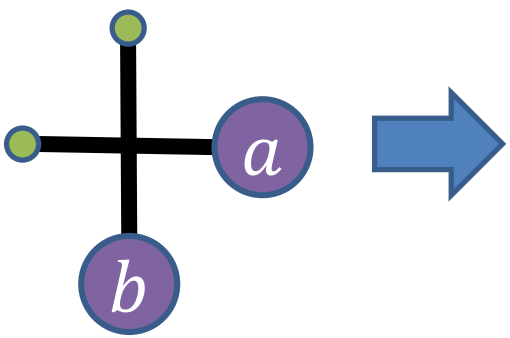
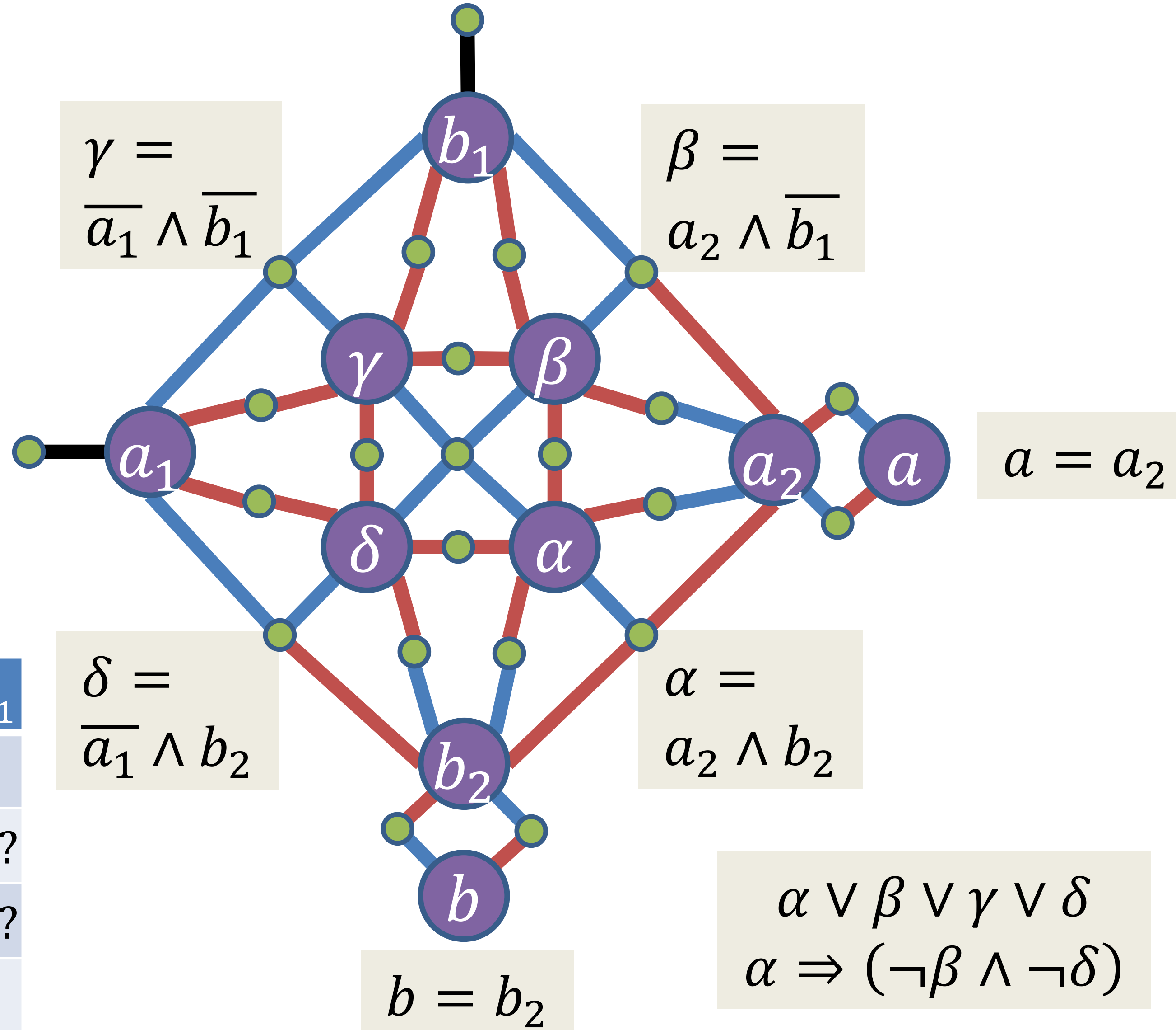
Planar 3SAT is NP-hard

[Lichtenstein 1982]

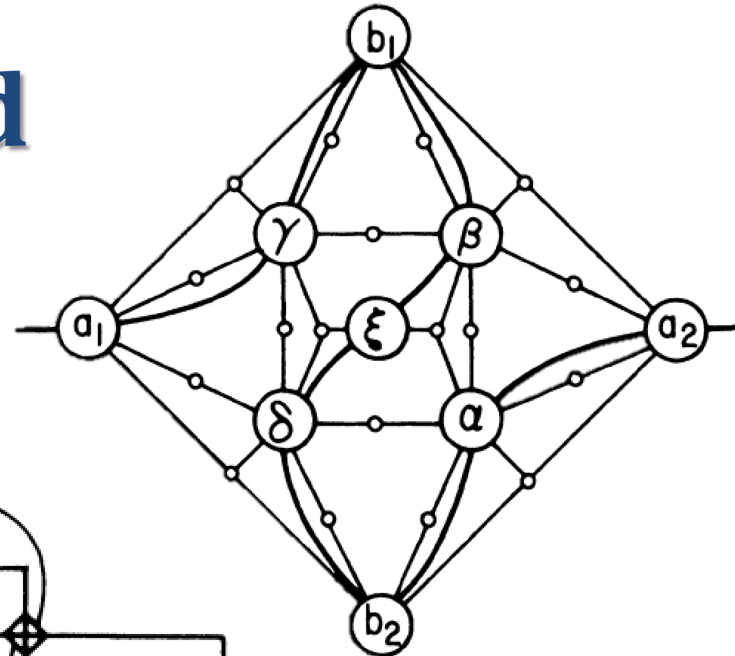


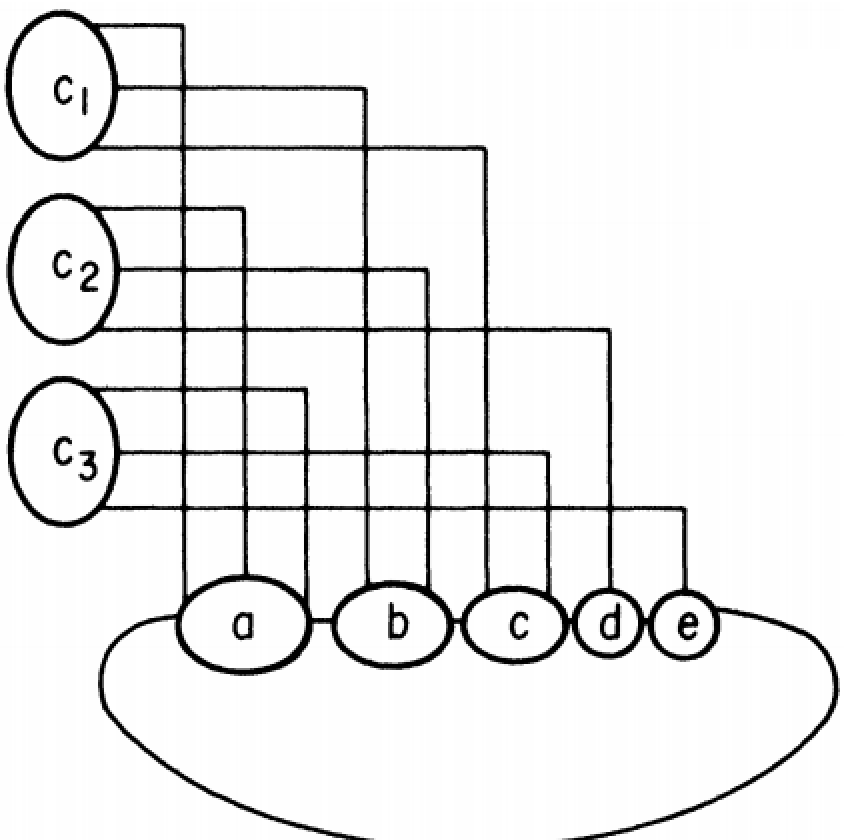


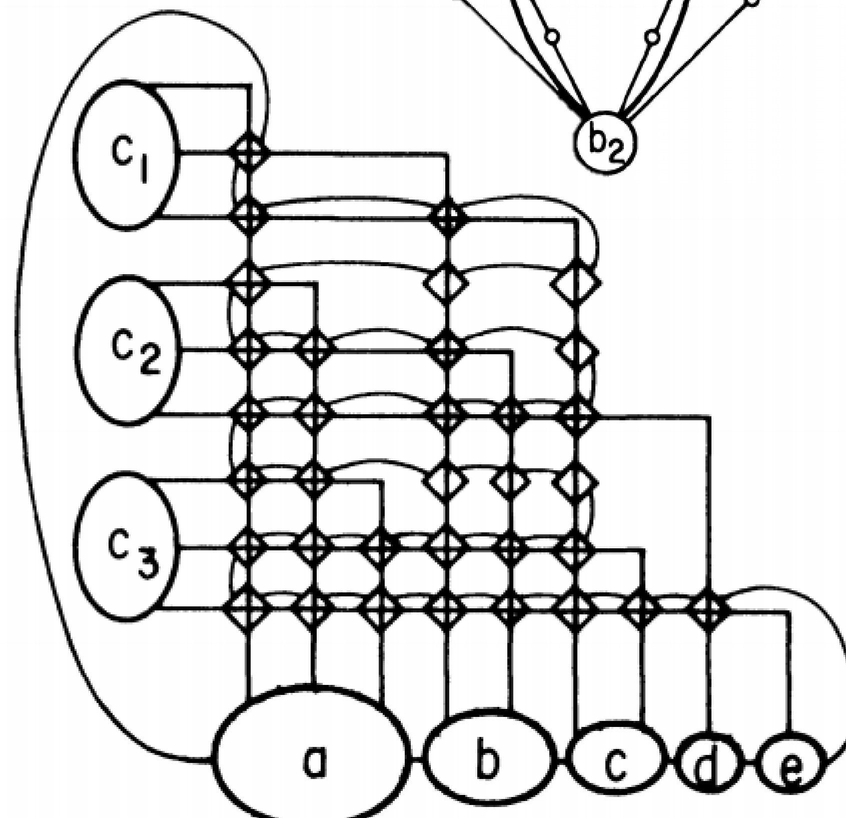
a_2	b_2	α	β	δ	γ	a_1	b_1
0	0	0	0	0	1	0	0
0	1	0	0		1?	0?	0?
1	0	0		0	1?	0?	0?
1	1	1	0	0		1	1

Planar 3SAT is NP-hard

[Lichtenstein 1982]

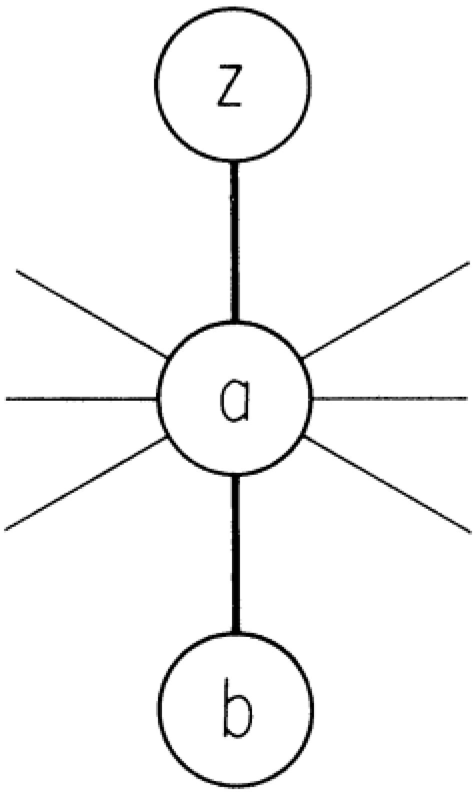


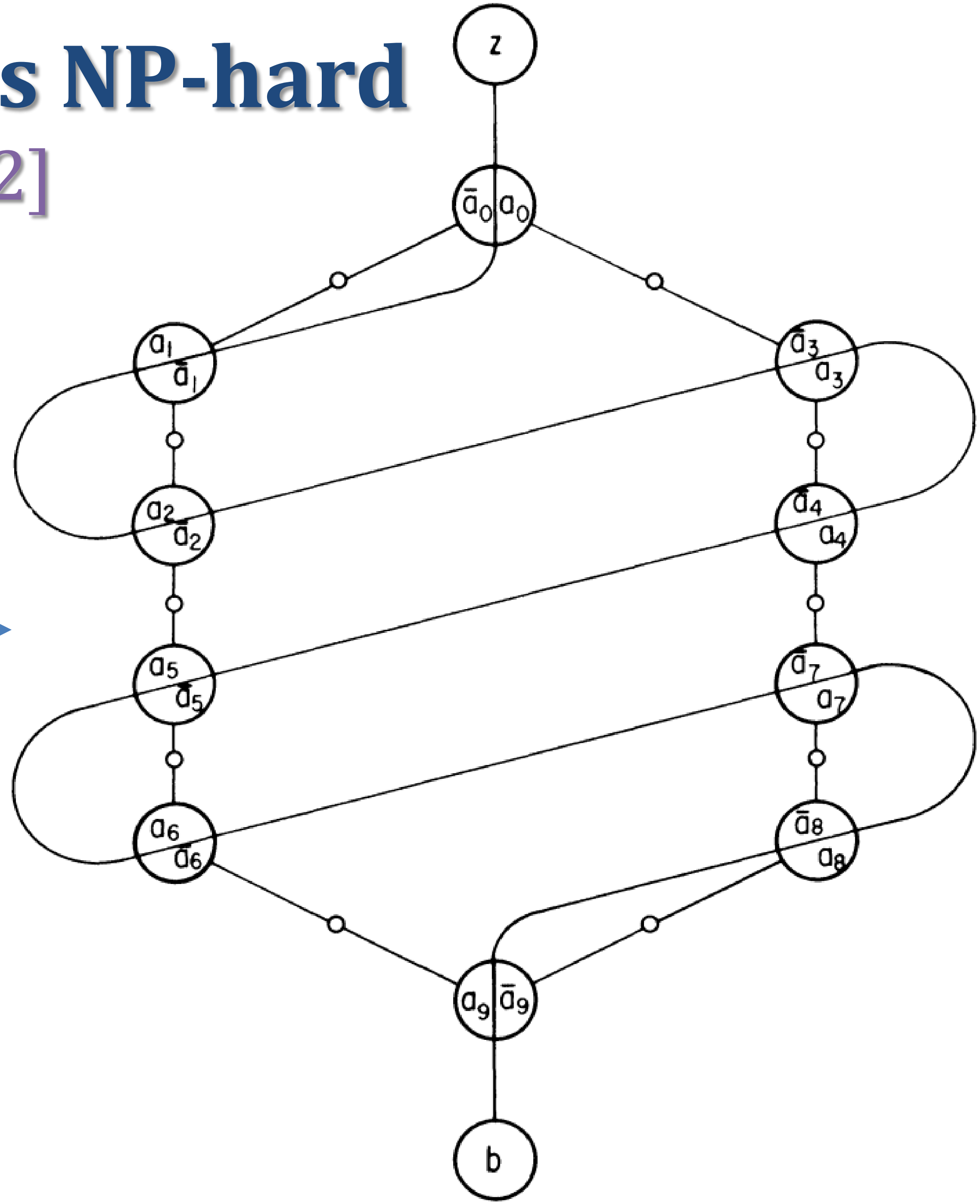




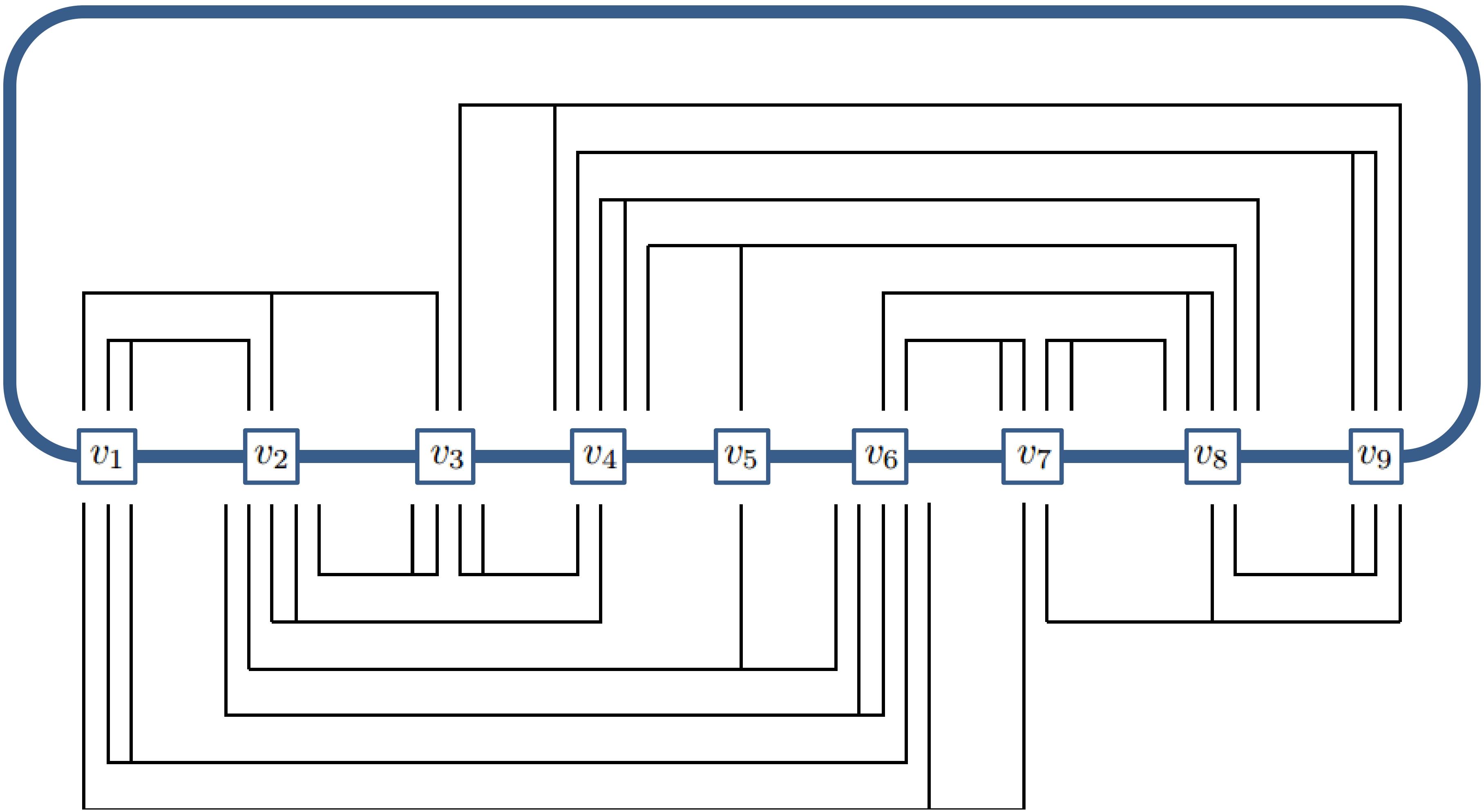
Planar 3SAT is NP-hard

[Lichtenstein 1982]





