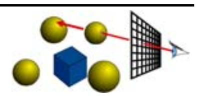
Ray Casting II

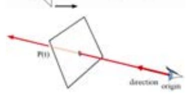
Administrivia

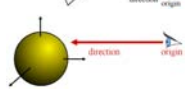
- Assignment 3 due tomorrow

Last Time?

- Ray Casting / Tracing
- Orthographic Camera
- Ray Representation
  - P(t) = origin + t * direction
- Ray-Sphere Intersection
- Ray-Plane Intersection
- Implicit vs. Explicit Representations

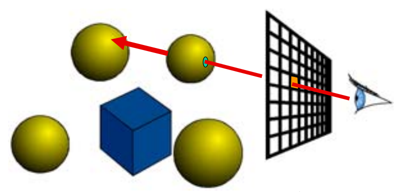






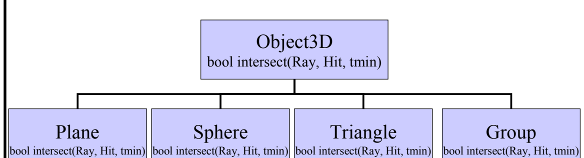
Ray Casting: Object oriented design

```
For every pixel
  Construct a ray from the eye
  For every object in the scene
     Find intersection with the ray
     Keep if closest
```



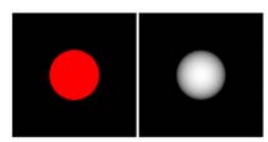
Object-Oriented Design

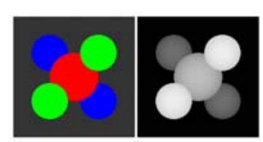
- We want to be able to add primitives easily
  - Inheritance and virtual methods
- Even the scene is derived from Object3D!



Assignment 4 & 5: Ray Casting/Tracing

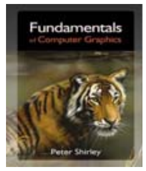
- Write a basic ray caster
  - Orthographic camera
  - Sphere Intersection
  - Main loop rendering
  - 2 Display modes: color and distance
- We provide:
  - Ray: origin, direction
  - Hit: t, Material, (*normal*)
  - Scene Parsing



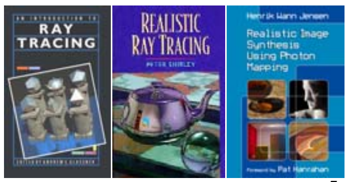


Books

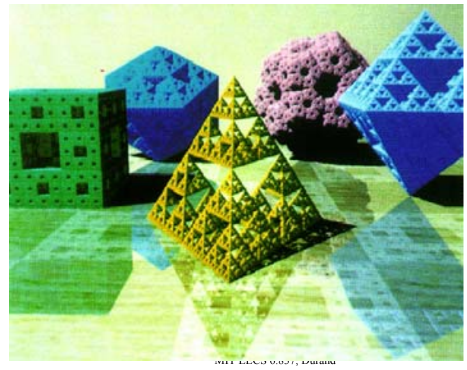
- Peter Shirley
  Fundamentals of Computer Graphics
  AK Peters



- Ray Tracing

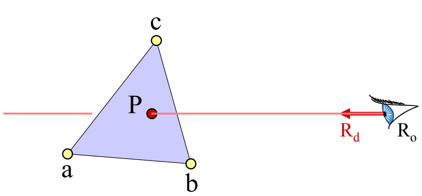


Questions?



Ray-Triangle Intersection

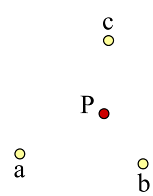
- Use general ray-polygon
- Or try to be smarter
  - Use barycentric coordinates



Barycentric Definition of a Plane

[Möbius, 1827]

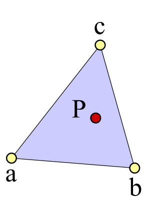
- $P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$
  with $\alpha + \beta + \gamma = 1$
- Is it explicit or implicit?



P is the *barycenter*: the single point upon which the plane would balance if weights of size α, β, & γ are placed on points a, b, & c.

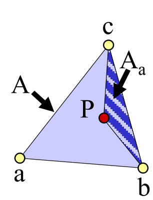
Barycentric Definition of a Triangle

- $P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$
  with $\alpha + \beta + \gamma = 1$
- AND $0 < \alpha < 1$ & $0 < \beta < 1$ & $0 < \gamma < 1$



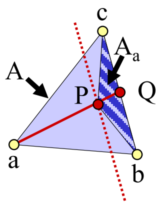
How Do We Compute α, β, γ ?

