6.885	Lecture 1	Sept. 5, 2007

6.885: Geometric Folding Algorithms
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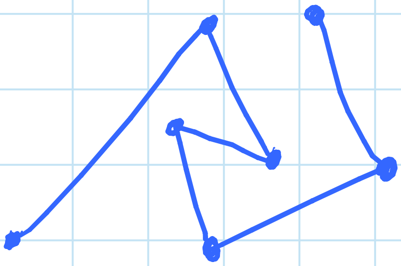
http://courses.csail.mit.edu/6.885/fall07/

In general: Mathematics & algorithms behind (un)folding of geometric objects

Applications/connections to:
- robotics: arm motion, reconfigurable, ...
- graphics: morphing, animation, ...
- mechanics: steam engines, ...
- manufacturing: sheet-metal & tube bending, nanomanufacturing, optics, ...
- medical: stents
- aerospace: telescope deployment, ...
- biology: protein folding & design, ...
- sculpture: origami, interactive sculpture, ...
- architecture: dynamic architecture, deployable/collapsible structures, ...

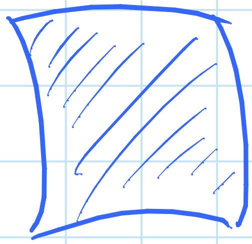
Geometric objects & rules for folding:

(I) linkage



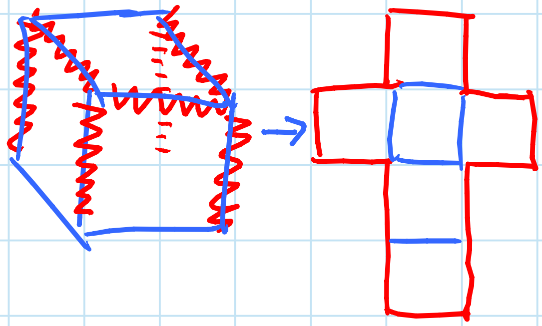
↳ rigid bars
↳ [don't cross]

(II) paper



↳ don't stretch
↳ don't tear
↳ don't cross

(III) polyhedron



↳ cut surface
↳ one piece
↳ no overlap

Questions:

- What structures can fold at all or in a particular way? FOLDABILITY
- What shapes, or other properties, are possible by folding? DESIGN

Results:

- Everything is foldable! (& here's an algorithm to do it) UNIVERSALITY
- Efficient algorithm to decide foldability DECISION
- Computationally intractable to decide foldability HARDNESS

The Class:

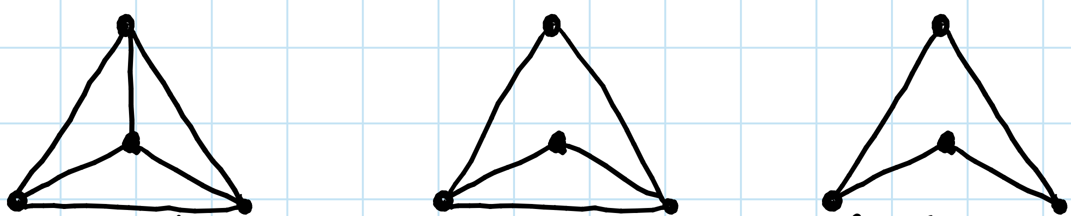
- lectures (mandatory attendence)
- problem sets (not a lot)
- project & presentation
  - implement algorithm/illustration/tool
  - sculpture/design
  - pose an open problem
  - survey a subfield
  - try to solve an open problem
- open problem session (optional)

I LINKAGES: first, allowing intersection

Early motivation: convert linear motion
(steam engines) $\Leftrightarrow$ circular motion
- Watt parallel motion [1784]
- Peaucellier inversor [1864]
- Kempe's *How To Draw a Straight Line* [1877]

Universality: [Kempe 1876; Kapovich & Millson 2002; King 1998; Abbott, Barton, Demaine, O'Rourke]
- there's a linkage signing your name (tracing any polynomial curve) Erik
- stronger results on topology of motion space
- $\boxed{\text{OPEN}}$: what's possible forbidding crossings?

Rigidity: which linkages fold at all?