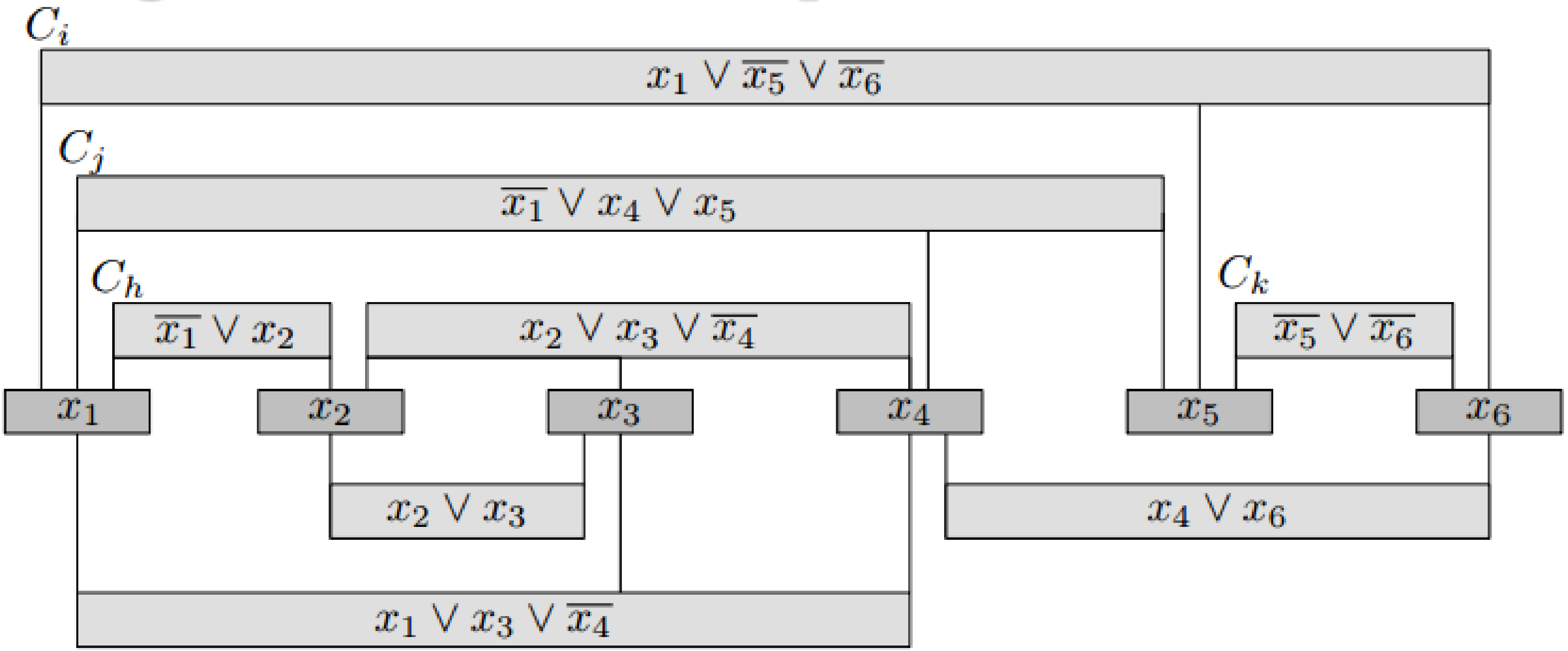
Planar Rectilinear 3SAT

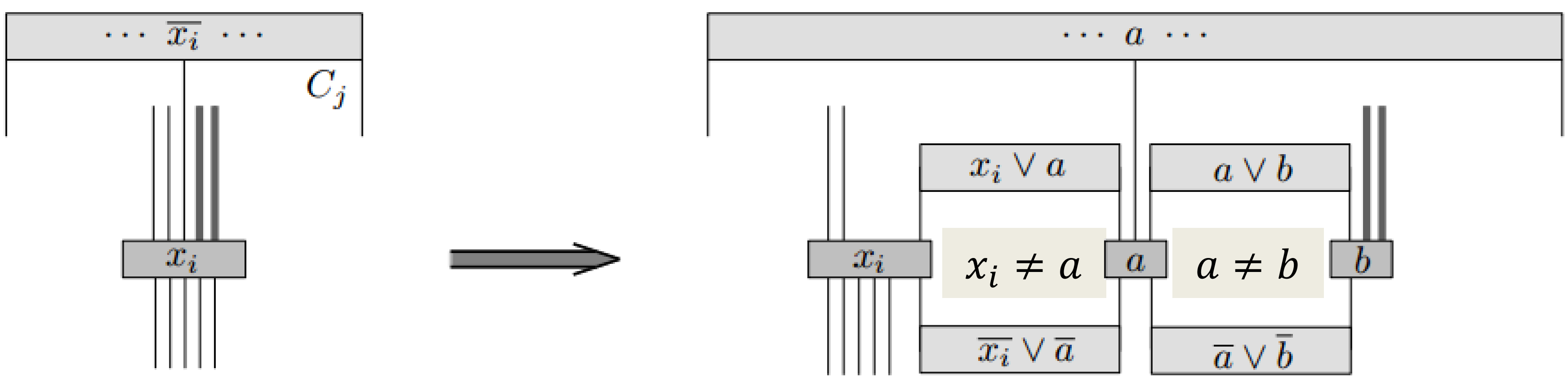


[Knuth & Raghunathan 1992]

Planar Monotone Rectilinear 3SAT

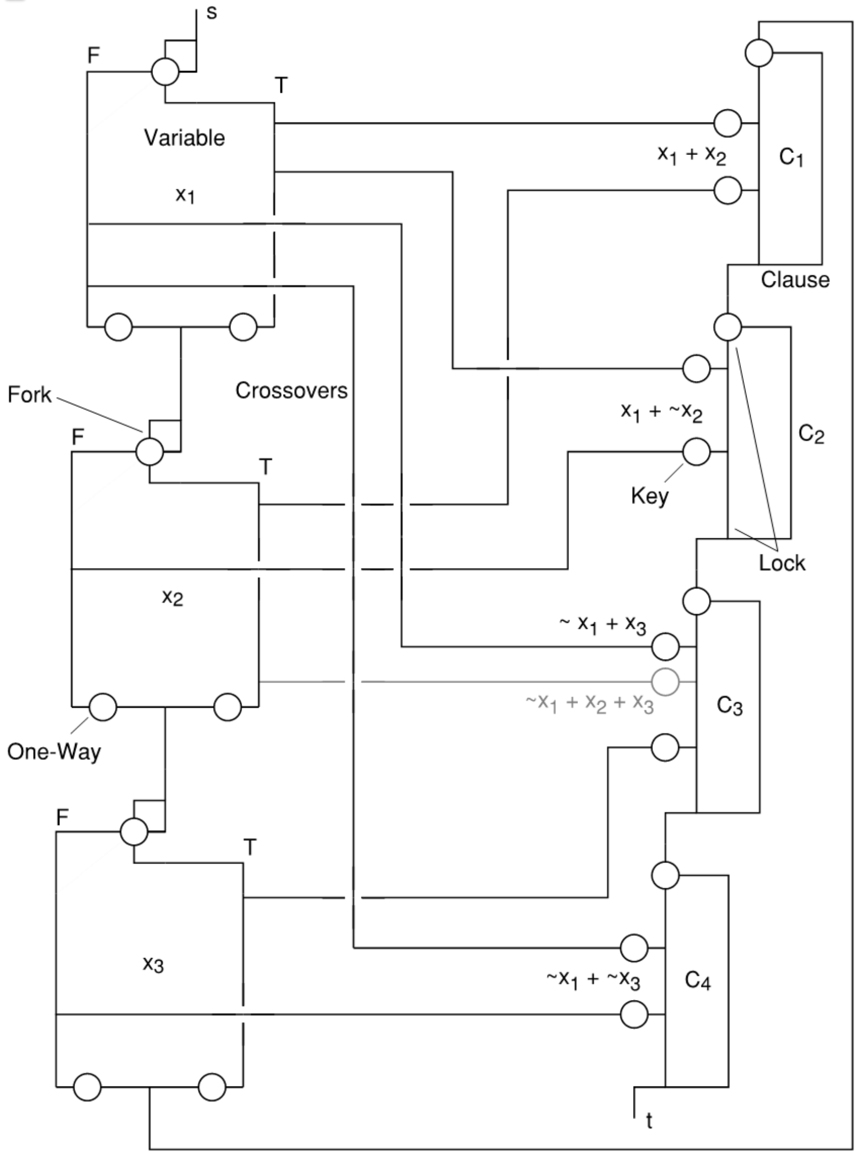
[de Berg & Khosravi 2010]

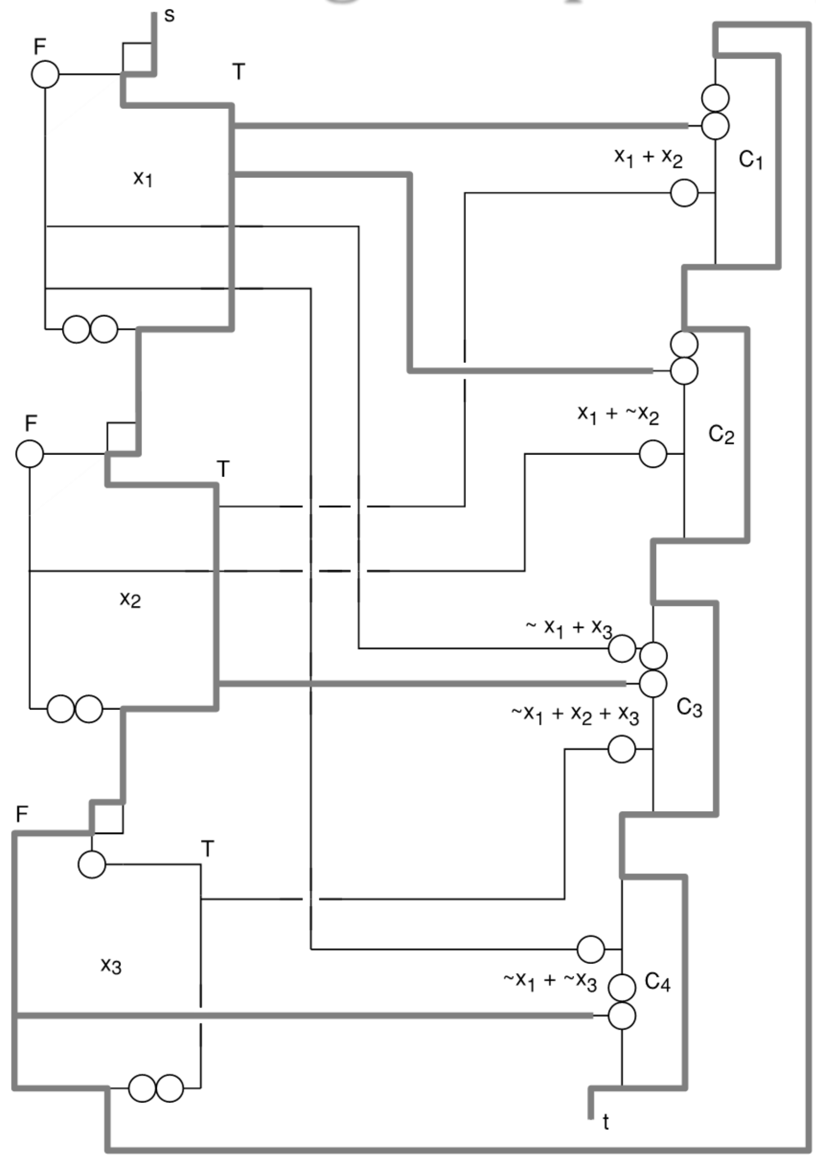




PushPush-1 is NP-hard in 3D

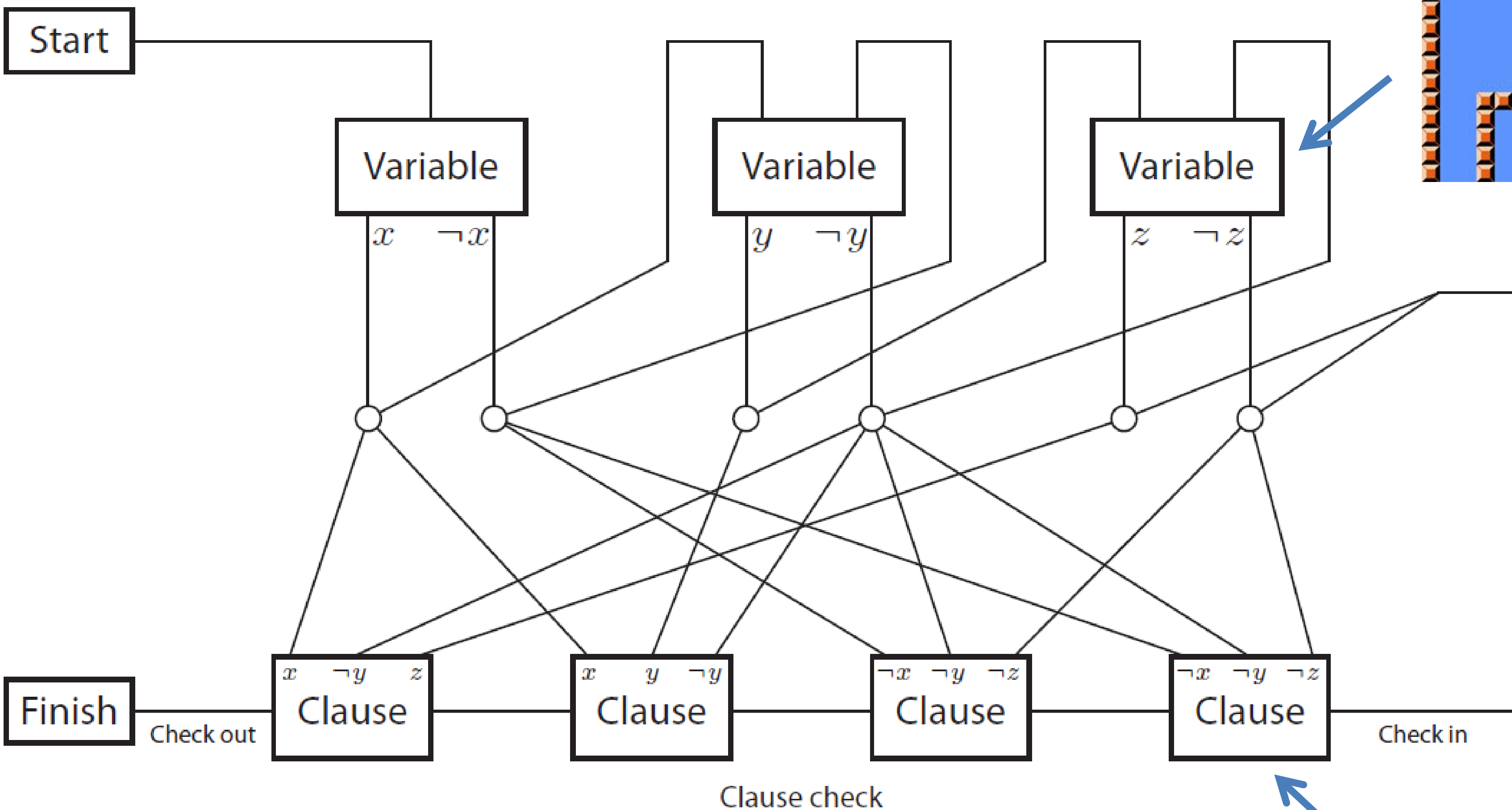
[O'Rourke & Smith Problem Solving Group 1999]

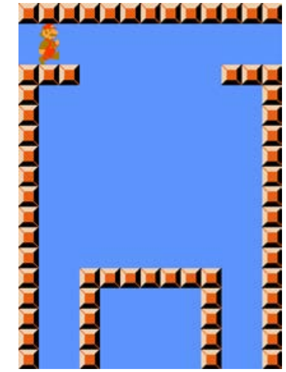




Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

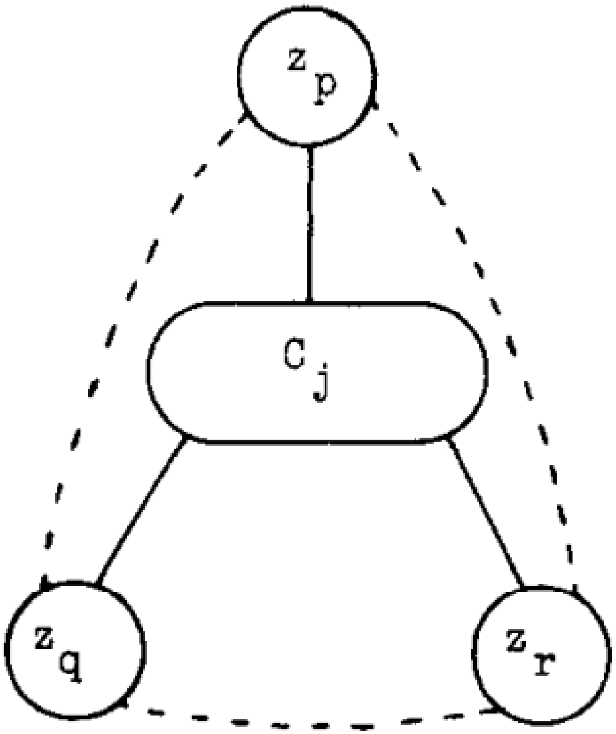


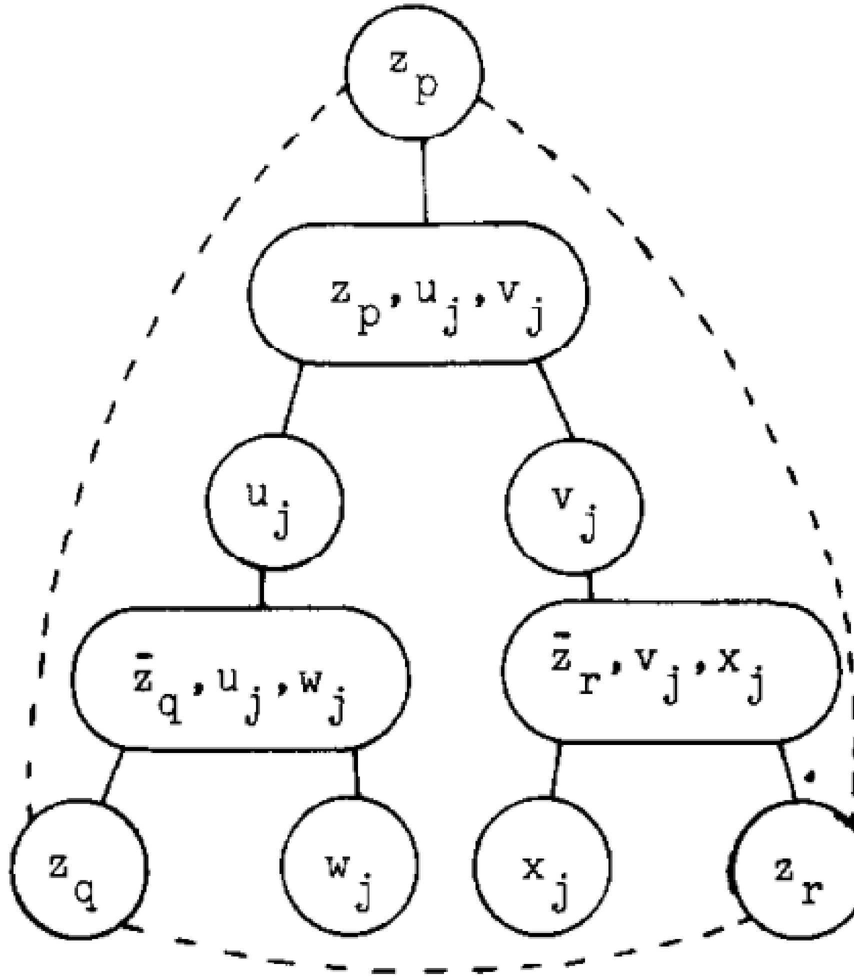


$(x \text{ OR } \neg y \text{ OR } z)$ & $(x \text{ OR } y \text{ OR } \neg y)$ &
$(\neg x \text{ OR } \neg y \text{ OR } \neg z)$ & $(\neg x \text{ OR } \neg y \text{ OR } \neg z)$