- Ratio of opposite sub-triangle area to total area
  - $\alpha = A_a/A$ $\beta = A_b/A$ $\gamma = A_c/A$
- Use signed areas for points outside the triangle



Intuition Behind Area Formula

- P is barycenter of a and Q
- A_a is the interpolation coefficient on $\overline{aQ}$
- All points on lines parallel to $\overline{bc}$ have the same α (All such triangles have same height/area)

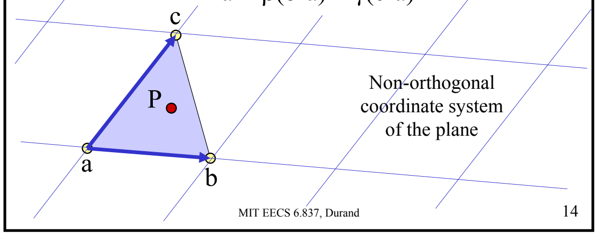


Simplify

- Since $\alpha + \beta + \gamma = 1$, we can write $\alpha = 1 - \beta - \gamma$

$$P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$$
$$P(\beta, \gamma) = (1-\beta-\gamma)a + \beta b + \gamma c$$
$$= a + \beta(b-a) + \gamma(c-a)$$

rewrite



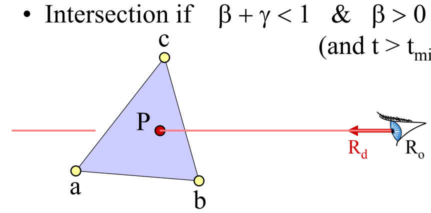
Questions?

Intersection with Barycentric Triangle

- Set ray equation equal to barycentric equation

$$P(t) = P(\beta, \gamma)$$
$$R_o + t * R_d = a + \beta(b-a) + \gamma(c-a)$$

- Intersection if $\beta + \gamma < 1$ & $\beta > 0$ & $\gamma > 0$ (and $t > t_{min}$ …)



Intersection with Barycentric Triangle

- $R_o + t * R_d = a + \beta(b-a) + \gamma(c-a)$

$$R_{ox} + tR_{dx} = a_x + \beta(b_x - a_x) + \gamma(c_x - a_x)$$
$$R_{oy} + tR_{dy} = a_y + \beta(b_y - a_y) + \gamma(c_y - a_y)$$
$$R_{oz} + tR_{dz} = a_z + \beta(b_z - a_z) + \gamma(c_z - a_z)$$

3 equations, 3 unknowns

- Regroup & write in matrix form:

$$\begin{bmatrix} a_x - b_x & a_x - c_x & R_{dx} \\ a_y - b_y & a_y - c_y & R_{dy} \\ a_z - b_z & a_z - c_z & R_{dz} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} a_x - R_{ox} \\ a_y - R_{oy} \\ a_z - R_{oz} \end{bmatrix}$$

Cramer's Rule

- Used to solve for one variable at a time in system of equations

$$\beta = \frac{\begin{vmatrix} a_x - R_{ox} & a_x - c_x & R_{dx} \\ a_y - R_{oy} & a_y - c_y & R_{dy} \\ a_z - R_{oz} & a_z - c_z & R_{dz} \end{vmatrix}}{|A|} \qquad \gamma = \frac{\begin{vmatrix} a_x - b_x & a_x - R_{ox} & R_{dx} \\ a_y - b_y & a_y - R_{oy} & R_{dy} \\ a_z - b_z & a_z - R_{oz} & R_{dz} \end{vmatrix}}{|A|}$$

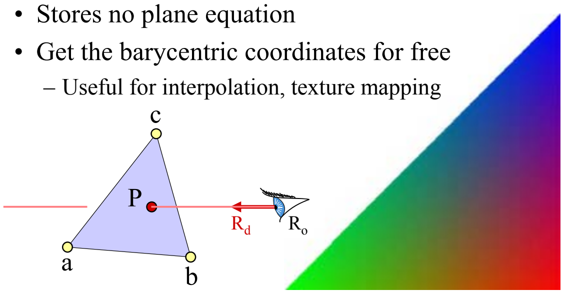
$$t = \frac{\begin{vmatrix} a_x - b_x & a_x - c_x & a_x - R_{ox} \\ a_y - b_y & a_y - c_y & a_y - R_{oy} \\ a_z - b_z & a_z - c_z & a_z - R_{oz} \end{vmatrix}}{|A|}$$

| | denotes the determinant

Can be copied mechanically into code

Advantages of Barycentric Intersection

- Efficient
- Stores no plane equation
- Get the barycentric coordinates for free
  - Useful for interpolation, texture mapping

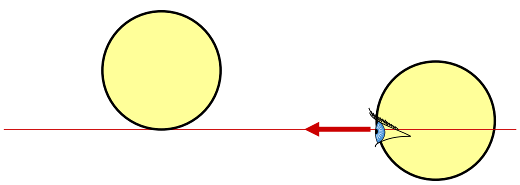


Questions?

