6.5440: Algorithmic Lower Bounds, Fall 2023

Prof. Erik Demaine, Josh Brunner, Lily Chung, Jenny Diomidova

Problem Set 4

Due: Monday, October 2, 2023 at noon

Problem 4.1 [Planar Positive NAE EU3SAT].

Let ϕ be an instance of Not-All-Equal SAT satisfying the following properties:

Positive: Negations are not allowed; ϕ contains only positive literals.

EU3: Every clause contains exactly three distinct variables.

Planar: The following graph G is planar: the graph with a vertex for each variable and each clause, and edges connecting vertices to the clauses they appear in.

Prove that ϕ is always satisfiable. **You must include a drawing or diagram in your submission.**

Hint: Use the Four-Color Theorem, which states that the vertices of any planar graph can be colored with four colors so that adjacent vertices have different colors. You may need to modify graph G (e.g., by adding extra edges) before coloring it in order to make the coloring more useful.