Planar 1-in-3SAT

[Dyer & Freeze 1986]

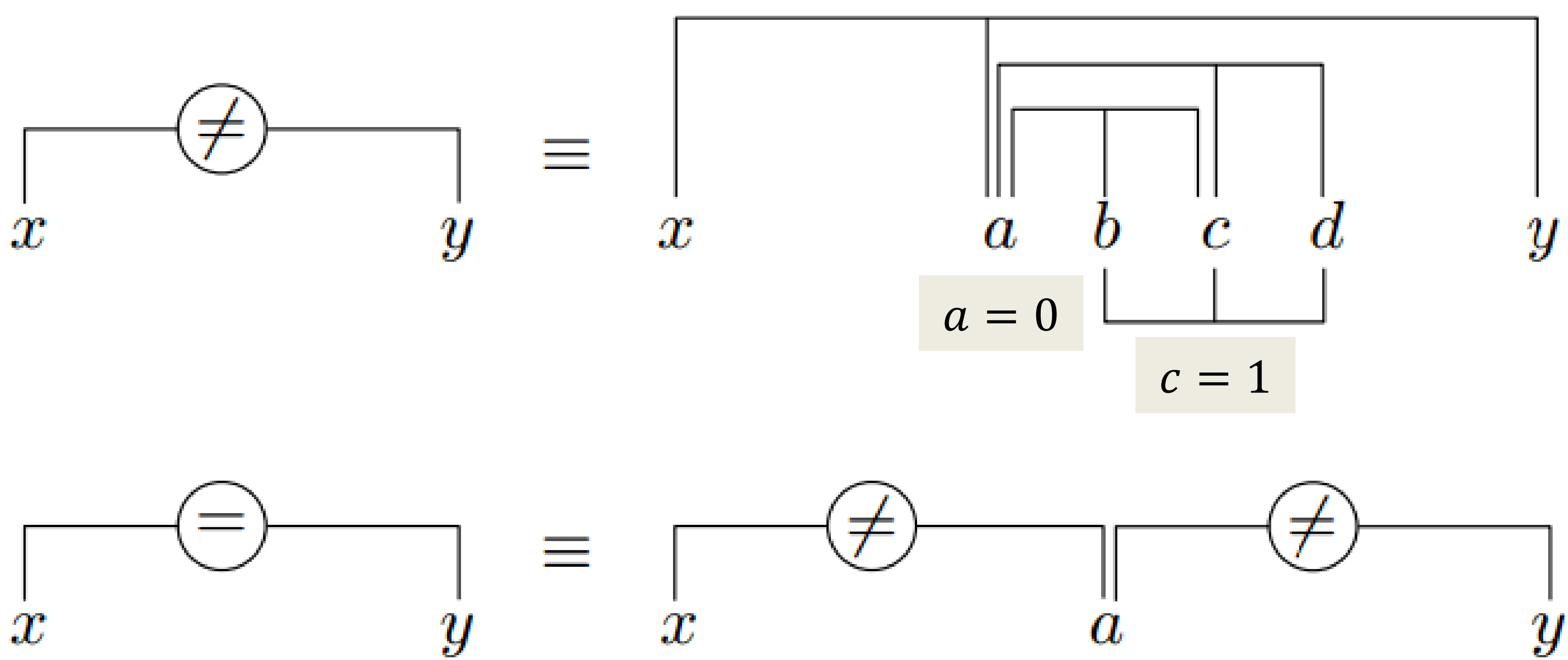




$C_j = \{z_p, z_q, z_r\}$ $\quad \{z_p, u_j, v_j\}, \{\bar{z}_q, u_j, w_j\}, \{\bar{z}_r, v_j, x_j\}$

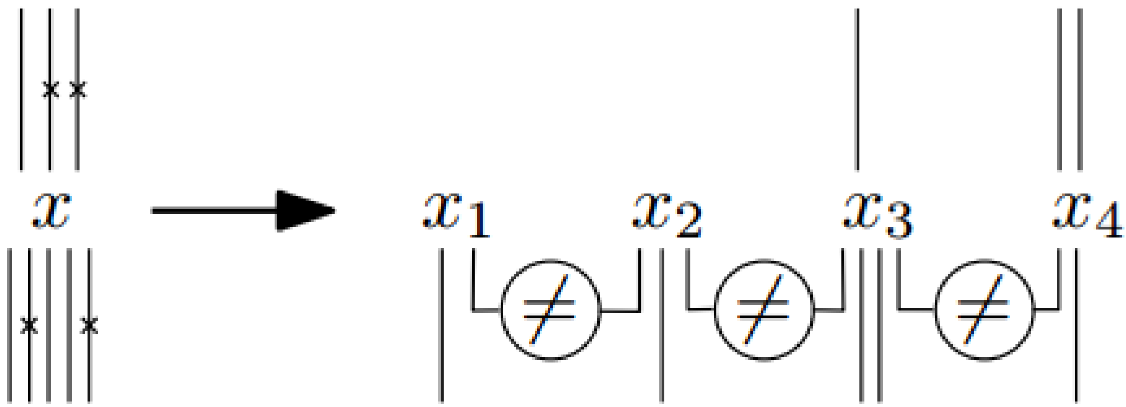
Planar Positive Rectilinear 1-in-3SAT

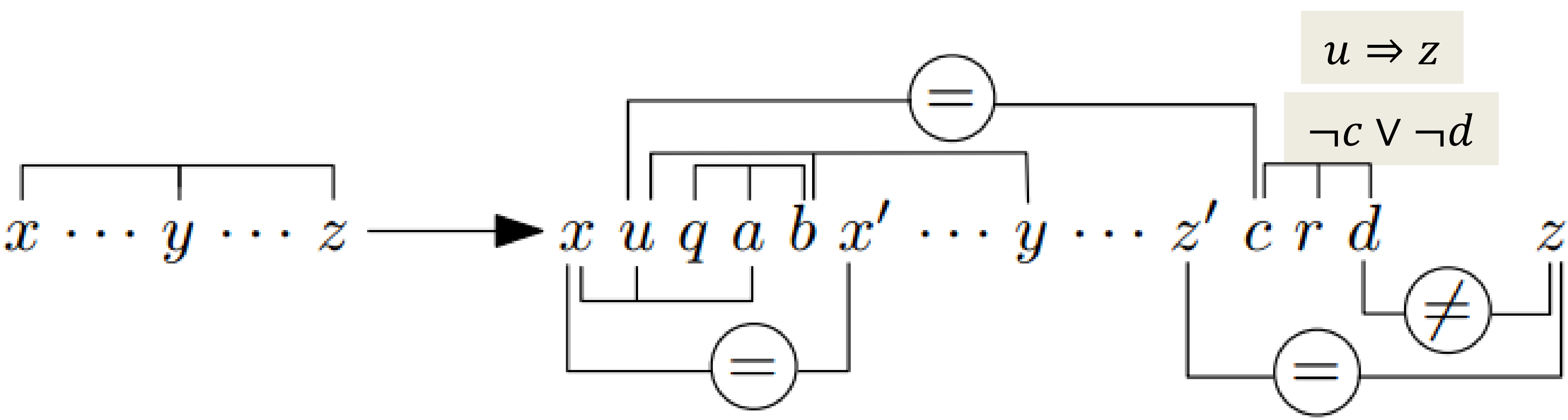
[Mulzer & Rote 2008]



Planar Positive Rectilinear 1-in-3SAT

[Mulzer & Rote 2008]

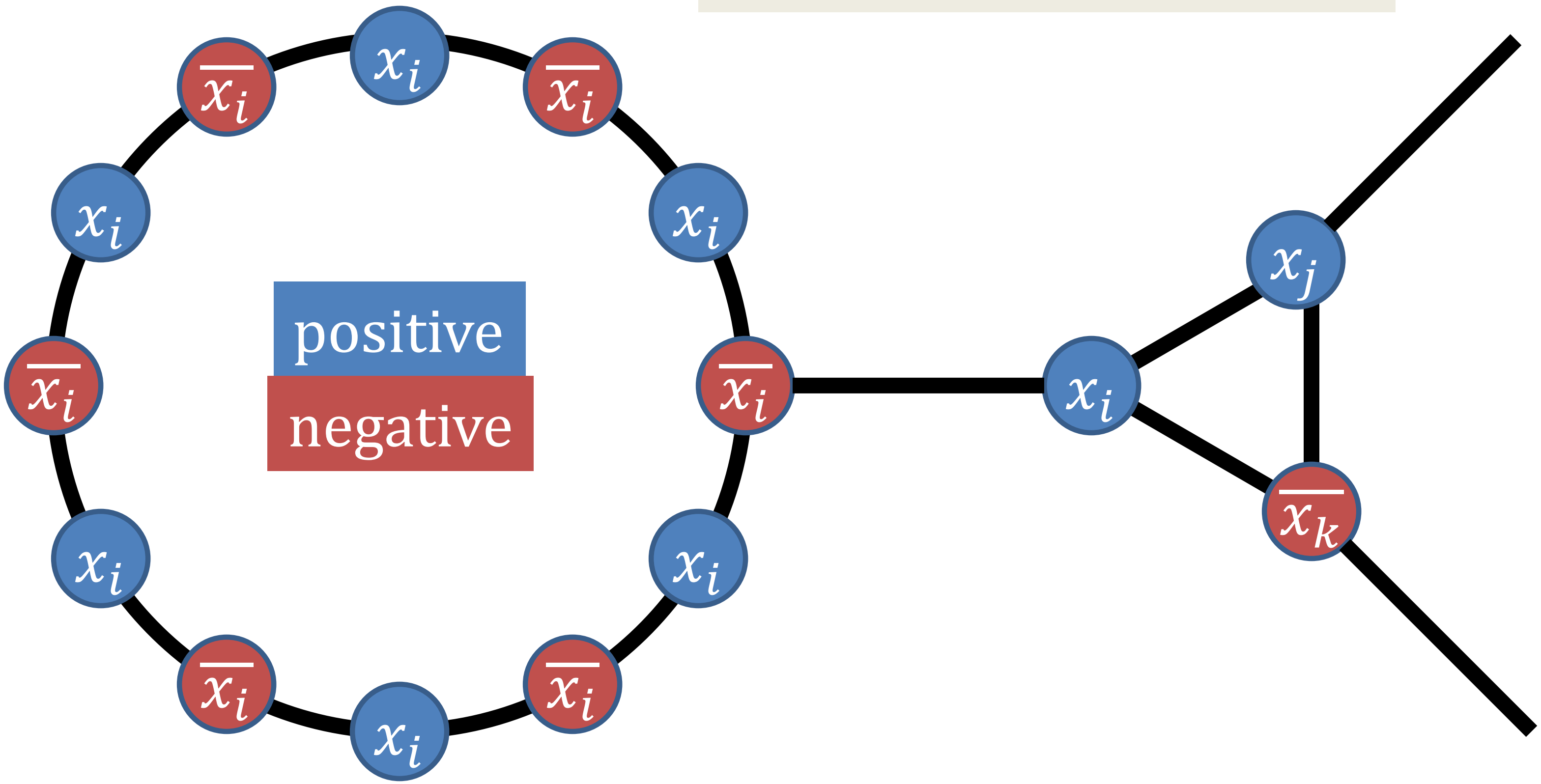




Planar NAE 3SAT is Polynomial

[Moret 1988]

reduction to Max Cut



cut size = $2n_i$

cut size = 5

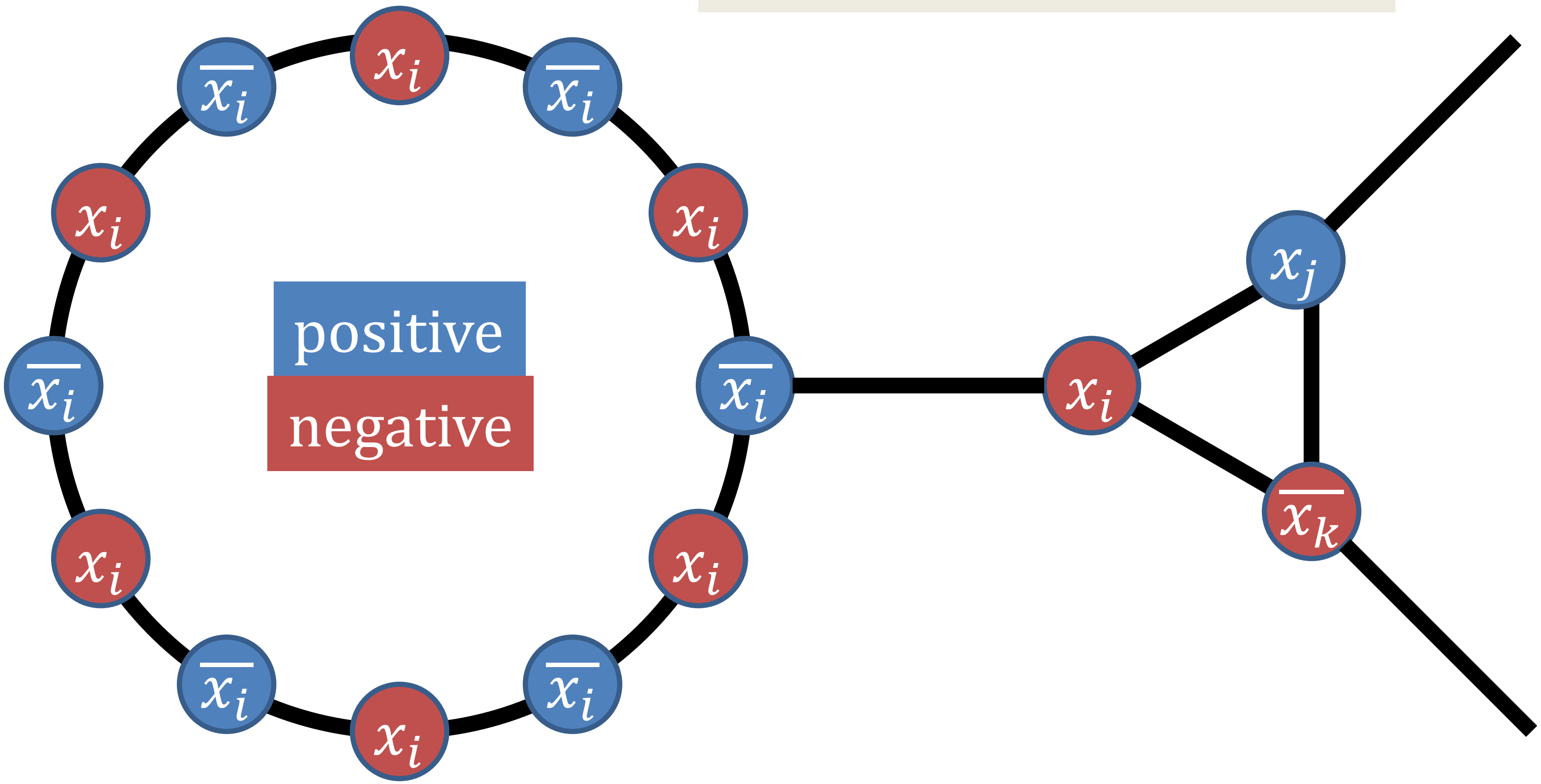
variable

NAE clause

Planar NAE 3SAT is Polynomial

[Moret 1988]

reduction to Max Cut



cut size = $2n_i$

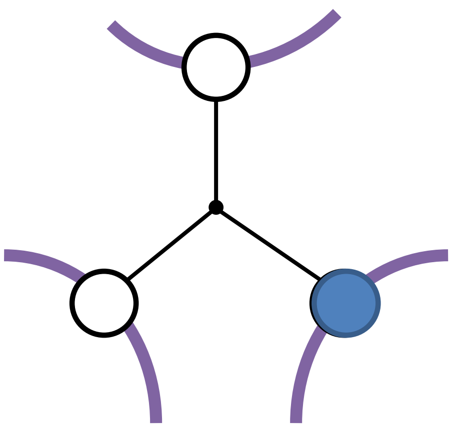
cut size = 5

variable

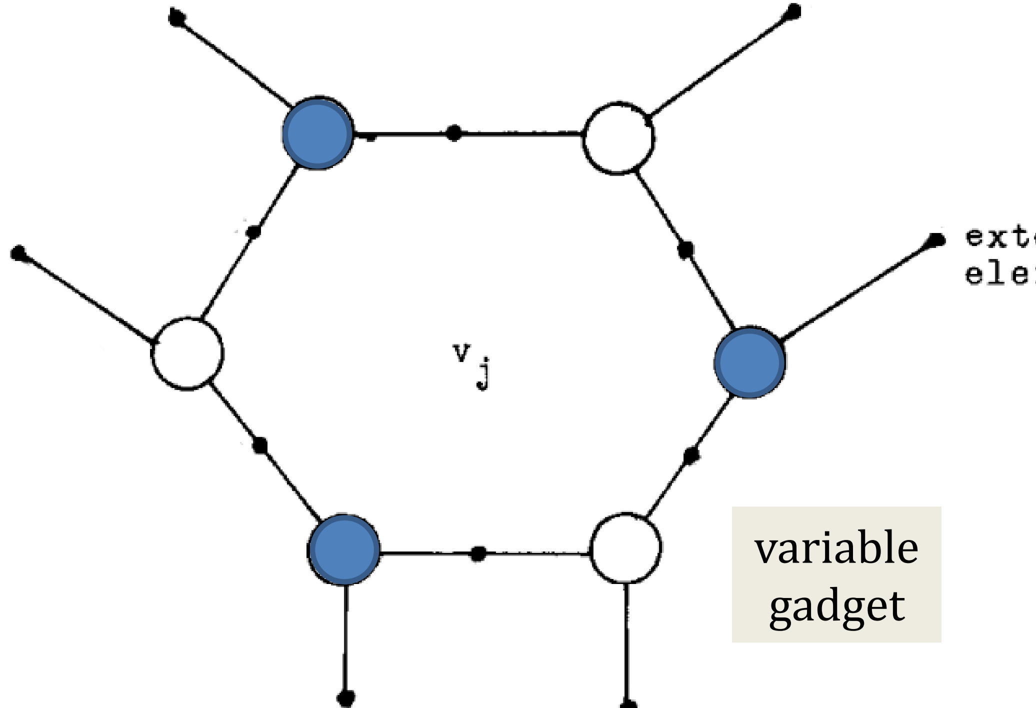
NAE clause

Planar X3C

[Dyer & Freeze 1986]