- Image computed using the RADIANCE system by Greg Ward

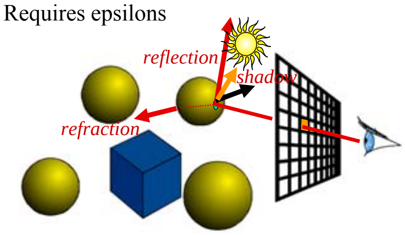
Precision

- What happens when
  - Origin is on an object?
  - Grazing rays?
- Problem with floating-point approximation



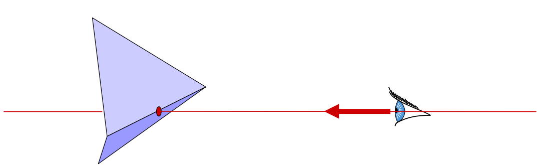
The evil ε

- In ray tracing, do NOT report intersection for rays starting at the surface (no false positive)
  - Because secondary rays
  - Requires epsilons



The evil ε: a hint of nightmare

- Edges in triangle meshes
  - Must report intersection (otherwise not watertight)
  - No false negative

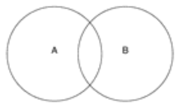


Questions?

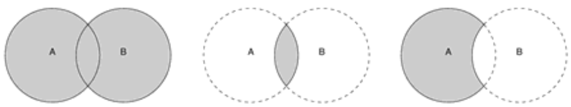
Image by Henrik Wann Jensen

Constructive Solid Geometry (CSG)

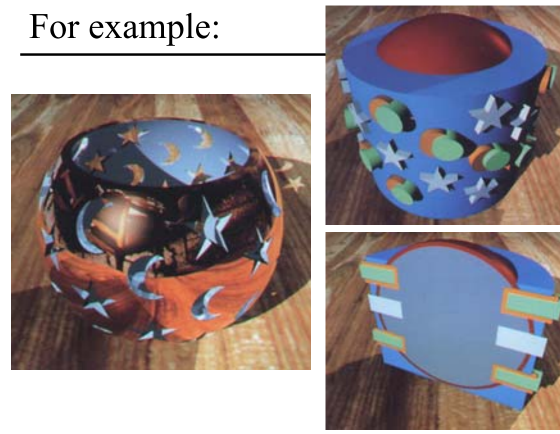
Given overlapping shapes A and B:



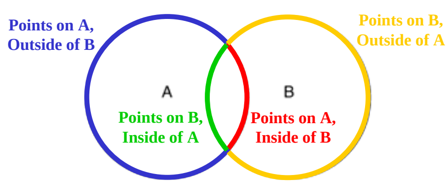
Union | Intersection | Subtraction



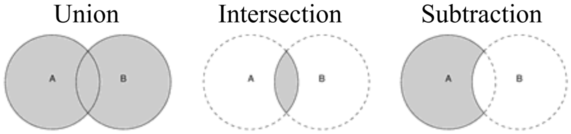
For example:



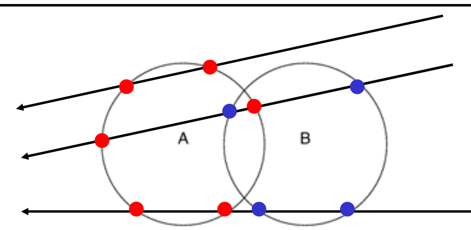
How can we implement CSG?



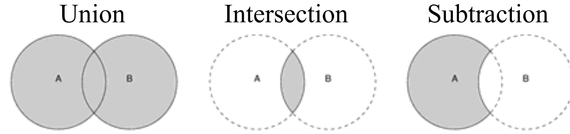
Union | Intersection | Subtraction



Collect all the intersections

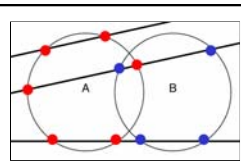


Union | Intersection | Subtraction

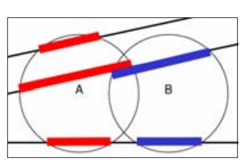


Implementing CSG

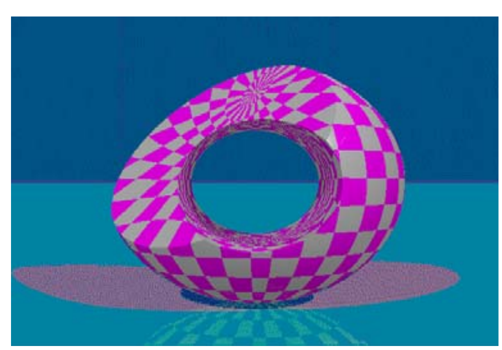
1. Test "inside" intersections:
   - Find intersections with A, test if they are inside/outside B
   - Find intersections with B, test if they are inside/outside A



2. Overlapping intervals:
   - Find the intervals of "inside" along the ray for A and B
   - Compute union/intersection/subtraction of the intervals



My early CSG raytraced Image

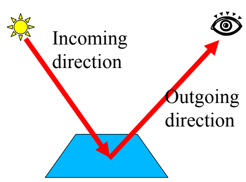


Questions?

