6.5440: Algorithmic Lower Bounds, Fall 2023

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Problem Set 5

Due: Tuesday, October 10, 2023 at noon

Problem 5.1 [$1 \times n$ edge-matching].

Recall that an (unsigned) ***edge-matching puzzle*** consists of mn tiles, where each tile is a unit square whose sides are each labeled with a color. The goal is to place the mn given tiles into an $m \times n$ rectangle such that tiles match in color on shared edges.

Give a reduction from Hamiltonicity in Directed 3-Regular Graphs to show that edge-matching puzzles are NP-hard even when the following two conditions hold simultaneously:

- $m = 1$, i.e., the puzzle is $1 \times n$; and
- tiles can be freely rotated, reflected, and translated.

By contrast, in Problem Set 2, you were asked to prove NP-hardness of $2 \times n$ edge matching ($m = 2$) when tiles *cannot* be rotated or flipped.

You must include a drawing or diagram in your submission.