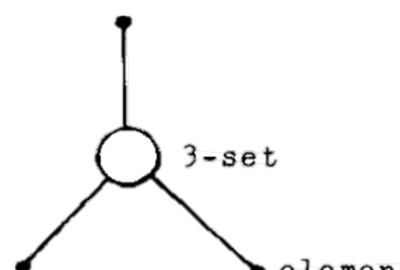


simple clause gadget



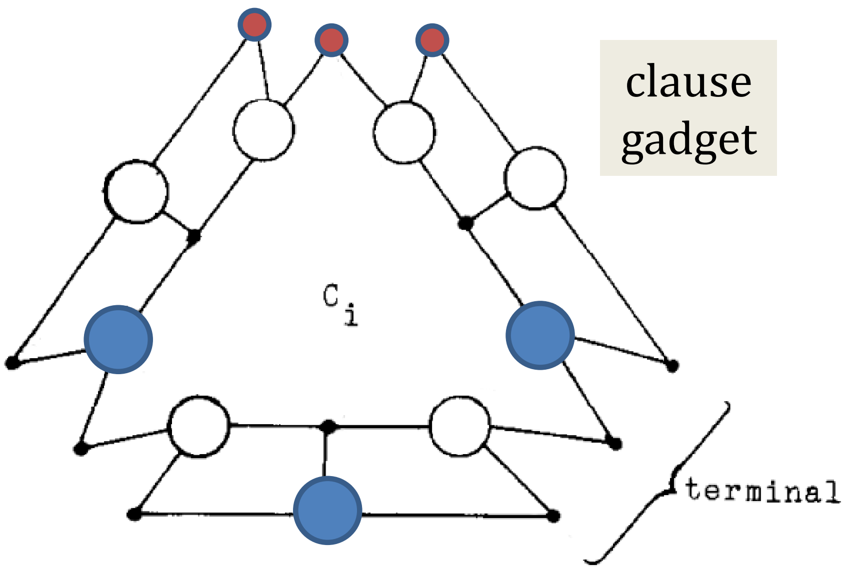
variable gadget

size of cover $= \sum_i \frac{1}{2} n_i$

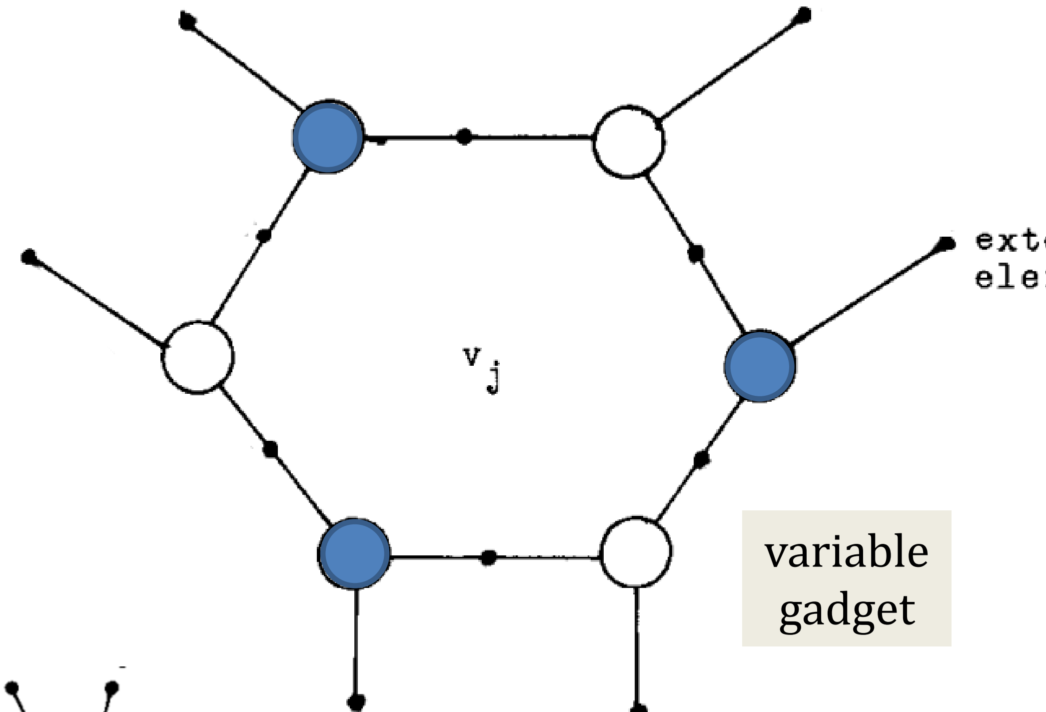


Planar X3C

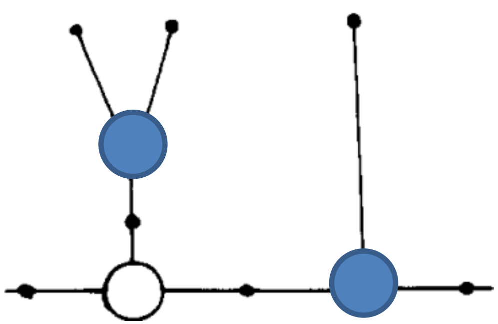
[Dyer & Freeze 1986]



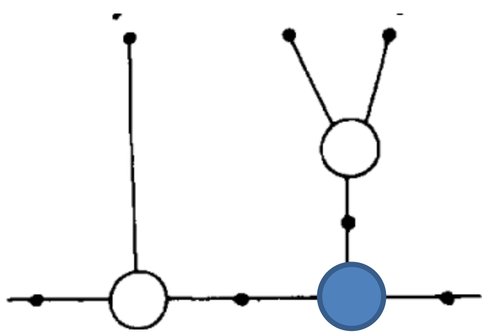
clause gadget



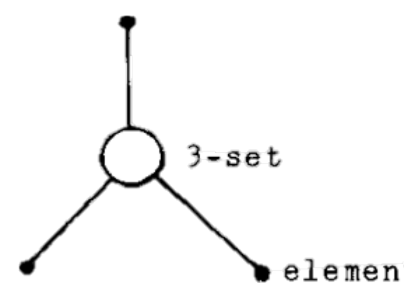
variable gadget



positive connector

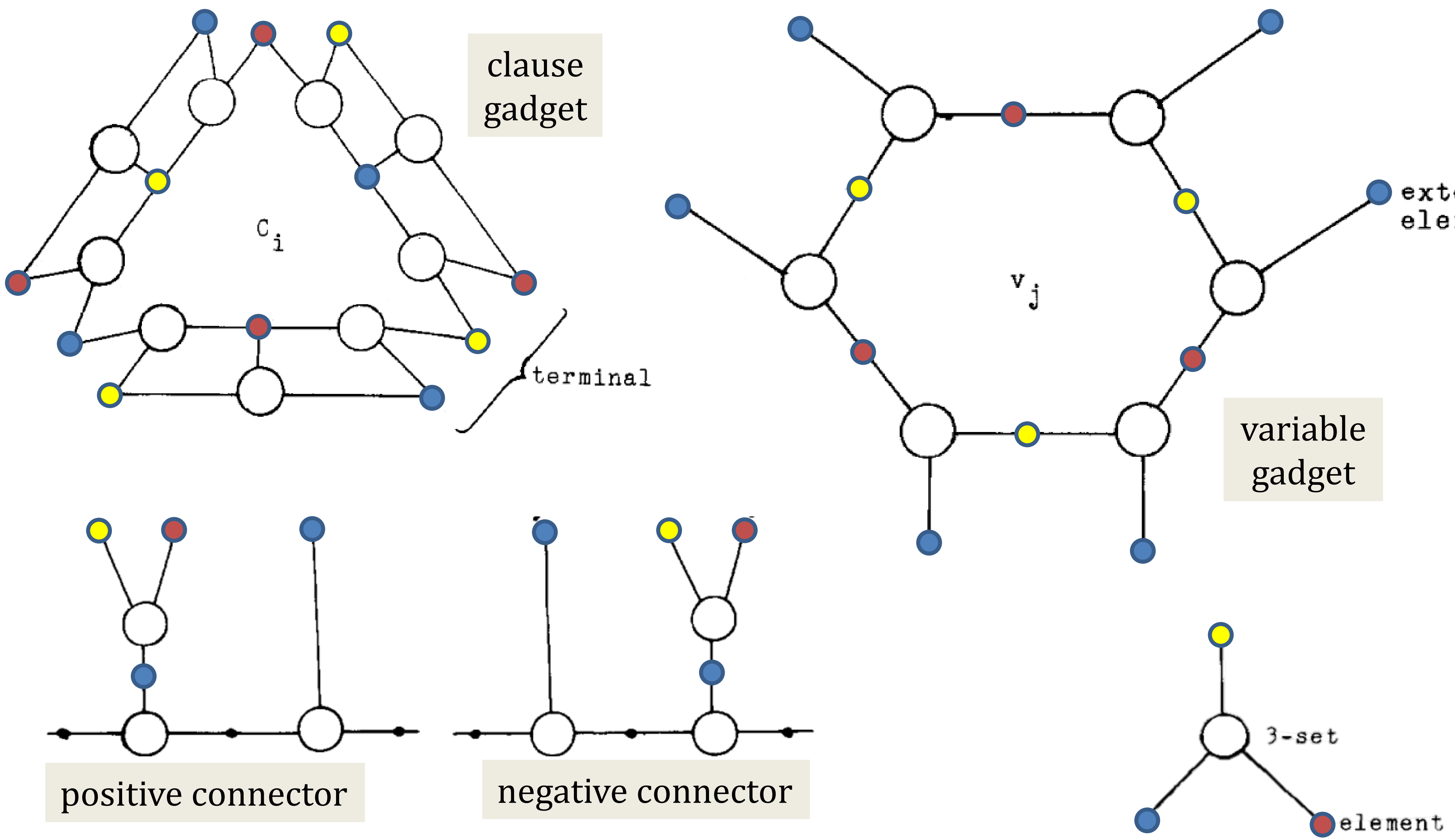


negative connector



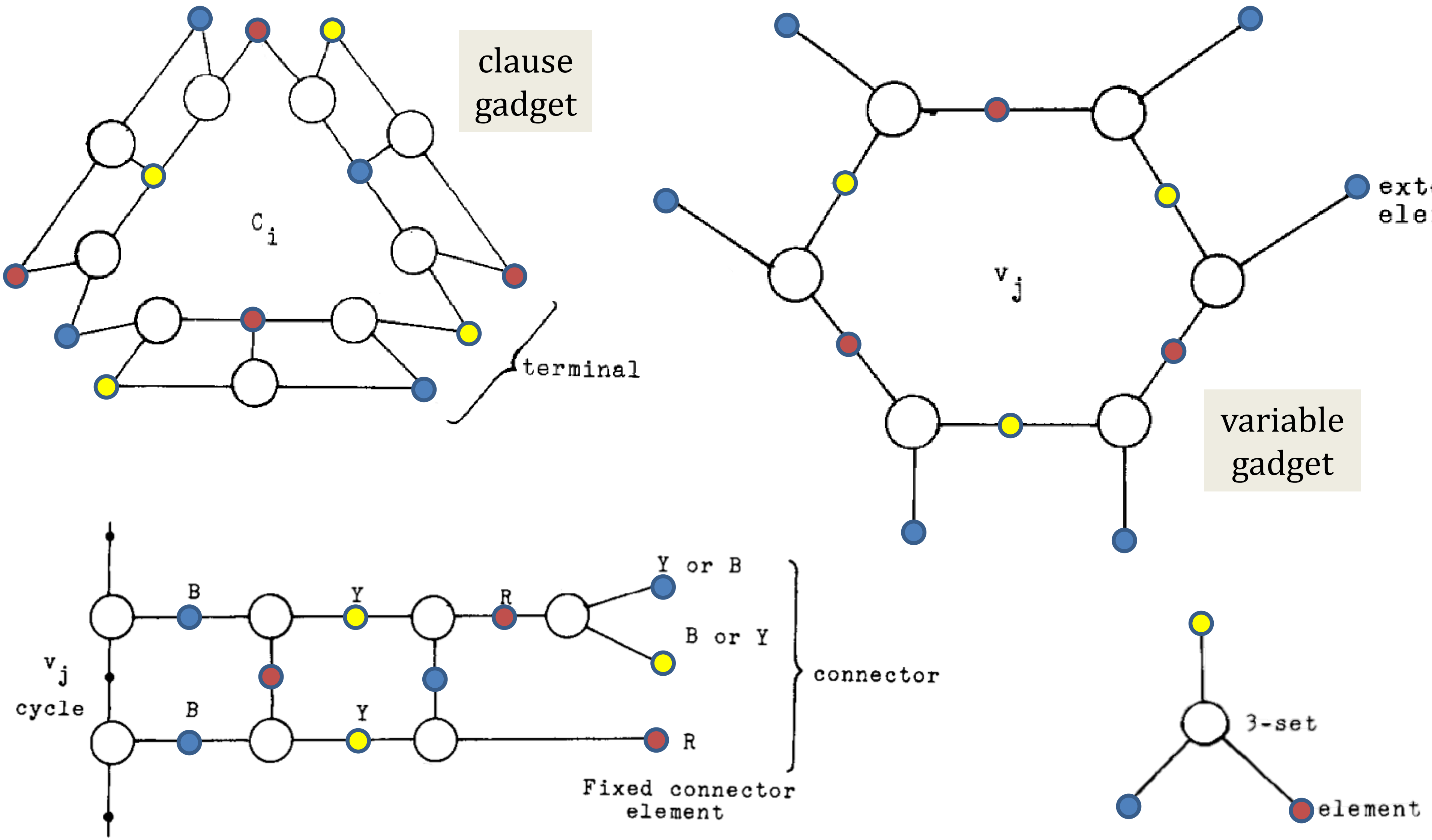
Planar 3DM

[Dyer & Freeze 1986]



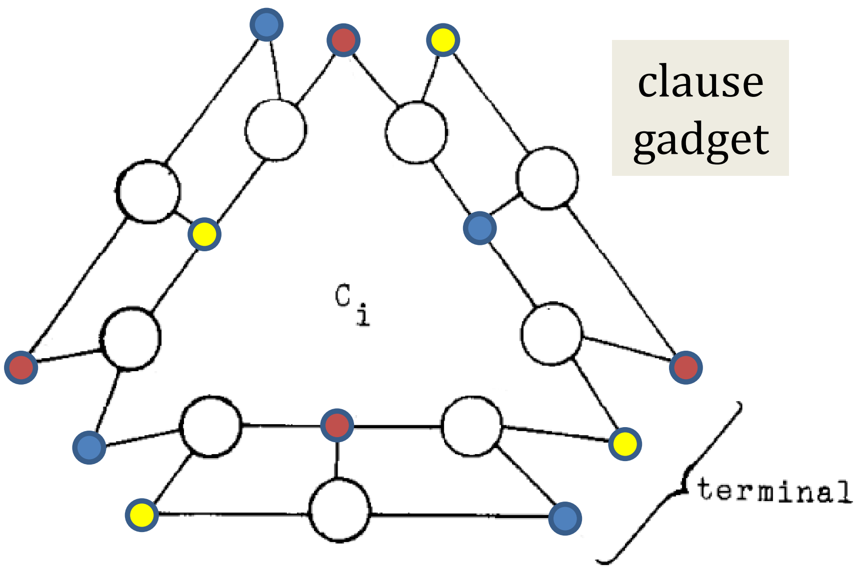
Planar 3DM

[Dyer & Freeze 1986]

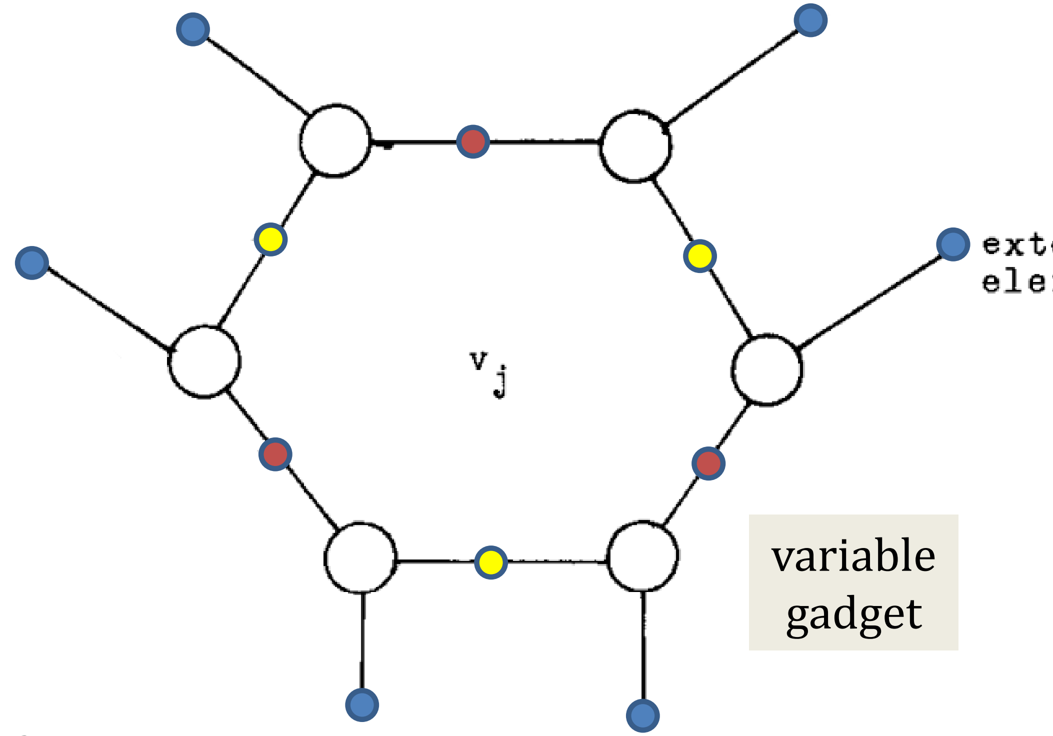


Planar 3DM

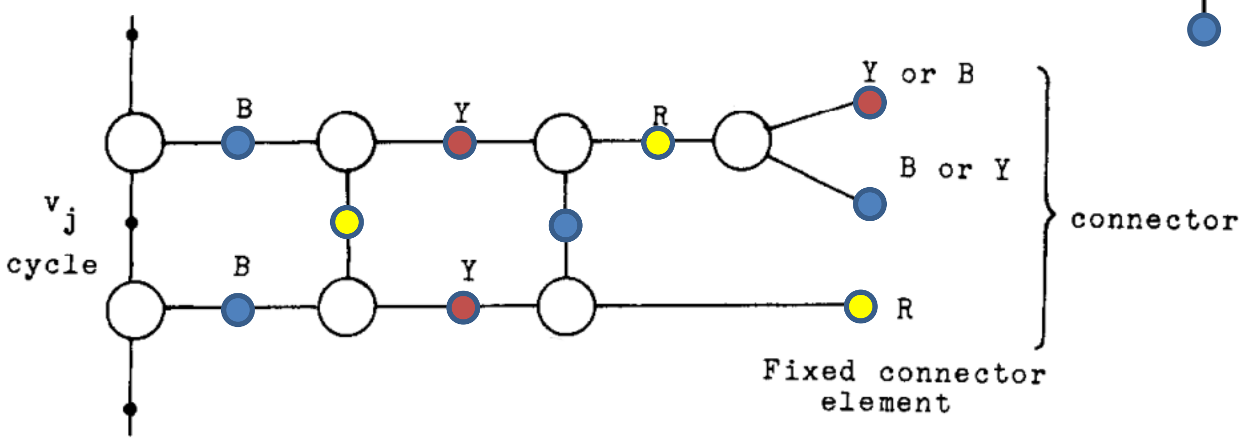
[Dyer & Freeze 1986]

