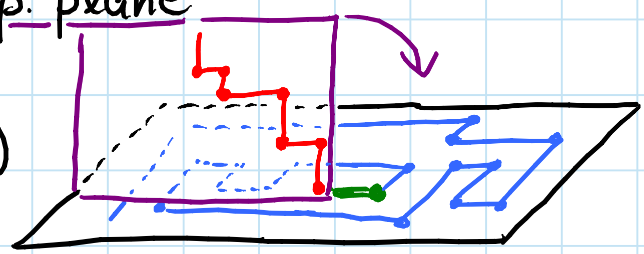
- canonical form: staircase (trans config. from L16)
  (alternate $\pm 90^\circ$ turns)



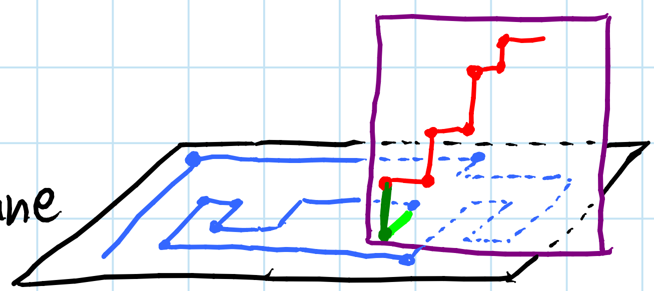
- lift a flat state into canonical form:



  ⓪ induction hypothesis:
  - half of chain remains in plane
  - half of chain in canonical form in perp. plane



  ① rotate canonical half (and its containing plane) so that next edge makes a larger staircase



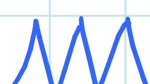
  ② rotate larger staircase (around following edge) to lift into staircase plane

  ③ repeat
- FedEx via canonical form

Nonacute open chains: similar

- canonical state = z-monotone ($\Rightarrow$ never hit $z=0$)

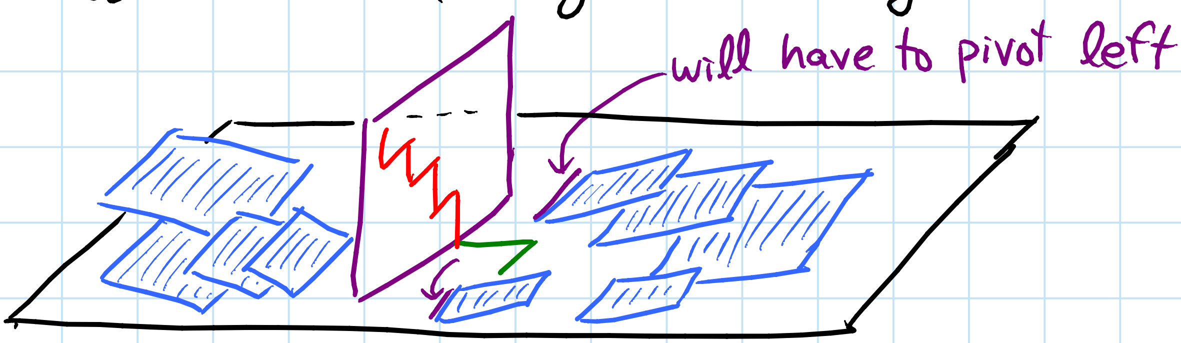
Equal acute chains: similar

- canonical state = zig-zag  ($\Rightarrow$ lifting harder)

$\boxed{\text{OPEN}}$: general chains?

Multiple pinned orthogonal chains are flat-state connected

- k chains with initial link pinned in xy plane
- two kinds of chains:
  1. initial edge parallel to x axis "x-edge"
  2. initial edge parallel to y axis "y-edge"
- canonical form:
  - x-chains staircase in xz planes below xy plane
  - y-chains staircase in yz planes above xy plane
  - non-self-intersecting assuming distinct x & y coords.
- canonicalization algorithm processes x- & y-chains separately
- to lift two links of y-chain into staircase:
  - invariant: next link is y-link & staircase extends
  - so pick up one link trivially
  - next swing other y-chains' planes down to almost-horizontal, away from this y-chain



  - pivot around vertical edge to incorporate next x-edge
  - lift & repart planes around this x coord.
  - pivot around vertical edge to restore invariant

[also covered first three pages of L18]