Prof. Erik Demaine, Josh Brunner, Lily Chung, Jenny Diomidova

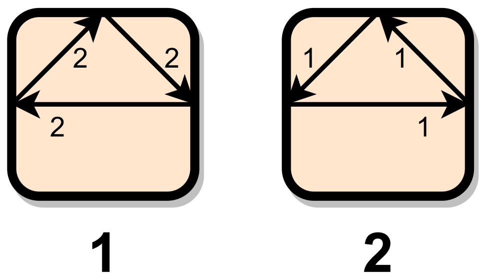
Problem Set 6 Solution

Due: Monday, October 23, 2023 at noon

Problem 6.1 [3-spinner hardness].

A ***k*-*spinner*** is a deterministic gadget with k locations and two states, 1 and 2. When the gadget is in state 1, the agent can enter at any location and exit at the *clockwise* next location, while switching the gadget to state 2. When the gadget is in state 2, the agent can enter at any location and exit at *counterclockwise* next location, while switching the gadget to state 1.

For example, here is the state diagram of a 3-spinner:



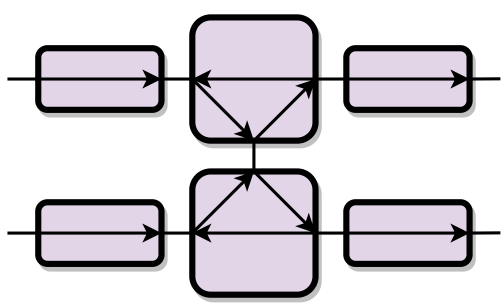
You can also see an example of a 4-spinner in action in this video: https://youtu.be/QjfiRNrAmIo?t=1097.

Prove PSPACE-completeness of reachability (one-player motion planning) with 3-spinners.

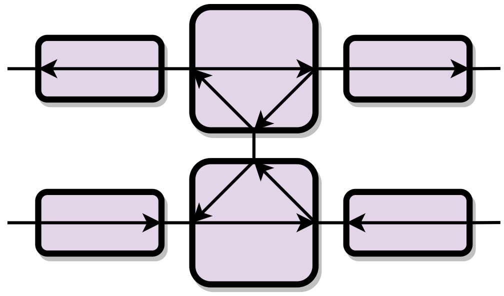
You must include a drawing or diagram in your submission.

Solution:

We reduce from motion planning with locking 2-toggles. First note that if we only use two locations of a 3-spinner it acts as a 1-toggle. Next connect a pair of 3-spinners with and surround them with 1-toggles like this:



In this state the agent can either traverse from bottom-left to top-right or from top-left to bottom-right. After traversing from top-left to bottom-right, we get the following:



In this state, the only possible traversal is from bottom-right to top-left, undoing the previous transition. Similarly, if the first traversal was bottom-left to top-right, the next traversal must undo it. Therefore this is a locking 2-toggle.

Fun fact: this problem is still PSPACE-hard even without branching hallways, and when the 3-spinners are placed on a hexagonal grid! However, this proof is a lot more complicated; see “Recognizing the Repeatable Configurations of Time-Reversible Generalized Langton’s Ant Is PSPACE-Hard” by Tatsuie Tsukiji and Takeo Hagiwara, *Algorithms* 4(1):1–15, 2011.