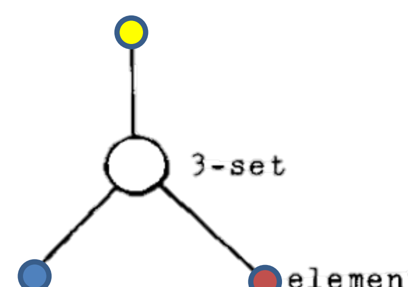


clause gadget



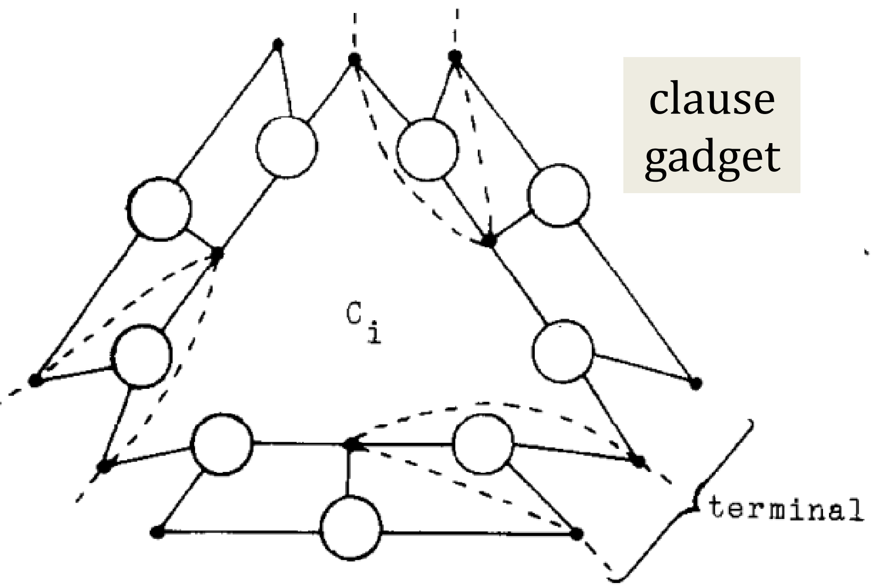
variable gadget



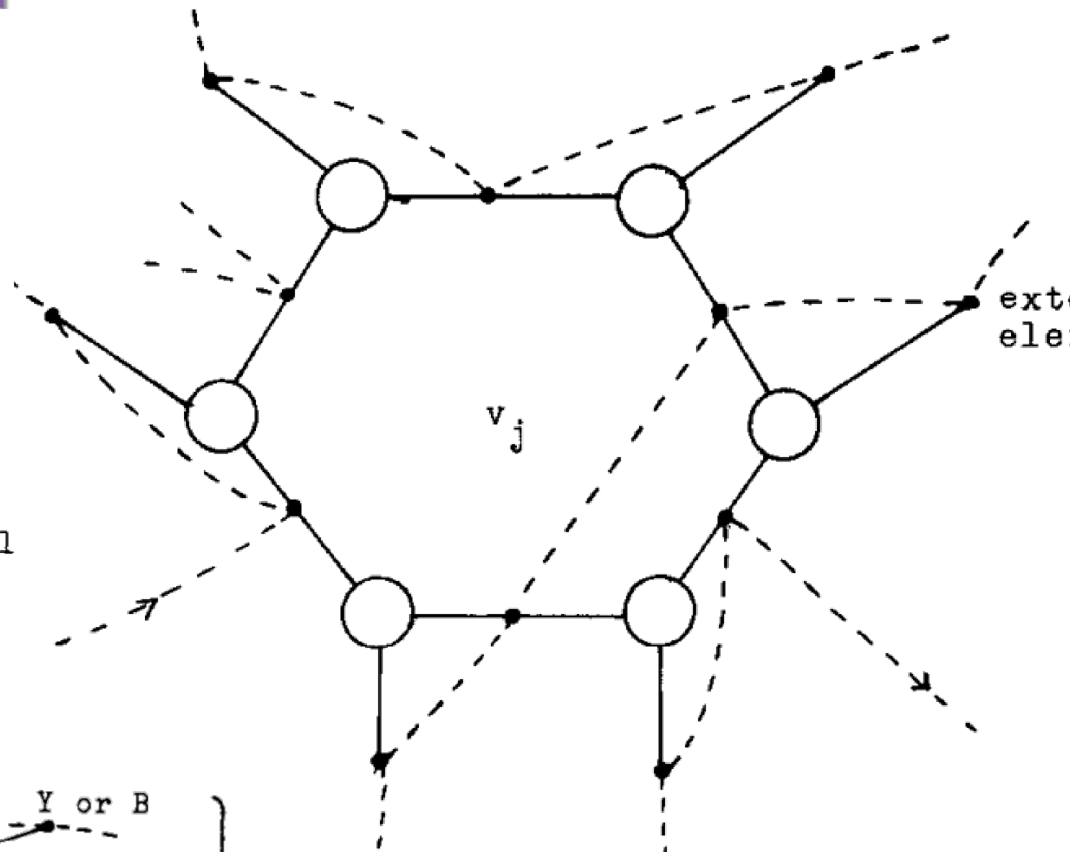


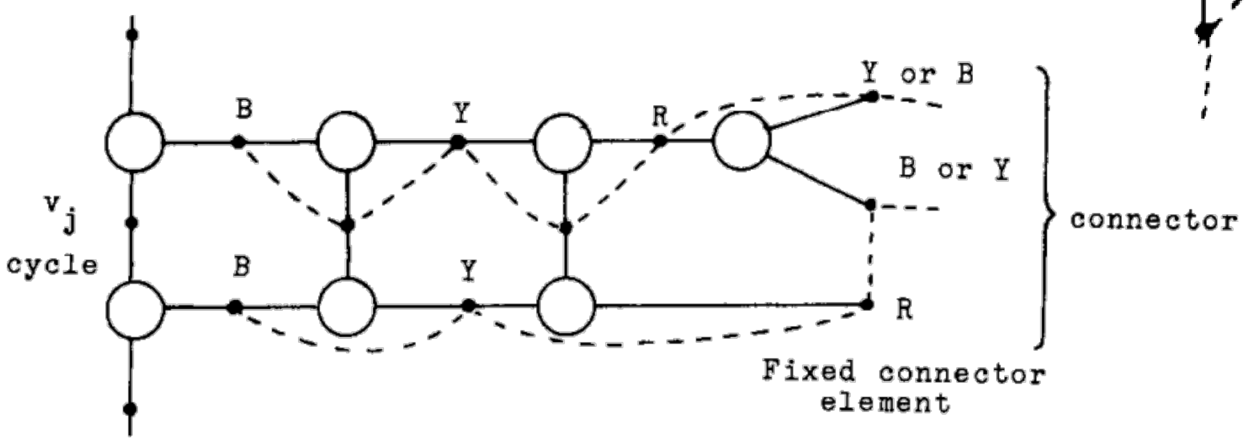
Planar 3DM with Element Cycle

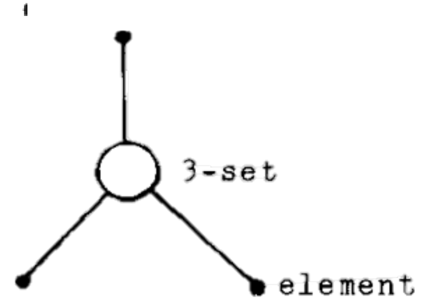
[Dyer & Freeze 1986]



clause gadget

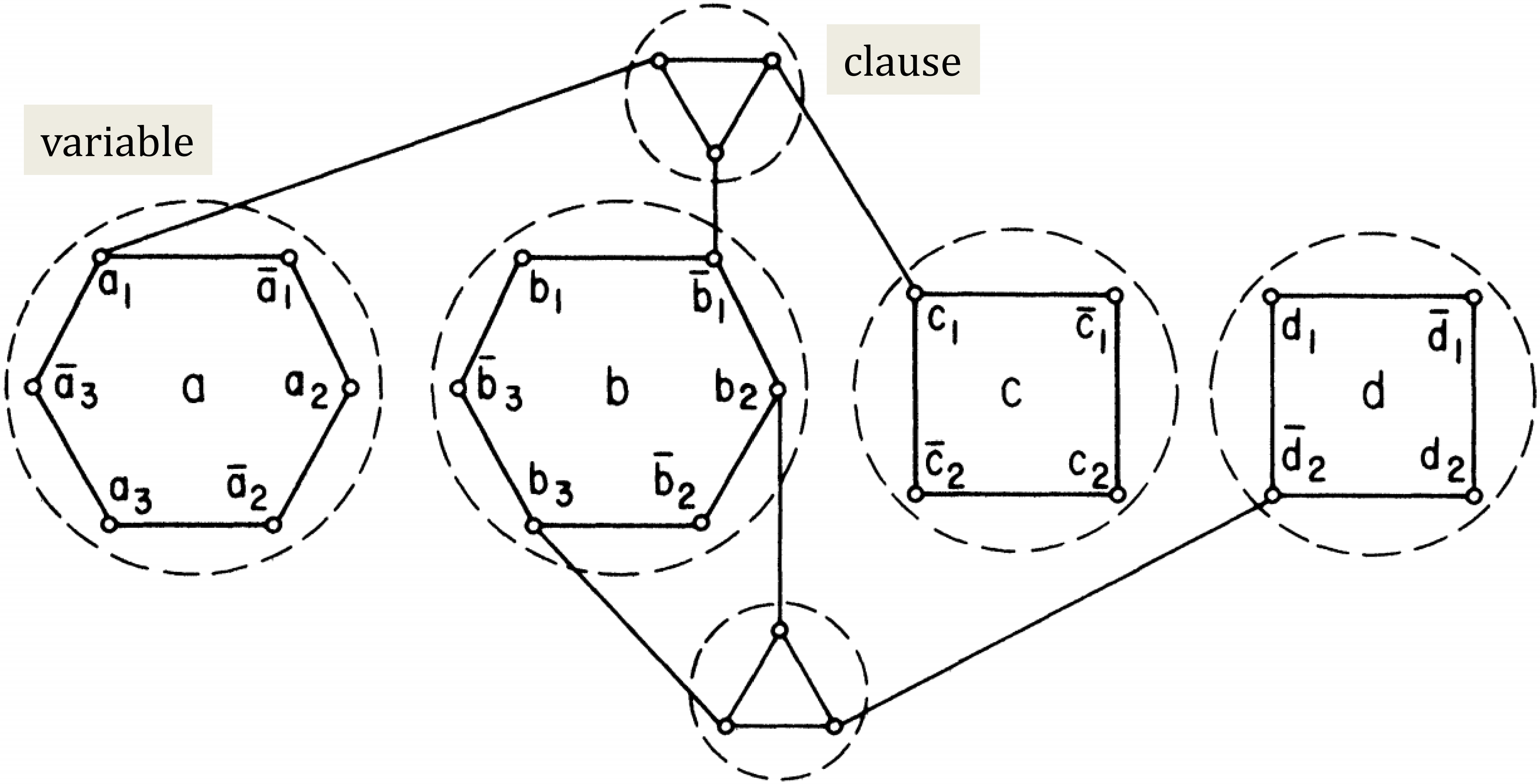






Planar Vertex Cover

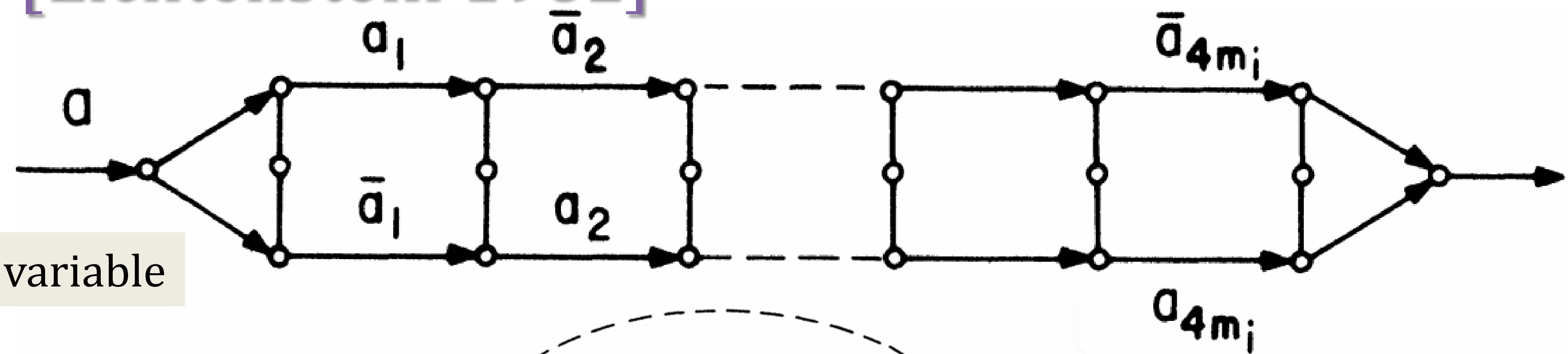
[Lichtenstein 1982]



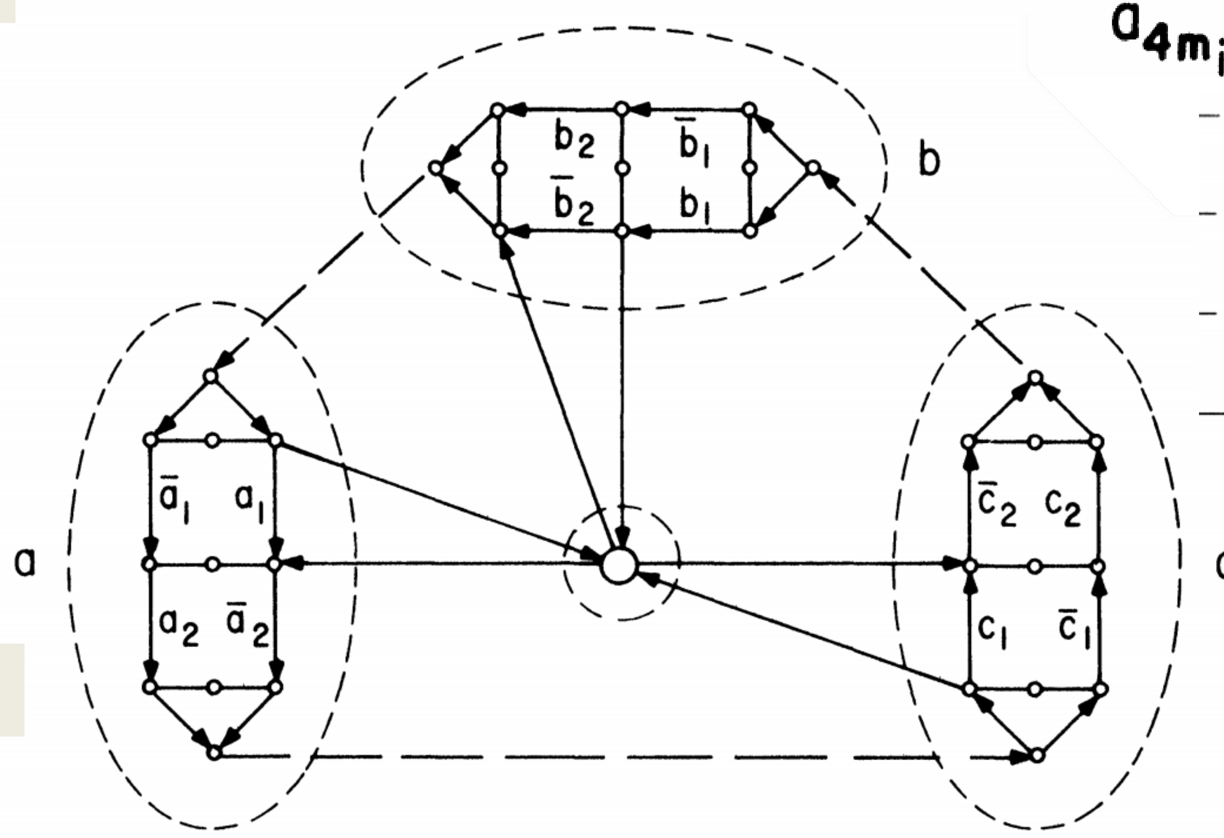
Example : $B = (a + \bar{b} + c)(b + b + \bar{d})$

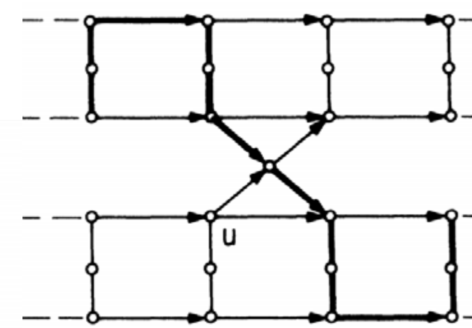
Planar (Directed) Hamiltonian Cycle

[Lichtenstein 1982]