Local Illumination

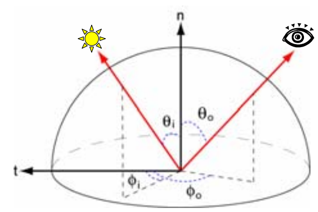
BRDF

- Ratio of light coming from one direction that gets reflected in another direction
- Bidirectional Reflectance Distribution Function



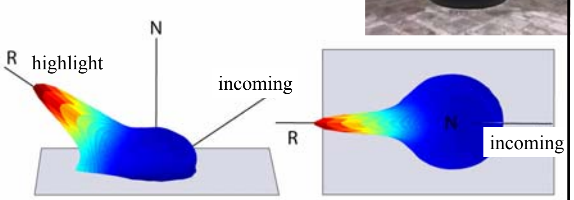
BRDF

- Bidirectional Reflectance Distribution Function
  - 4D
    - 2 angles for each direction
  - $R(\theta_i, \phi_i; \theta_o, \phi_o)$



Slice at constant incidence

- 2D spherical function



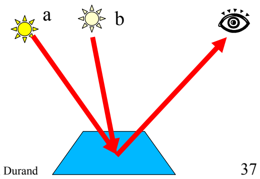
Example: Plot of "PVC" BRDF at 55° incidence

Unit issues - radiometry

- We will not be too formal in this lecture
- Typical issues:
  - Directional quantities vs. integrated over all directions
  - Differential terms: per solid angle, per area, per time
  - Power, intensity, flux
- Should be per wavelength
  - Usually simplified into brand-X RGB

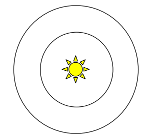
Light sources

- Today, we only consider point light sources
- For multiple light sources, use linearity
  - We can add the solutions for two light sources
    - I(a+b)=I(a)+I(b)
  - We simply multiply the solution when we scale the light intensity
    - I(s a) = s I(a)



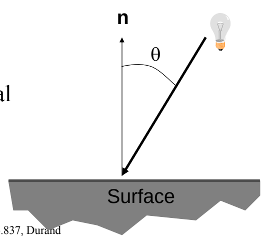
Light intensity

- $1/r^2$ falloff
  - Why?
  - Same power in all concentric circles



Incoming radiance

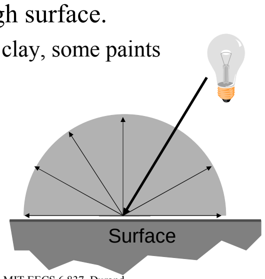
- The amount of light received by a surface depends on incoming angle
  - Bigger at normal incidence
    - Similar to Winter/Summer difference
- By how much?
  - Cos θ law
  - Dot product with normal
  - This term is sometimes included in the BRDF, sometimes not



Questions?

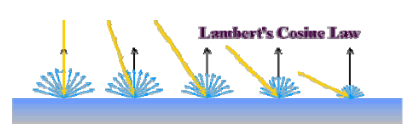
Ideal Diffuse Reflectance

- Assume surface reflects equally in all directions.
- An ideal diffuse surface is, at the microscopic level, a very rough surface.
  - Example: chalk, clay, some paints



Ideal Diffuse Reflectance

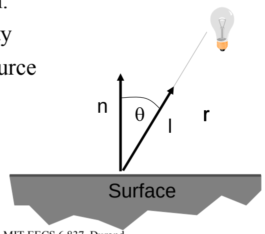
- Ideal diffuse reflectors reflect light according to Lambert's cosine law.



Ideal Diffuse Reflectance

- Single Point Light Source
  - k_d: diffuse coefficient.
  - **n**: Surface normal.
  - **l**: Light direction.
  - L_i: Light intensity
  - r: Distance to source

$$L_o = k_d (\mathbf{n} \cdot \mathbf{l}) \frac{L_i}{r^2}$$



Ideal Diffuse Reflectance – More Details

- If n and l are facing away from each other, n • l becomes negative.
- Using max((n • l),0) makes sure that the result is zero.
  - From now on, we mean max() when we write •.
- Do not forget to **normalize your vectors** for the dot product!

Questions?