6.5440: Algorithmic Lower Bounds, Fall 2023

Prof. Erik Demaine, Josh Brunner, Lily Chung, Jenny Diomidova

Problem Set 11

Due: Monday, November 27, 2023 at noon

Problem 11.1 [Parameterized Steiner Tree].

Let $G = (V, E)$ be an undirected graph, and let $T \subseteq V$ be a set of ***terminal vertices***. A ***Steiner subgraph*** on T is a connected subgraph $H = (V', E')$ of G such that $T \subseteq V'$, that is, H connects all of the terminals.

The k-Nonterminal Steiner Tree problem has k as a parameter and G, T as inputs. It asks whether there exists a Steiner subgraph $H = (V', E')$ on T such that $|V' \setminus T| \leq k$. In other words, the question is whether there exists a set of k or fewer nonterminal vertices N such that $T \cup N$ induces a connected subgraph of G.

Prove that this problem is W[2]-hard.

You must include a drawing or diagram in your submission.

Hint: You may find it helpful to reduce from the W[2]-complete problem k-Set Cover: given a set U of elements, a collection $S \subseteq 2^U$ of subsets of U, and a parameter k, is there is a subcollection $S' \subseteq S$ of size $|S'| \leq k$ such that every element of U is contained in some member of S'?