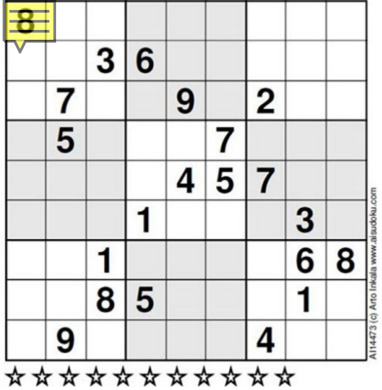
variable



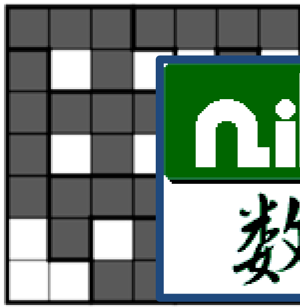


clause

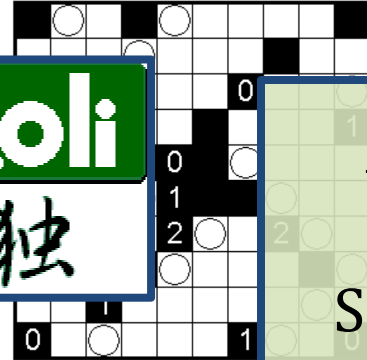
$a \vee \bar{b} \vee c$



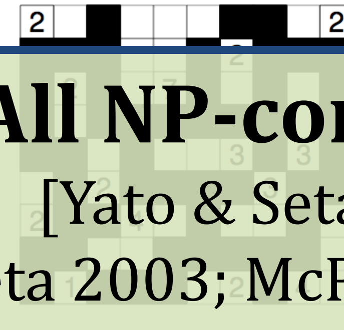
Sudoku



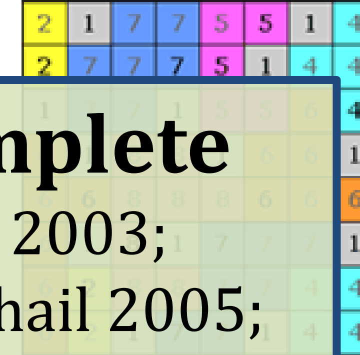
Lits



Light Up



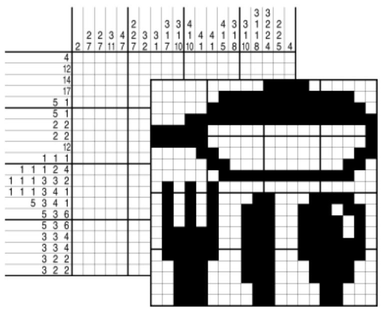
Nurikabe



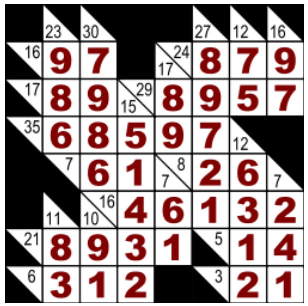
Filomino

All NP-complete

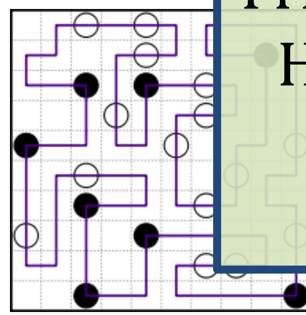
[Yato & Seta 2003;
Seta 2003; McPhail 2005;
Ueda & Nagao 1996;
Friedman 2002; Hearn 2008;
Holzer, Klein, Kutrib 2004;
Andersson 2007;
Holzer & Ruepp 2007]



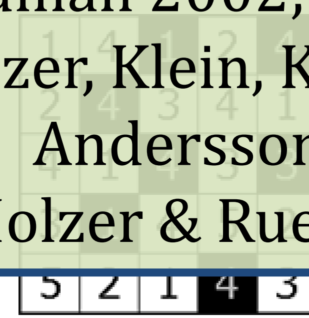
Nonogram
(Paint By Numbers)



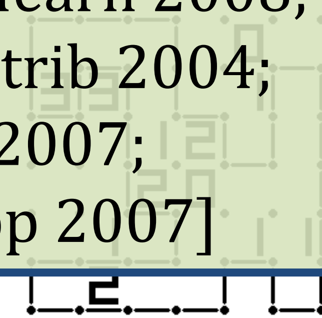
Kakuro
(Cross Sum)



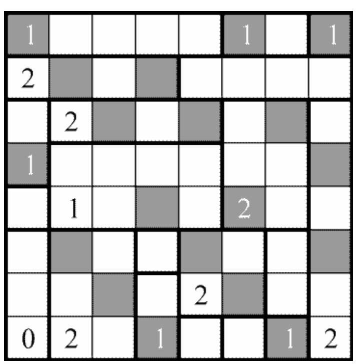
Masyu
(Pearl Puzzle)



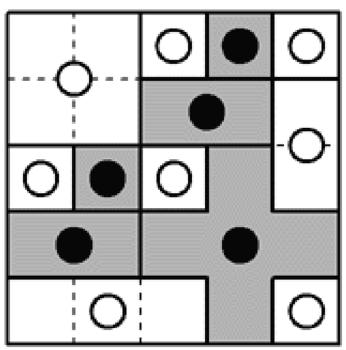
Hitori



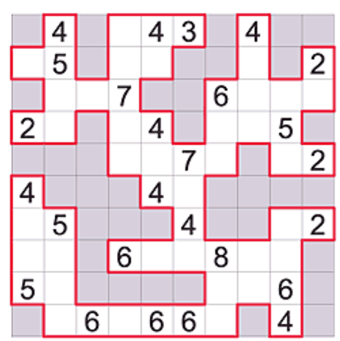
Slitherlink
(Fences)



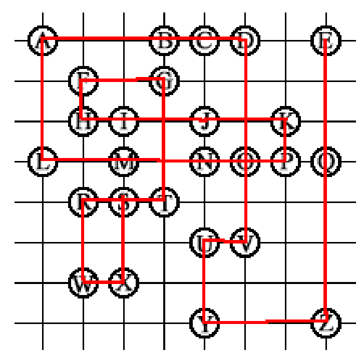
Heyawake



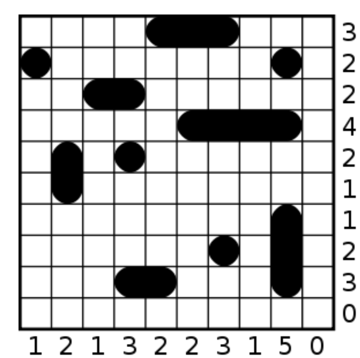
Tentai Show
(Spiral Galaxies)



Bag
(Corral Puzzle)

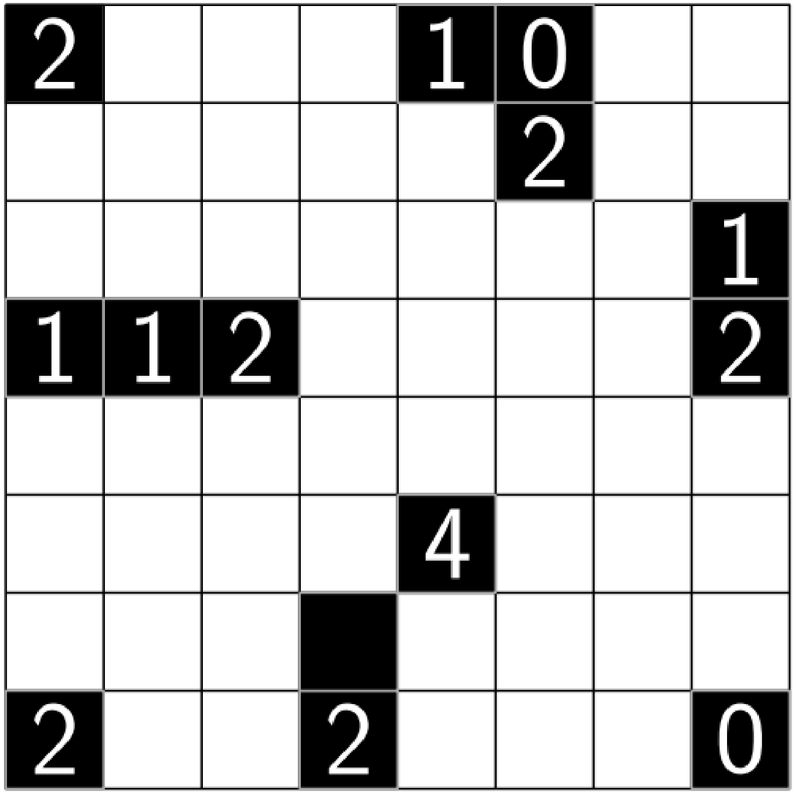


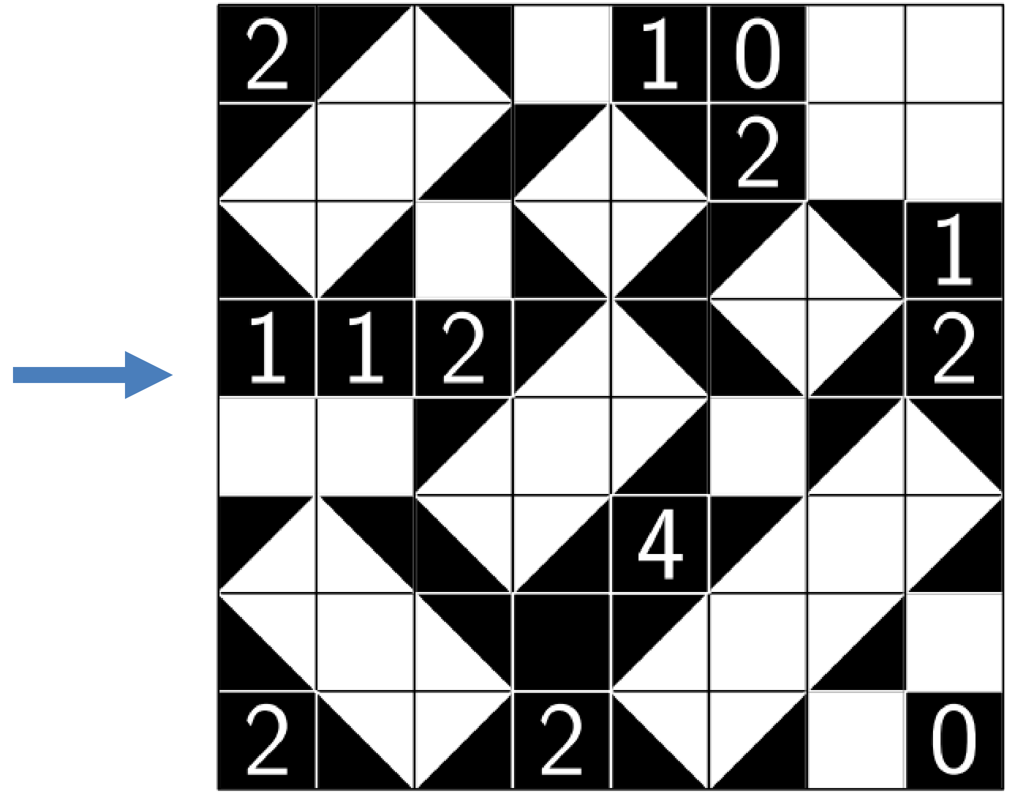
Hiroimono
(Goishi Hiroi)



Battleships

Shakashaka [Guten 2008; Nikoli 2012–]



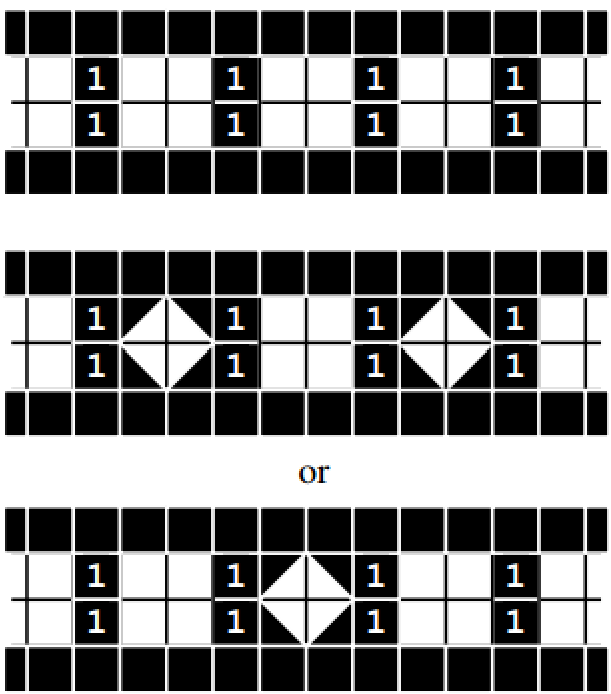


- White squares
- Black squares
  - Labeled 0, 1, 2, 3, 4, or nothing

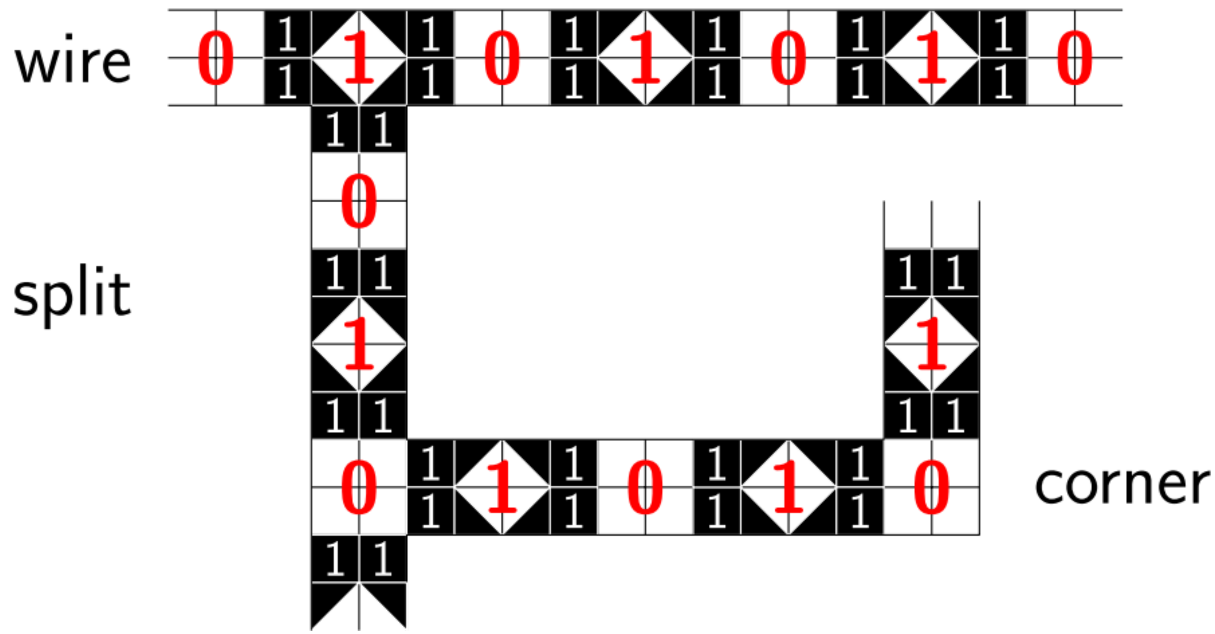
- Half-fill some white squares
- Labels specify number of half-filled neighbors
- Rectangular white regions

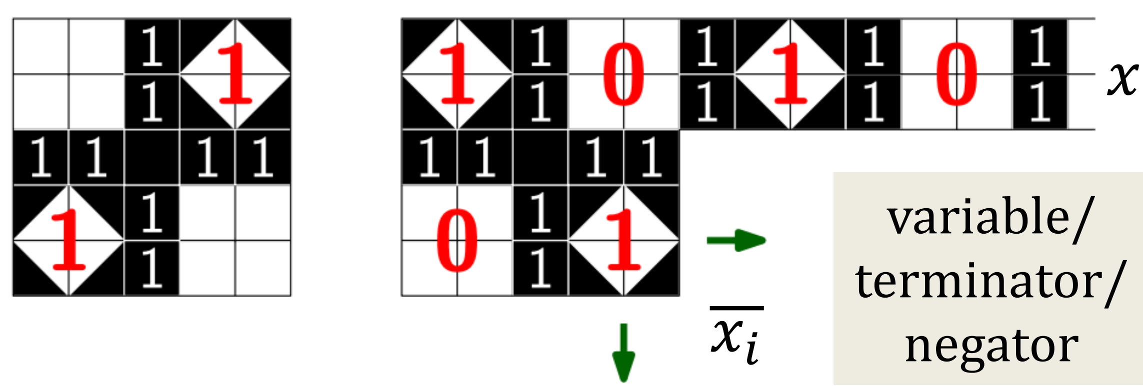
Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]



wire





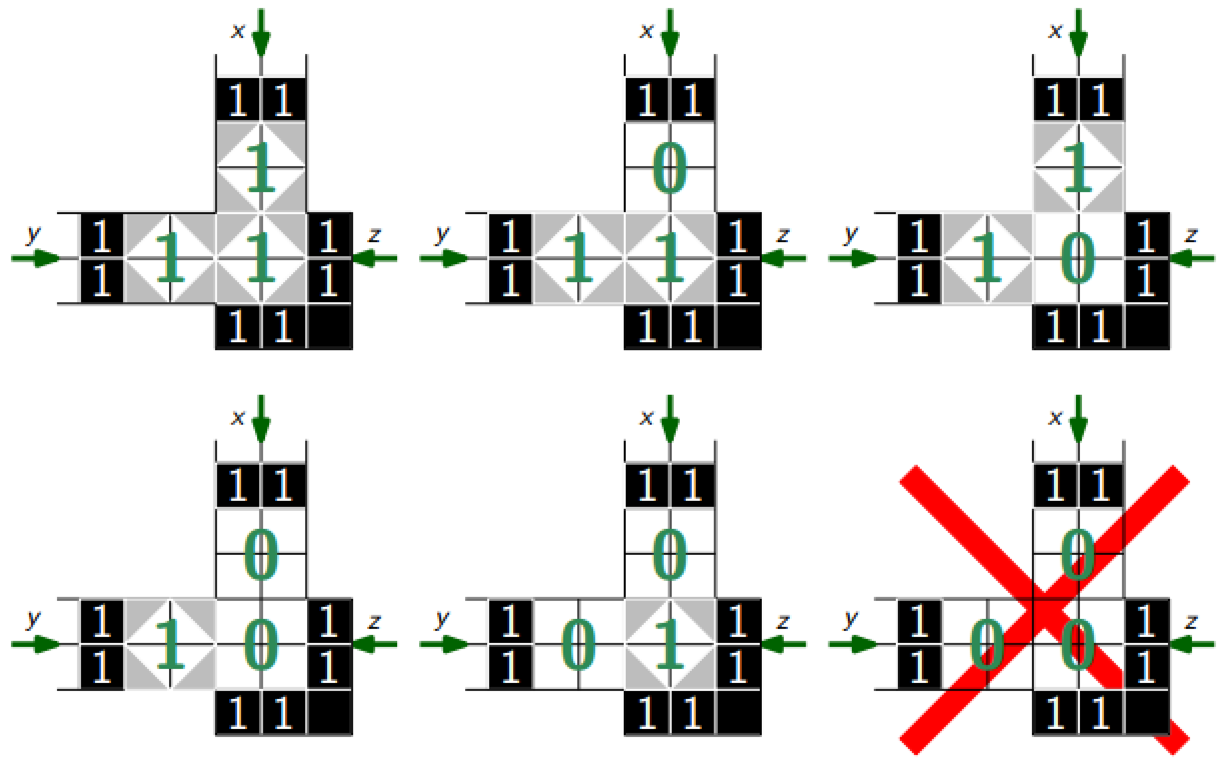
variable/
terminator/
negator

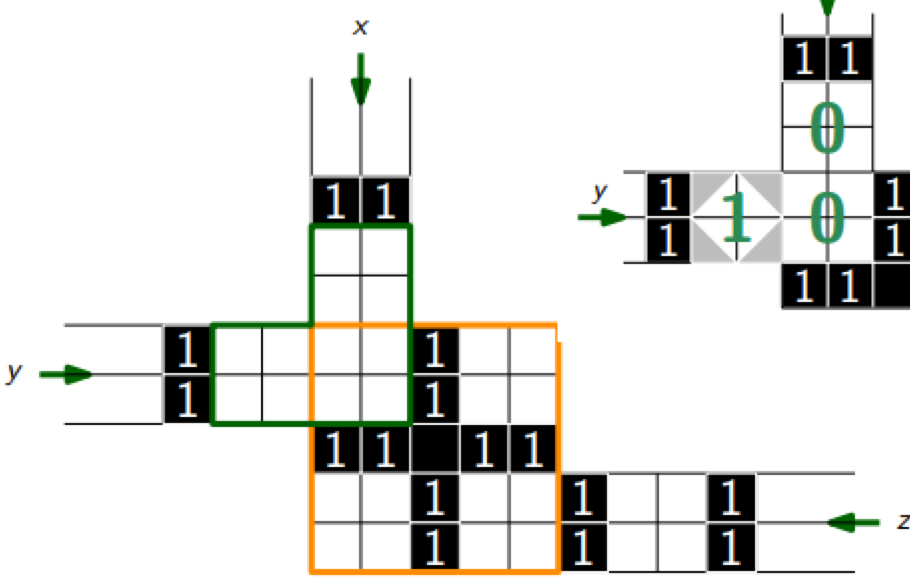
Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]

