6.5440: Algorithmic Lower Bounds, Fall 2023

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Problem Set 8

Due: Monday, November 6, 2023 at noon

Problem 8.1 [Acyclic Geography].

In ***directed vertex geography*** (introduced in Lecture 13), players take turns moving a single token along the edges of a directed graph, without repeating previously visited vertices. The loser is the first player who is unable to move. In general directed graphs, determining the winner is PSPACE-complete.

Prove that directed vertex geography is P-hard when played on a ***directed acyclic graph***, that is, a graph with no directed cycles. (In fact, this problem is P-complete.) Remember that P-hardness reductions need to be NC algorithms. (Alternatively, it suffices to guarantee your reduction uses logarithmic space.)

You must include a drawing or diagram in your submission.