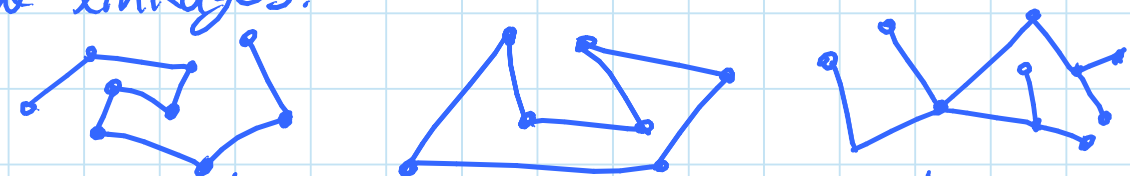
rigid in 2D & 3D | rigid in 2D not 3D | flexible in 2D

- efficient characterization in 2D
- 3D $\boxed{\text{OPEN}}$
- generalization to tensegrities
  - bars: fixed
  - struts: lengthen
  - cables: shorten

(I) LINKAGES: forbidding intersection

Reconfiguration: fold from config. A to config. B — always possible?

- roadmap algorithm: [Canny et al.]
  exponential time, polynomial space
- PSPACE-hard [Alt, Knauer, Rote, Whitesides 2004]
- special linkages:



open chain, closed chain, tree

	Chains	Trees
2D	Always [Connelly, Demaine, Rote 2000]	Not [Biedl et al. 1998]
3D	Not [Cantarella & Johnston 1998]	Not
4D+	Always [Cocan & O'Rourke 2001]	Always [ditto]

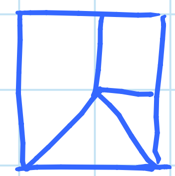
- algorithms for 2D chains
  [Streinu 2000; Cantarella, Demaine, Iben, O'Brien 2004]
- deciding whether 3D chain / 2D tree is locked
  or can be folded between any two configs.
  is OPEN (but A→B is PSPACE-complete)
- tools for proving locked [Connelly, Demaine, Rote 2002]
- interlocked short chains
  [Demaine, Langerman, O'Rourke, Snoeyink 2002&3]
- protein folding leads to many cool problems
- flips & other reconfigurations

II PAPER:

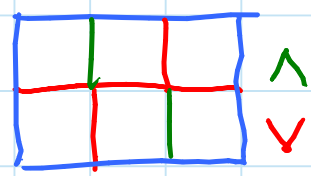
Definitions: folded states & motions

- subtlety: overlapping layers
- every folded state is reachable by a motion [Demaine, Devadoss, Mitchell, O'Rourke 2004]

Foldability: which crease patterns fold flat?



- NP-hard [Bern & Hayes 1996]
- efficient characterization for single vertex [Kawasaki; Justin; Hull 2003]
- rectangular maps: $\boxed{\text{OPEN}}$
  - $1\times n$ solved, $2\times n$ open
  - polynomial for "simple folds"
  - NP-hard with diagonal creases (45°)

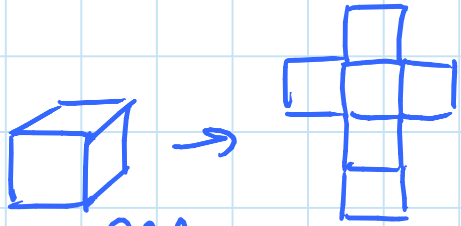


Design: what shapes can be folded?

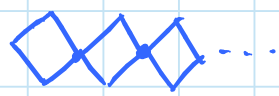
- any 2D polygon, 3D polyhedron, 2-color pattern (inefficiently) [Demaine, Demaine, Mitchell 2001]
- TreeMaker: efficient "tree base" [Lang & Demaine]
- fold & cut: any set of line segments can be aligned by flat folding [Demaine, Demaine, Lubiw 1998]
- flattening: any polyhedron can be folded flat (without tearing) [Demaine, Demaine, Lubiw; Bern & Hayes]
- tessellations [Bateman; Lang; Palmer]
- curved creases [Resch; Huffman]

III POLYHEDRA:

Unfolding:



- OPEN: edge-unfolding convex polyhedra [Dürer 1525]
  - no conjectured counterexamples [Schlickenrieder 1997; Lucier 2006]
  - every attempted algorithm fails
  - false for triangulated nonconvex [Bern et al. 2001]
- general unfolding
  - possible for convex [Agarwal et al. 1997; Sharir & Schorr 1986]
  - possible for orthogonal [Damian, Flatland, O'Rourke 2006]
  - OPEN in general
- vertex unfolding



  - possible for triangulated [Demaine, Eppstein, Erickson, Hart, O'Rourke 2002]
  - OPEN for convex

Folding: glue polygon boundary to make convex polyhed.

- exponential algorithm to list all gluings [Demaine, Demaine, Lubiw, O'Rourke 2002]
  - output can be that large
- poly. time for bounded sharpness
- OPEN: poly. time decision / 1 shape? [Lubiw & O'Rourke 1998]
  - possible for "edge-to-edge" gluing
- reconstructing resulting 3D shapes [Bobenko & Izmestiev 2006]