clause

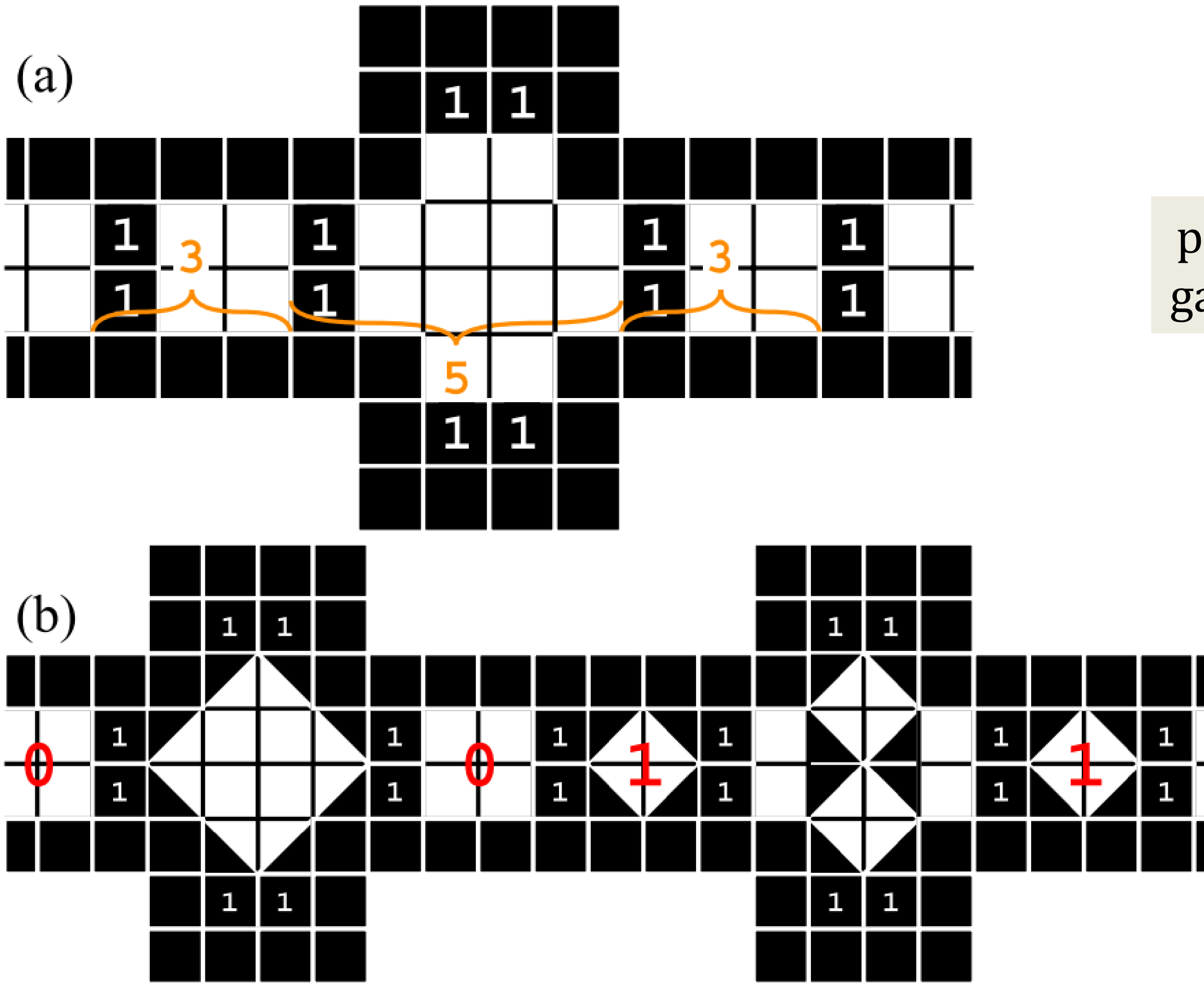
$x \lor y \lor z$





Shakashaka is NP-complete

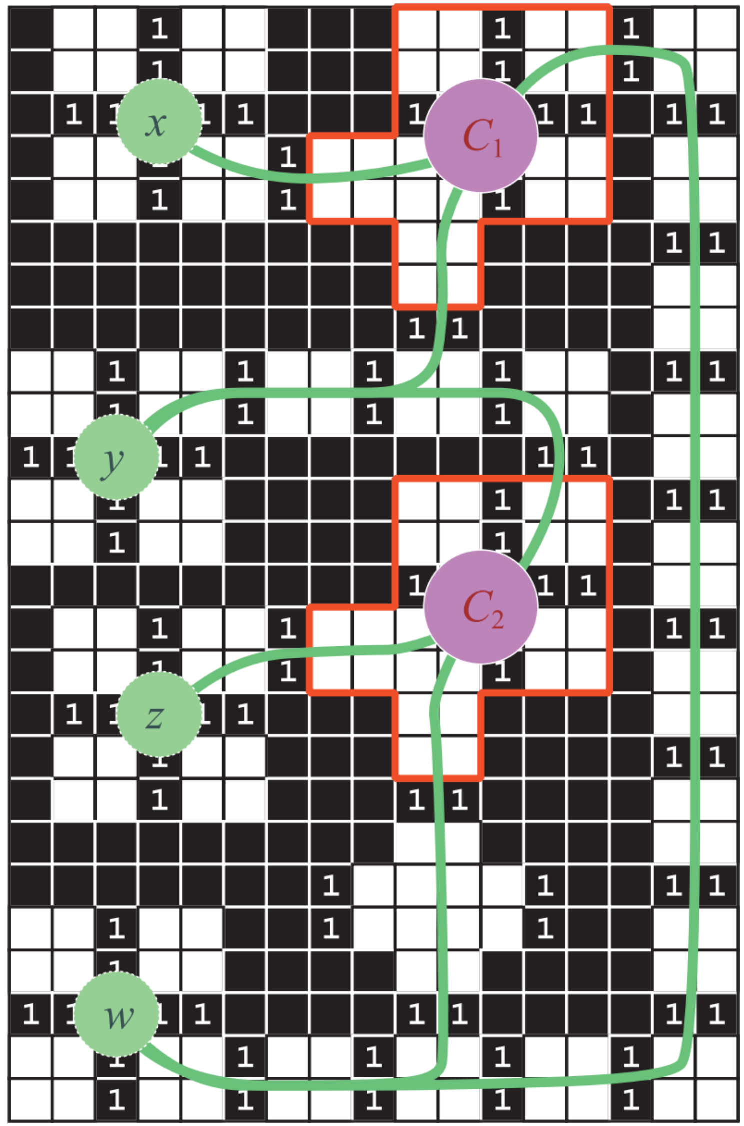
[Demaine, Okamoto, Uehara, Uno 2013]



parity gadget

Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]



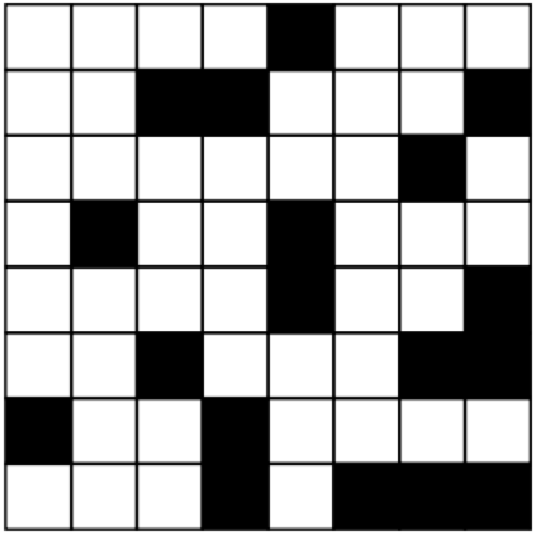
Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]

- Integer programming can solve small examples

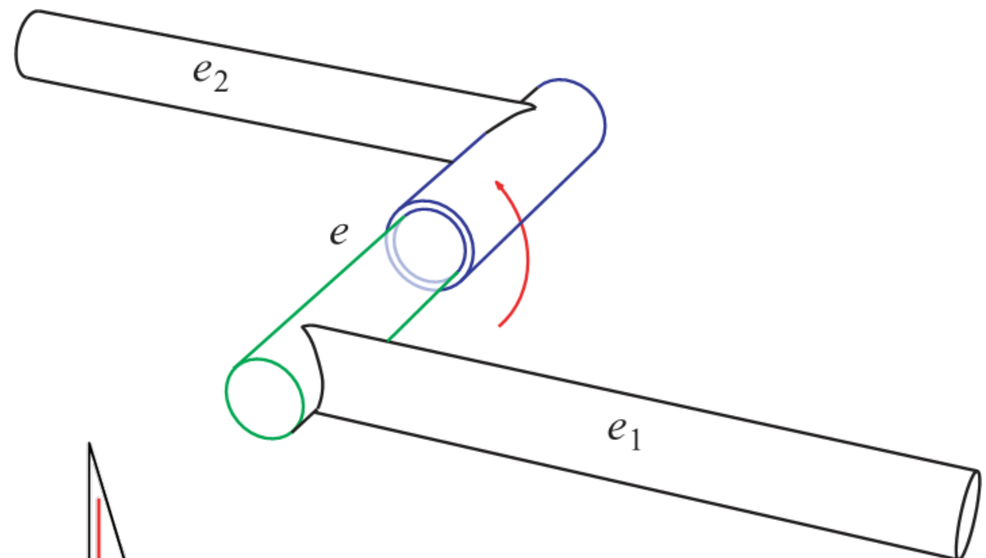
Problem	Size	Level	# of white squares	Time (sec)
1	10×10	Easy	76	0.02
2	10×10	Easy	77	0.03
3	10×10	Easy	82	0.03
4	10×18	Easy	131	0.07
5	10×18	Medium	156	0.09
6	10×18	Medium	144	0.07
7	14×24	Medium	297	0.21
8	14×24	Hard	295	0.19
9	20×36	Hard	645	0.84
10	20×36	Hard	632	0.91

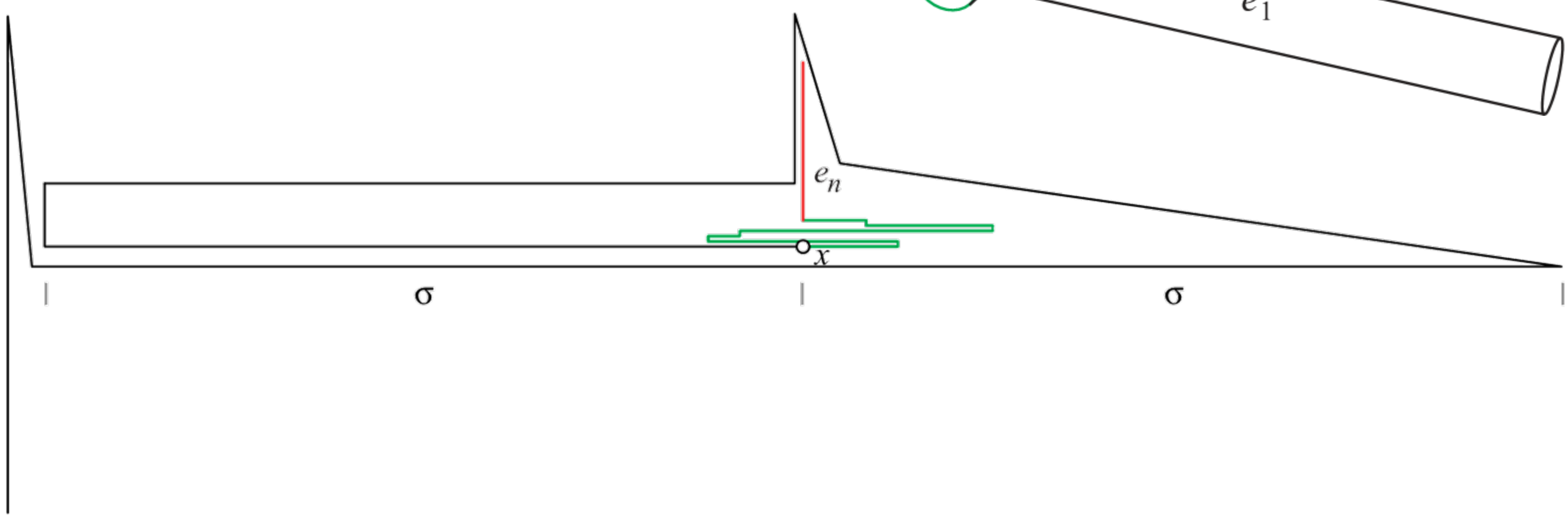
- Open: No labels



Flattening Fixed-Angle Chains

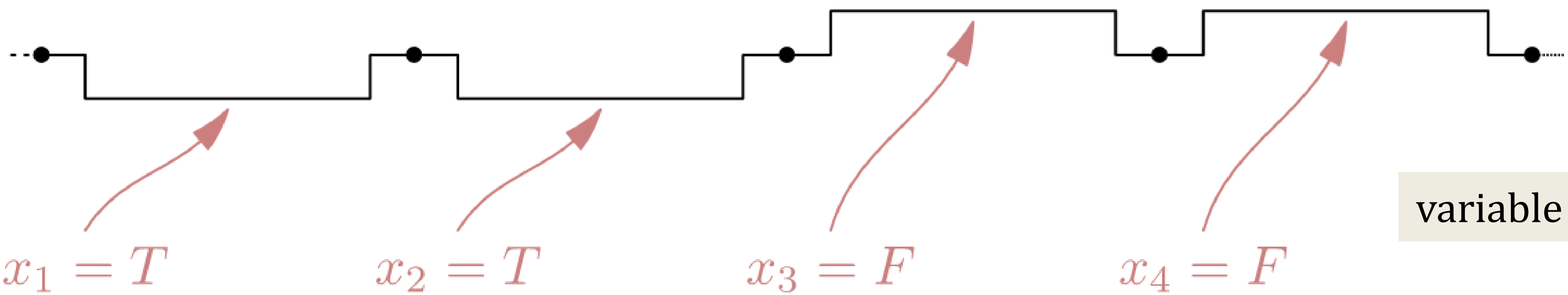
[Soss & Toussaint 2000]





Flattening Fixed-Angle Chains

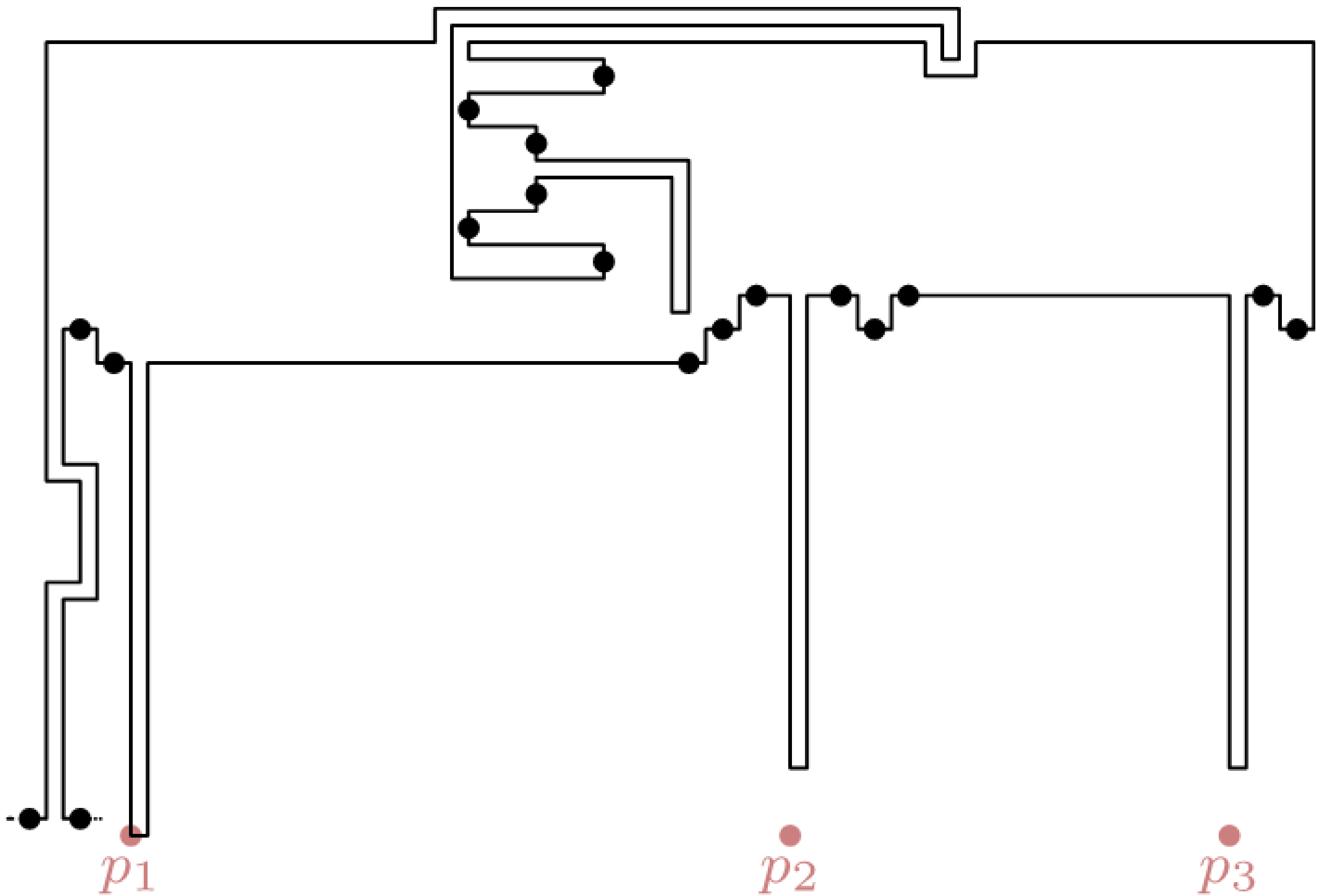
[Demaine & Eisenstat 2011]



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]

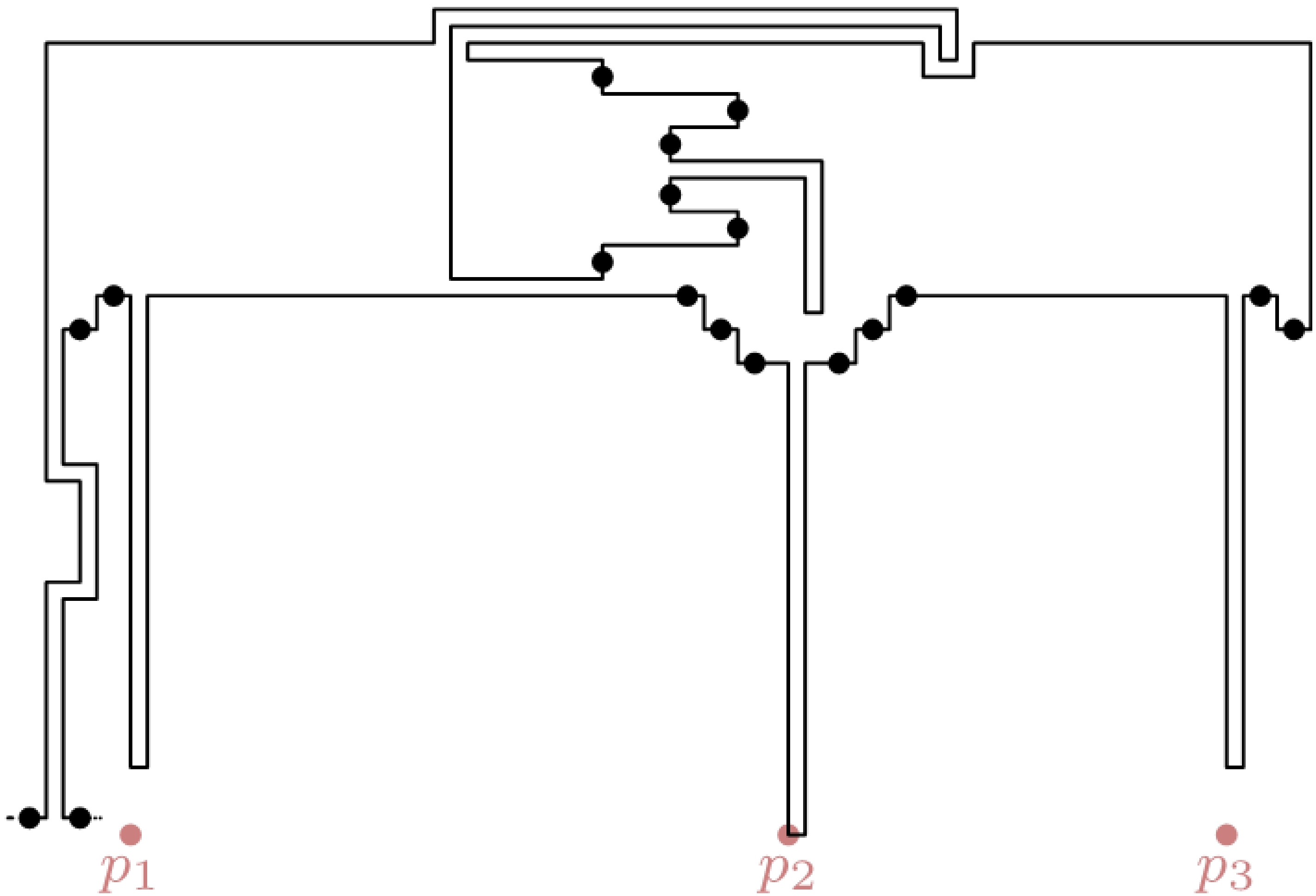
clause



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]

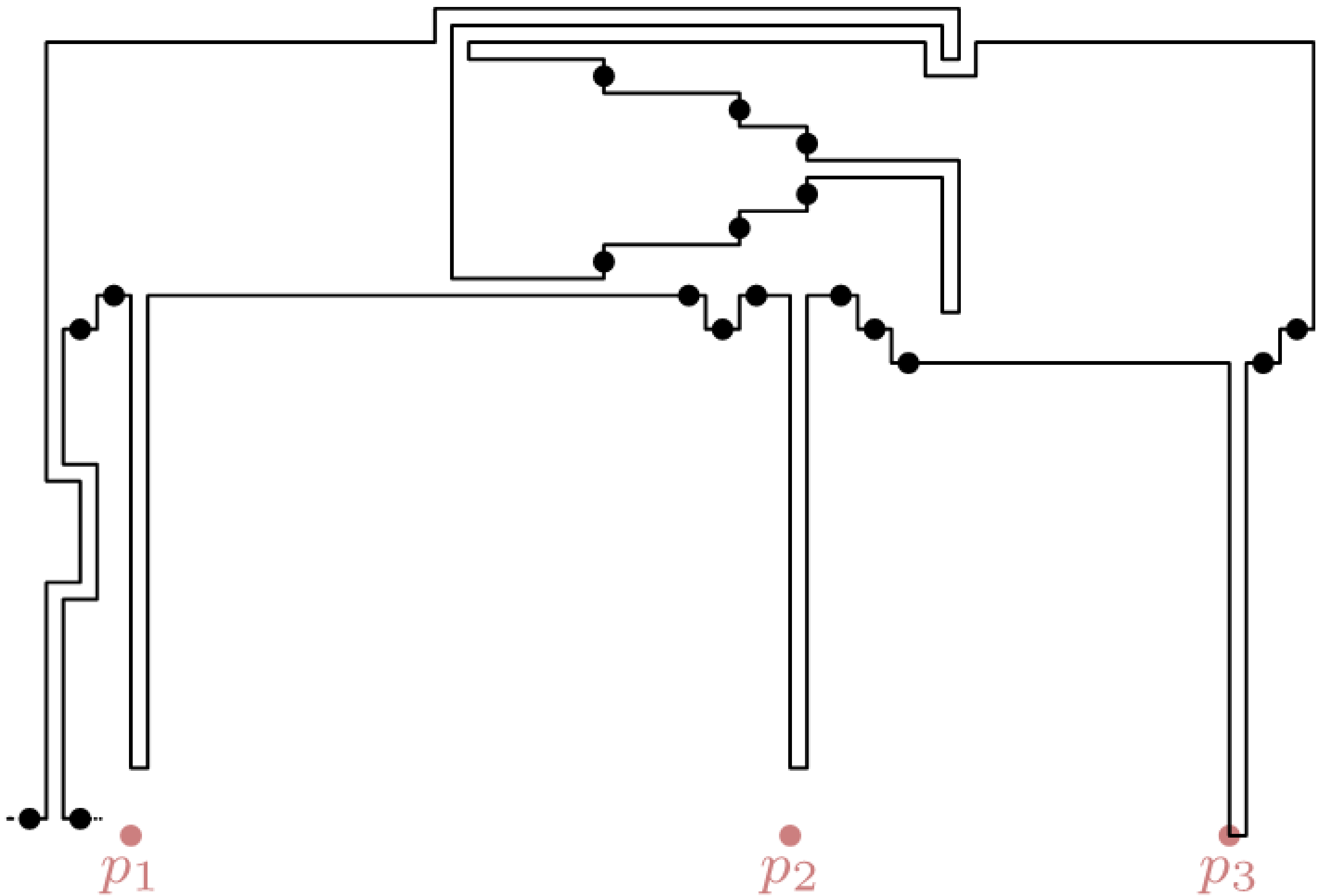
clause



Flat Folding of Fixed-Angle Chains

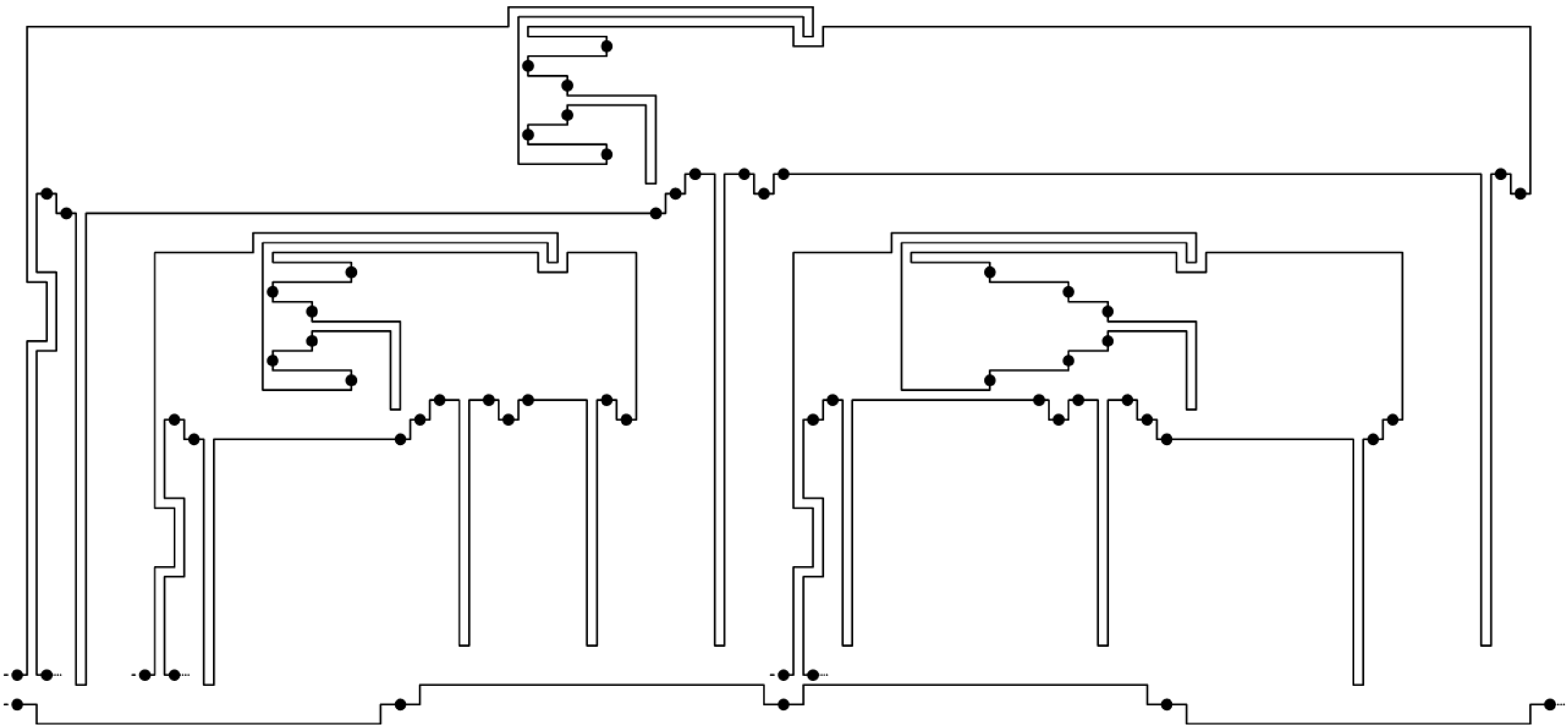
[Demaine & Eisenstat 2011]

clause



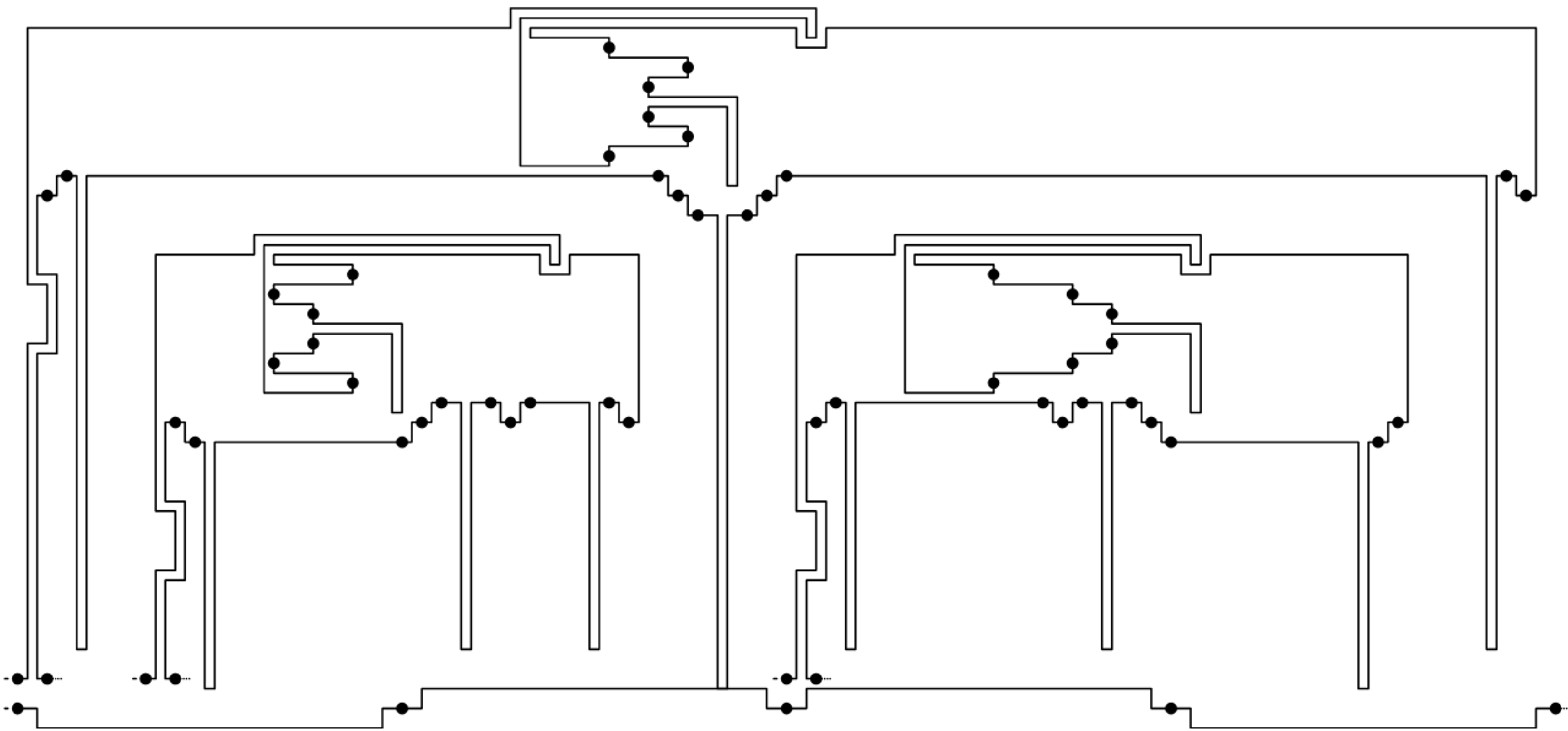
Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]



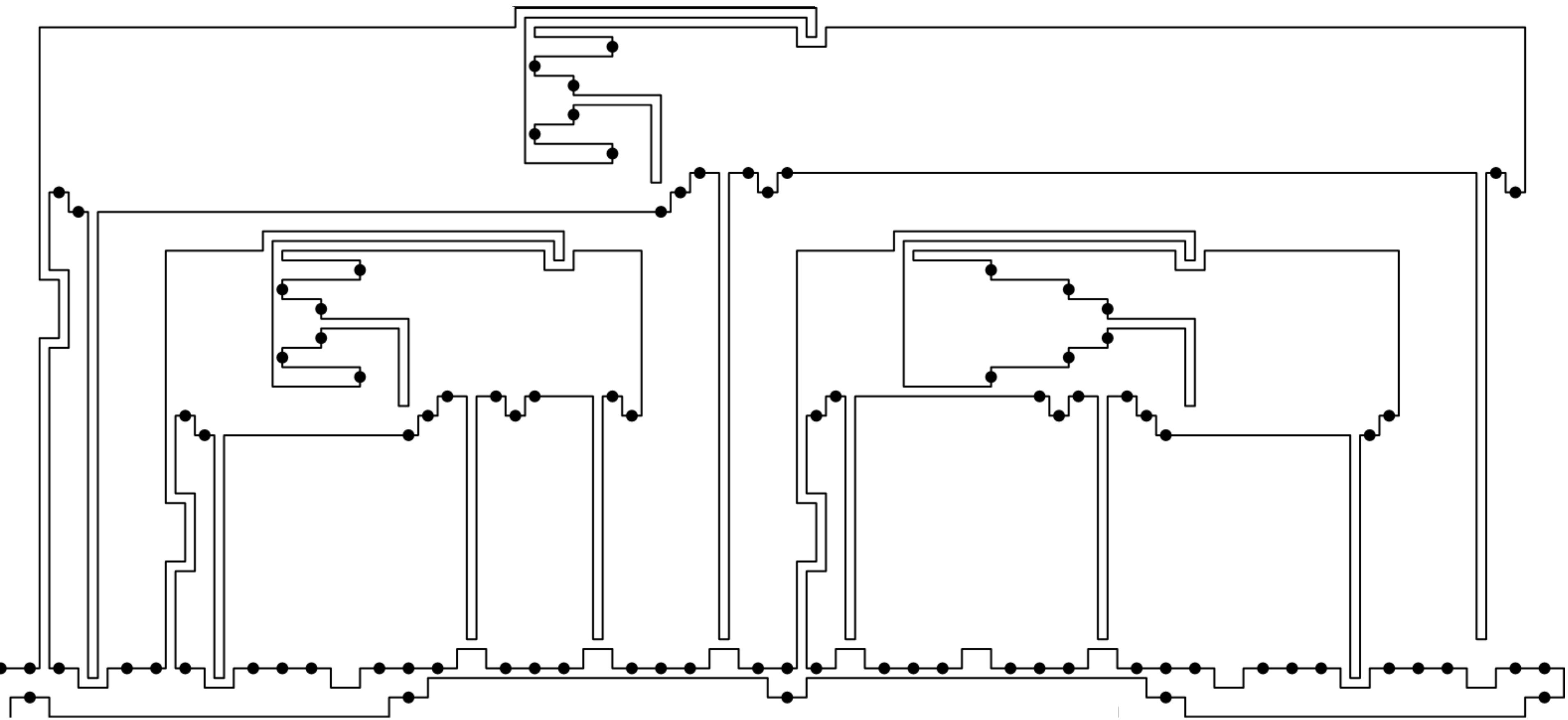
Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]